1)

a) Forcing frequency: $f = \frac{1}{\tau} = \frac{1}{\frac{L}{v}} = \frac{1}{\frac{5\ m}{70\times\frac{1000}{60\times 60}\ m/s}} = 3.889\ Hz$

Natural frequency: $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{95000\ N/m}{200\ kg}} = 3.469\ Hz$

Frequency Ratio: $\frac{f}{f_n} = \frac{3.889\ Hz}{3.469\ Hz} = 1.121$

Critical Damping Coefficient: $c_c = 2m\sqrt{\frac{k}{m}} = 2 \times 200\ kg \times \sqrt{\frac{95000\ N/m}{200\ kg}} = 8718\ Ns/m$

Damping Factor: $\delta = \frac{damping\ coefficient}{c_c} = \frac{2000\ Ns/m}{8718\ Ns/m} = 0.2294$

Damped Transmissibility: $T = \sqrt{\frac{1+4\left(\frac{f}{f_n}\right)^2\delta^2}{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+4\left(\frac{f}{f_n}\right)^2\delta^2}} = \sqrt{\frac{1+4\times1.121^2\times0.2294^2}{(1-1.121^2)^2+4\times1.121^2\times0.2294^2}} = 1.956$

$$T = \frac{resultant\ amplitude}{forcing\ amplitude} = \frac{8\ mm}{forcing\ amplitude} = 1.956$$

∴ Forcing Amplitude: $\frac{8\ mm}{1.956} = 4.090\ mm$

b) Forcing angular velocity: $\omega = 2\pi f = 2\pi \times 30\ Hz = 188.5\ rad/s$

Maximum acceleration: $a_{max} = maximum\ displacement \times \omega^2$

$$= 0.004\ m \times (188.5\ rad/s)^2 = 142.1\ m/s^2$$

Maximum force: $F_{max} = ma_{max} = 20\ kg \times 142.1\ m/s^2 = 2842\ N = 2.842\ kN$

c) Isolation: $0.94 = 1 - T$

∴ Transmissibility: $T = 0.06$

Using transmissibility graph, when $\xi = 0.1$, $\frac{f}{f_n} = 5$ (less accurate, but faster if no time).

Or otherwise, use $T = \sqrt{\frac{1+4\left(\frac{f}{f_n}\right)^2\delta^2}{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+4\left(\frac{f}{f_n}\right)^2\xi^2}}$ formula. Substitute $T = 0.06$ and $\xi = 0.1$ results in $\frac{f}{f_n} = 4.669$ (more accurate, if you have time).

$$\frac{f}{f_n} = \frac{30\ Hz}{f_n} = 4.669$$

∴ Natural frequency of isolator – mass system: $f_n = \frac{30\ Hz}{4.669} = 6.425\ Hz$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{total}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_{total}}{20\ kg}} = 6.425\ Hz$$

∴ Total spring constant of 4 springs/isolators (in parallel):

$$k_{total} = 20\ kg \times (2\pi \times 6.425\ Hz)^2 = 32590\ N/m$$

Stiffness/spring constant of 1 spring/isolator: $k = \frac{32590\ N/m}{4} = 8148\ N/m$

(For this part of this question, I'm unsure. Confirm with Prof.)

Transmissibility: $T = \frac{resultant\ max\ displacement}{forcing\ max\ displacement} = \frac{d_{max}}{4\ mm} = 0.06$

∴ Resultant maximum displacement: $d_{max} = 0.06 \times 4\ mm = 0.24\ mm$

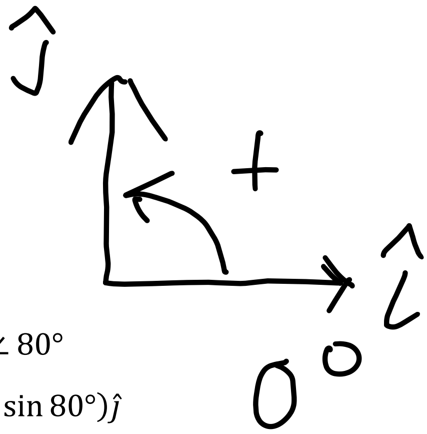
Mass/specimen will vibrate at natural frequency, NOT forcing frequency.

Maximum force on mass: $F_{max} = ma_{max} = md_{max}\omega_n^2 = md_{max} \times (2\pi f_n)^2$

$$= 20\ kg \times 0.00024\ m \times (2\pi \times 6.425\ Hz)^2 = 7.82\ N$$

2)

a) Take positive as counter-clockwise from phase mark, assume pointing left. Let $\hat{\imath}$ be horizontal direction and $\hat{\jmath}$ be vertical direction.



