NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2016-2017

MA2002 – THEORY OF MECHANISM

November/December 2016 Time Allowed: $2^1/_2$ hours

SEAT NUMBER: | | | | |

MATRICULATION NUMBER: | | | | | | | | | |

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **TWENTY-THREE (23)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is an **OPEN-BOOK** examination.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

For examiners:

Questions	**1** (25)	**2** (25)	**3** (25)	**4** (25)	**Total** (100)
Marks					

86

Q1 (25 marks) /25

(a) The schematic of the entire linkage for a large power shovel used in strip mining is shown in Figure 1. Note that Links 3, 4, 5 are connected through revolute joints. Links 8, 9, 14 and 16 are connected through revolute joints. Links 15, 16 and 17 are connected through revolute joints. Link 7 is connected to Link 8 with a revolute joint.

(i) Identify the types of links in the linkage as shown in Figure 1.

(ii) Determine its degrees of freedom.

(7 marks)

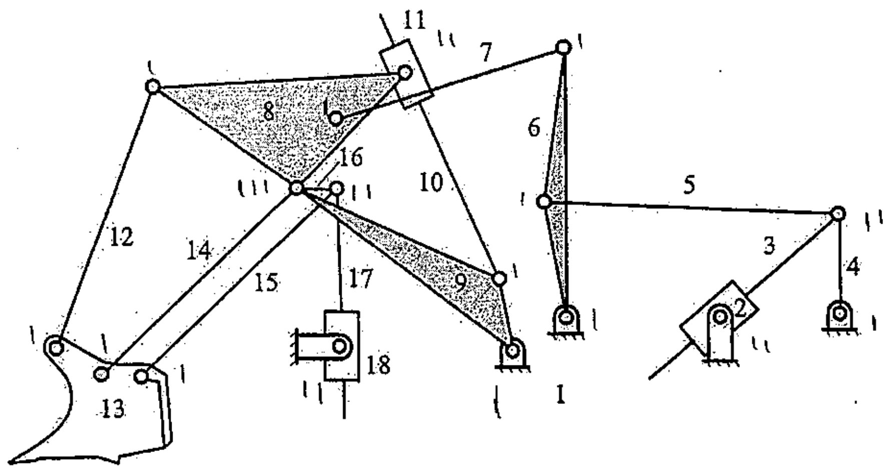


Figure 1

Solution

Binary links: 2, 3, 4, 5, 7, 10, 11, 12, 14, 15, 16, 17, 18

Ternary links: 6, 9, 13,

Quaternary links: 8,

Pentagonal links: 1

ii) $n_L = 18$

$n_J' = 24$

$n_J'' = 0$

$\therefore DOF = 3(n_L - 1) - 2n_J' - n_J''$

$= 3(18-1) - 2(24)$

$= 3$ ※

Q1 (continued)

(b) A set of links are given with the following dimensions:

$L_1 = 10$ mm, $L_2 = 20$ mm, $L_3 = 38$ mm, $L_4 = 47$ mm, $L_5 = 62$ mm

Determine the set of links that can form Grashof 4-bar linkages.

(6 marks)

For Grashof 4-bar linkages, it must follow

$$L_{max} + L_{min} < L_a + L_b$$

① $L_5 + L_1 = 72$ cm (max + min)

$L_2 + L_3 = 58$ (rejected)

$L_2 + L_4 = 67$ (rejected)

$L_3 + L_4 = 85$ (accepted)

$\therefore$ L_1, L_3, L_4, L_5

Similarly

② L_2, L_3, L_4, L_5

③ L_1, L_2, L_3, L_4

Q1 (continued)

(c) A compound planetary gear train system is shown in Figure 2. Gear 1 is the input A running at 120 rpm (cw). Carrier 12 is the input B running at 100 rpm (ccw). Gears 1, 2, 3, and 4 form a reverted gear train. (Gear 3 and 4 form an internal gear pair.) Gear 5 is the sun gear connected to ring gear 4. Planet gears 6 and 7 are connected and supported by carrier 12. Sun gears 8 and 9 are rigidly connected together. Gear 10 is a planet gear. Gear 11 a ring gear fixed to the wall. Carrier 13 is the output. The tooth numbers are given as:

$N_1 = 16, N_2 = 40, N_3 = 20, N_4 = 76, N_5 = 48, N_6 = 12, N_7 = 24, N_9 = 30, N_{10} = 24.$

All the gears are standard full-depth spur gears with a pressure angle of 20° and have the same module of 3 mm.