Remember to set calculator to degrees mode.

$$\vec{V}_U = 0.3\ mm \angle 80°$$

$$\vec{W} = 50\ g \angle 0°$$

$$\vec{V}_{U+W} = 0.2\ mm \angle 40°$$

$$\vec{V}_W = \vec{V}_{U+W} - \vec{V}_U = 0.2\ mm \angle 40° - 0.3\ mm \angle 80°$$

$$= (0.2\cos 40° - 0.3\cos 80°)\hat{\imath} + (0.2\sin 40° - 0.3\sin 80°)\hat{\jmath}$$

$$= 0.1011\hat{\imath} - 0.1669\hat{\jmath} = 0.1951\ mm \angle 58.79°$$

$$\vec{B} = -\vec{U} = -\frac{\vec{V}_U}{\vec{V}_W}\vec{W} = -\frac{0.3\ mm \angle 80°}{0.1951\ mm \angle 58.79°} \times 50\ g \angle 0°$$

$$= -\frac{0.3\ mm}{0.1951\ mm} \times 50\ g \angle(80° - 58.79° + 0°) = -76.88\ g \angle 21.21°$$

$$= 76.88\ g \angle (21.21 + 180)° = 76.88\ g \angle 201.2°$$

b) Must solve for 6 different parts. Too much work for 12 marks, I'll do other questions first.

$m_p = 0\ kg\ (assume\ counterbalanced\ crank)\ , m_c = 1\ kg\ , l = 0.4\ m\ , r = 0.1\ m$

$$\omega = 3000 \times \frac{2\pi}{60}\ rad/s = 314.2\ rad/s$$

$\alpha_1 = 120°\ , \alpha_2 = 0°\ , \alpha_3 = 240°$, remember to set calculator to degrees mode.

$s_1 = -0.5\ m\ , s_2 = 0\ m\ , s_3 = 0.5\ m$ (Remember: Origin is from middle crank!)

i) Primary x force.

$$F_{xp} = \sum_{i=1}^{3} (m_p + m_c)_i r\omega^2 \cos(\omega t + \alpha_i)$$

$$(m_p + m_c)_i r\omega^2 = constant$$

Focus: $\sum_{i=1}^{3} \cos(\omega t + \alpha_i) = 0$ (Test this out by typing $\cos(x + 120°) + \cos x + \cos(\omega t + 240°)$ into your calculator. If it = 0 for any random value of x, it = 0.)

$$\therefore F_{xp} = 0\ N$$

ii) Secondary x force. (Easy: $m_p = 0\ kg$)

$$F_{xs} = \sum_{i=1}^{3} (m_p)_i \frac{r^2\omega^2}{l} \cos(2\omega t + 2\alpha_i) = 0\ N$$

iii) y force.

$$F_y = \sum_{i=1}^{3} -(m_c)_i r\omega^2 \sin(\omega t + \alpha_i)$$

$$-(m_c)_i r\omega^2 = constant$$

Focus: $\sum_{i=1}^{6} \sin(\omega t + \alpha_i) = 0$ (Do calculator test again.)

$$\therefore F_y = 0\ N$$

iv) Primary x moment. (Remember: $s_1 = 0\ m$)

$$(m_p + m_c)_i r\omega^2 = (0\ kg + 1\ kg) \times 0.1\ m \times (3.142\ rad/s)^2 = 0.9872$$

$$M_{xp} = \sum_{i=1}^{3} (m_p + m_c)_i r\omega^2 s_i \cos(\omega t + \alpha_i)$$

$$= 0.9872[-0.5\cos(\omega t + 120°) + 0.5\cos(\omega t + 240°)]$$

$$= 0.4936[\cos(\omega t + 240°) - \cos(\omega t + 120°)]$$

$$= 0.4936[(\cos\omega t \cos 240° - \sin\omega t \sin 240°) - (\cos\omega t \cos 120° - \sin\omega t \sin 120°)]$$

$$= 0.4936\left[\left(-0.5\cos\omega t + \frac{\sqrt{3}}{2}\sin\omega t\right) - \left(-0.5\cos\omega t - \frac{\sqrt{3}}{2}\sin\omega t\right)\right]$$

$$= 0.4936\left[-0.5\cos\omega t + \frac{\sqrt{3}}{2}\sin\omega t + 0.5\cos\omega t + \frac{\sqrt{3}}{2}\sin\omega t\right]$$

$$= 0.4936\sqrt{3}\sin\omega t\ \ Nm$$

v) Secondary x moment. (Easy: $m_p = 0\ kg$)

$$M_{xs} = \sum_{i=1}^{3} (m_p)_i \frac{r^2\omega^2}{l} s_i \cos(2\omega t + 2\alpha_i) = 0\ Nm$$

vi) y moment.

$$-(m_c)_i r\omega^2 = -1\ kg \times 0.1\ m \times \left(314.2\frac{rad}{s}\right)^2 = -0.9872$$

$$M_y = \sum_{i=1}^{3} -(m_c)_i r\omega^2 s_i \sin(\omega t + \alpha_i)$$

$$= -0.9872[-0.5\sin(\omega t + 120°) + 0.5\sin(\omega t + 240°)]$$

$$= 0.4936[\sin(\omega t + 120°) - \sin(\omega t + 240°)]$$

$$= 0.4936[(\sin\omega t \cos 120° + \cos\omega t \sin 120°) - (\sin\omega t \cos 240° + \cos\omega t \sin 240°)]$$

$$= 0.4936\left[\left(-0.5\sin\omega t + \frac{\sqrt{3}}{2}\cos\omega t\right) - \left(-0.5\sin\omega t - \frac{\sqrt{3}}{2}\cos\omega t\right)\right]$$

$$= 0.4936\left[-0.5\sin\omega t + \frac{\sqrt{3}}{2}\cos\omega t + 0.5\sin\omega t + \frac{\sqrt{3}}{2}\cos\omega t\right]$$

$$= 0.4936\sqrt{3}\cos\omega t\ \ Nm$$

3) (Not fully sure about sound/noise chapters; doing this 2.5 months after I last did this paper. Ask Prof for best answers.)

a) This looks like a silencer. Refer to silencer formulas.

$$kl = \frac{\omega l}{c} = \frac{2\pi f l}{c} = \frac{2\pi \times 0.425\ m}{340\ m/s} f = 0.007854f$$

Remember to set calculator to radians mode.

Transmission loss: $TL = 10\lg\left[1 + \frac{\left(\frac{A_1}{A_2} - \frac{A_2}{A_1}\right)^2}{4}\sin^2 kl\right]$

$$= 10\lg\left\{1 + \frac{\left[\left(\frac{30\ cm}{70\ cm}\right)^2 - \left(\frac{70\ cm}{30\ cm}\right)^2\right]^2}{4}\sin^2 0.007854f\right\}\ dB$$

Final decibel after silencer: $L_{P,final} = L_{P,initial} - TL = (100 - TL)\ dB$

Final Sound Pressure Level: $SPL_{final} = 20 \times 10^{-6} \times 10^{\frac{L_{P,final}}{20}}\ Pa$

f	$400\ Hz$	$600\ Hz$
kl	3.142	4.712
TL	$1.622 \times 10^{-9}\ dB$	$8.987\ dB$
$L_{P,final}$	$\approx 100\ dB$	$91.01\ dB$
SPL_{final}	$2\ Pa$	$0.7104\ Pa$

b) $SPL_{total} = \sqrt{SPL_{final,400}^2 + SPL_{final,600}^2} = \sqrt{(2\ Pa)^2 + (0.7104\ Pa)^2} = 2.122\ Pa$

c) To reduce sound pressure, we must increase TL. To maximise TL, we can maximise $\sin^2 kl$ to be 1. Using Math logic, $kl = \pi\left(n - \frac{1}{2}\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$, where n is a positive integer (1, 2, 3, 4, …).

$$kl = \frac{2\pi f l}{c} = \pi\left(n - \frac{1}{2}\right)$$

Condition 1: Length of silencer: $l = \frac{c}{2f}\left(n - \frac{1}{2}\right) = \frac{340\ m/s}{2f}\left(n - \frac{1}{2}\right)\ m$

ALSO, we don't want $\sin^2 kl = 0$, which means $kl = \pi n = \pi, 2\pi, 3\pi$ ...

$$kl = \frac{2\pi f l}{c} \neq \pi n$$

Condition 2: Length of silencer: $l \neq \frac{c}{2f} n = \frac{340\ m/s}{2f} n\ m$

∴ Find a length for the silencer that fits both conditions for both $f = 400\ Hz\ and\ 600\ Hz$. Then suggest changing the silencer length to this length.