(i) Determine the angular speed of gear 4. (3 marks)

(ii) Determine the number of teeth of gear 8 (N_8). (3 marks)

(iii) Determine the number of teeth of gear 11 (N_{11}). (3 marks)

(iv) Determine the angular speed of the output Carrier 13. (3 marks)

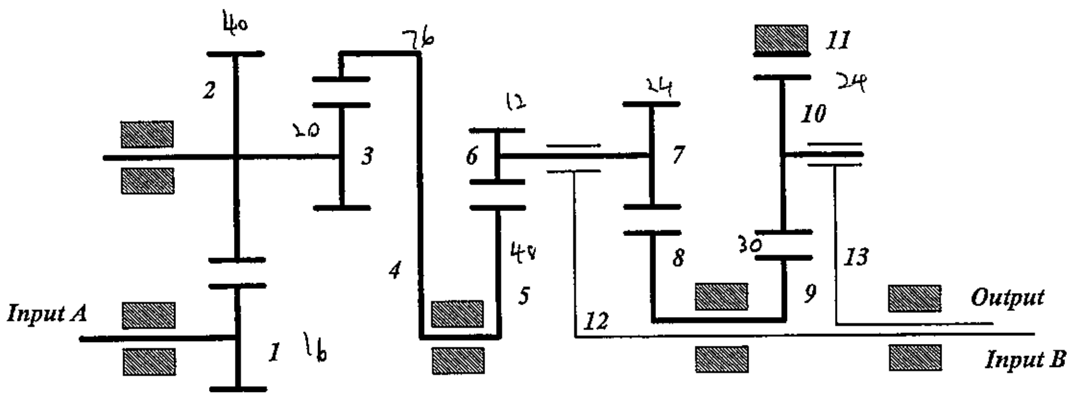


Figure 2

Solution:

Solution Q1(c)

$n_A = -120$ (cw)

$n_c = 100$ (ccw)

i) Reverted gear train

$$\frac{n_4}{n_1} = -\frac{N_1}{N_2}\left(\frac{N_3}{N_4}\right)$$

$$n_4 = 120\left(\frac{16}{40}\right)\left(\frac{20}{76}\right)$$

$$= 12.63 \quad (\text{ccw})$$

ii) Using geometry

$$N_5 + N_6 = N_7 + N_8$$

$$N_8 = 48 + 12 - 24$$

$$= 36$$

#

iii) $n_4 = n_5 = 12.63$

$$\frac{n_8 - n_c}{n_5 - n_c} = -\frac{N_5}{N_6}\left(\frac{N_7}{N_8}\right)$$

$$n_8 = (12.63 - 100)\left(-\frac{48}{12}\right)\left(\frac{24}{36}\right) + 100$$

$$= 332.99$$

(iii)

iv) $$\frac{n_4 - n_{13}}{n_9 - n_{13}} = -\frac{N_9}{N_{10}}\left(\frac{N_{10}}{N_{11}}\right)$$, from geometry

$$N_{11} = 2N_{10} + N_9$$

$$= 2(24) + 30$$

$$= 78$$

#

$$\frac{-n_{13}}{332.99 - n_{13}} = -\frac{30}{78}$$

$n_{13} = 92.5$ (cw) rpm #

28

Q2 (25 marks)

/25

A seven-bar 2-DOF-linkage mechanism is shown in Figure 3. Points B, C, and E are on a straight line on link 2. The first input is link 2 with a constant angular velocity ω_2 of 2 rad/s (cw). The second input is slider 5 with a velocity V_D of 20 mm/s and an acceleration A_D of 20 mm/s^2.

(a) Complete the velocity polygon according to the drawing scale given, and find the angular velocities ω_3, ω_4, and ω_6 of links 3, 4 and 6, the velocity V_C of point C, and the velocity V_F of point F.

(10 marks)

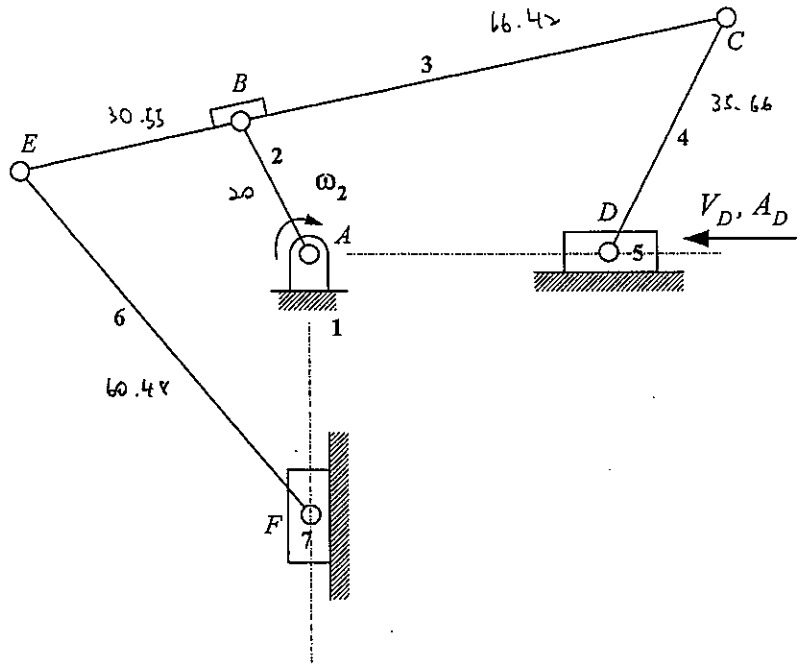
(b) Complete the acceleration polygon according to the drawing scale given, and find the angular accelerations α_3, α_4 and α_6 of links 3, 4 and 6, the acceleration A_C of point C, and the acceleration A_F of point F.