Table of possible values for l, in m.

	GOAL		AVOID	
	$\frac{340}{2f}\left(n-\frac{1}{2}\right)$		$\frac{340}{2f}n$	
n	$400\ Hz$	$600\ Hz$	$400\ Hz$	$600\ Hz$
1	$0.2125\ m$	$0.1417\ m$	$0.425\ m$	$0.2833\ m$
2	$0.6375\ m$	$0.425\ m$	$0.85\ m$	$0.5667\ m$

Coincidentally, the question uses $l = 0.425\ m$, highlighted in red! This is why the TL in part 3a is very low for $400\ Hz$, but high for $600\ Hz$. For $400\ Hz$, $\sin^2 kl = 0$ but for $600\ Hz$, $\sin^2 kl = 1$. To improve this, suggest any "Goal" value for l that isn't also in "Avoid". 0.1417 m will do fine.

Alternatively: We can also propose to change the areas too; for example, keeping $D_1 = 30\ cm$ but increasing $D_2 = 100\ cm$. (It makes more sense to change the silencer area instead of the original pipe area.) This should increase the TL. Show calculations by repeating the steps for 3a to get the new TL. (However, your $\sin^2 kl$ will forever remain at 0 for $400\ Hz$, ridiculously low, even though it is at 1 for $600\ Hz$. Not realising this fact may not be as impressive to the prof compared to other students who can.)

Still, this is a ridiculously long-winded explanation for 5 measly marks. When in doubt, skip.

4) (My answer doesn't feel long enough for 12 marks and 8 marks in 4a and 4b, yet too long for just 5 marks in 4c. Best is just ask Prof.)
   a) Room constant: $R = \frac{Area \times \alpha_{avg}}{1-\alpha_{avg}} = \frac{1800\ m^2 \times 0.1}{1-0.1} = 200$

Directional factor: $Q = 2$ (hemisphere)

Sound pressure level: $L_P = L_W + 10\lg\left(\frac{Q}{4\pi r^2} + \frac{4}{R}\right) = L_W + 10\lg\left[\frac{2}{4\pi \times (1\ m)^2} + \frac{4}{200}\right] = 90\ dB$

∴ Sound power level of machine source: $L_W = 90\ dB - 10\lg\left[\frac{2}{4\pi \times (1\ m)^2} + \frac{4}{200}\right] = 97.47\ dB$

   b) Sound pressure level due to reverberation/reflection:

$$L_P = L_W + 10\lg\frac{4}{R} = 97.47\ dB + 10\lg\frac{4}{200} = 80.48\ dB$$

c) The 80.48 dB calculated in 4b is only the sound pressure level due to reverberation from the walls and the floor. The 90 dB measured in the microphones also receive sound directly from the machine. After adding both direct and reverb sounds, the total sound pressure level should be 90 dB.

Sound pressure level due to direct sound:

$$L_P = L_W + 10\lg\left(\frac{Q}{4\pi r^2}\right) = 97.47\ dB + 10\lg\frac{2}{4\pi \times (1\ m)^2} = 89.49\ dB$$

Sound pressure level (in Pa): $SPL = 20 \times 10^{-6} \times 10^{\frac{L_P}{20}}\ Pa$

	Direct	Reverberation/Reflection
L_P	89.49 dB	80.48 dB
SPL	0.5964 Pa	0.2114 Pa

Total sound pressure level (in Pa):

$$SPL_{total} = \sqrt{SPL_{direct}^2 + SPL_{reverb}^2} = \sqrt{(0.5964\ Pa)^2 + (0.2114\ Pa)^2} = 0.6328\ Pa$$

Total sound pressure level (in dB): $L_{P,total} = 20\lg\frac{SPL_{total}}{20\times10^{-6}\ Pa} = 20\lg\frac{0.6328\ Pa}{20\times10^{-6}\ Pa} = 90.00\ dB$

This is as expected.

Alternatively: Use shortcut formula that immediately adds dB values.

Total sound pressure level (in dB):

$$L_{P,total} = 10\lg\left(\sum 10^{\frac{L_{P,i}}{10}}\right) = 10\lg\left(10^{\frac{89.49}{10}} + 10^{\frac{80.48}{10}}\right) = 90.00\ dB$$

This answer quality took way more than 2.5h, don't stress yourself.

Final answers highlighted in yellow.

Done by cgwee002@e.ntu.edu.sg.