(10 marks)

(c) The positions of link 2 and slider 5 are given as input. Write down the vector loop equations of the linkage for analytical position analysis and identify the unknowns in the equations. (DO NOT solve the equations.)

(5 marks)

Note: Round off all your answers to **two decimal places** for accuracy.

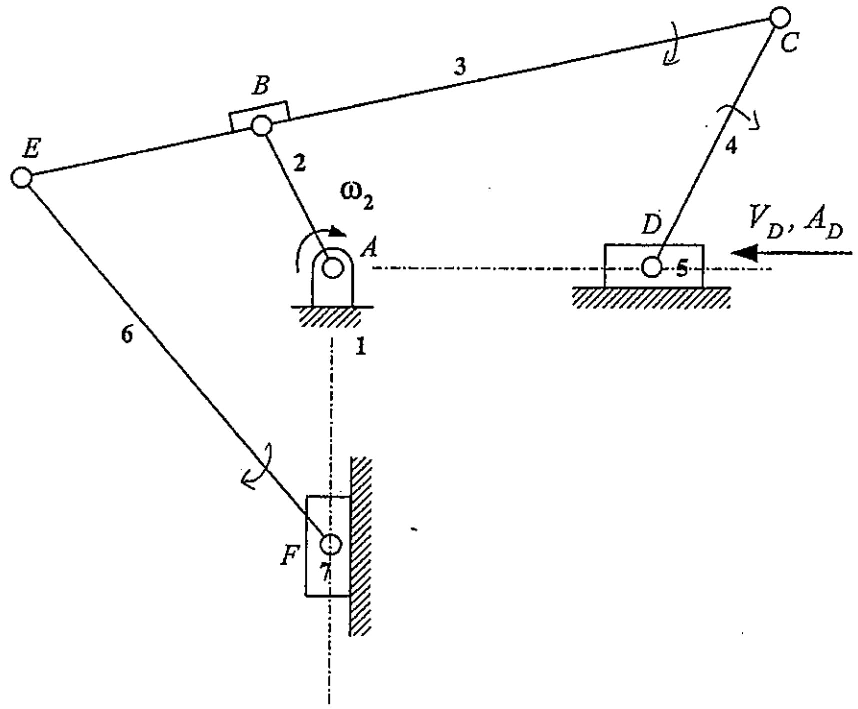


Dimensions:

AB = 20.00 mm
BC = 66.42 mm
BE = 30.55 mm
CD = 35.66 mm
EF = 60.48 mm

Figure 3

Q2(a) Velocity Analysis



Dimensions:

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BC = 66.42 mm
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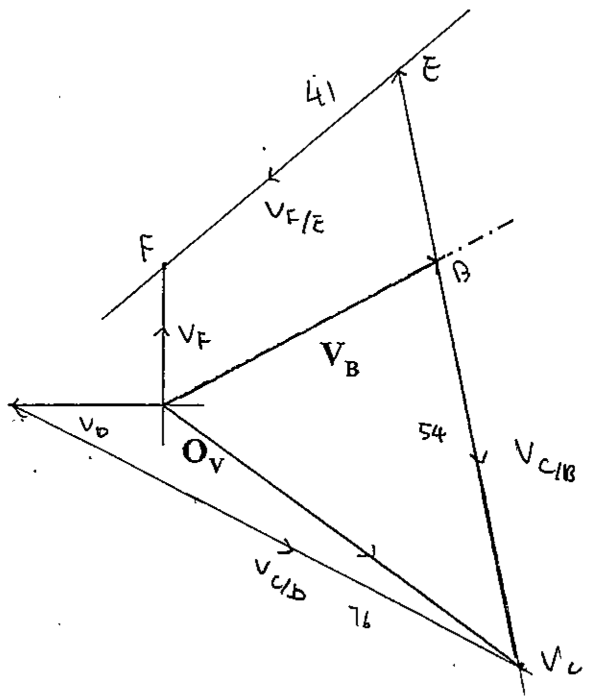

Velocity polygon
Scale = 1 mm : 1 mm/s

FILLING IN ANSWER HERE	
ω_3	0.81 rad/s (cw or ccw)*
ω_4	2.13 rad/s (cw or ccw)*
ω_6	0.68 rad/s (cw or ccw)*
V_C	58 mm/s
V_F	18 mm/s

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Solution Q2(a) Velocity Analysis (continued)

$\omega_2 = 2$ rad/s (cw)

$V_B = \omega r = 2 \times 20 = 40$ mm/s ($\nearrow$)

$V_3 = 20$ ($\leftarrow$)

i) $\left.\begin{array}{l} V_C = V_{C/B} + V_B \\ V_C = V_{C/D} + V_D \end{array}\right\}$

$\therefore V_C = 58$ mm/s #

EBC is a str line with ratio

$\frac{BE}{54} = \frac{30.55}{66.42}$

$BE = 24.84$

$V_F = V_{F/E} + V_E$

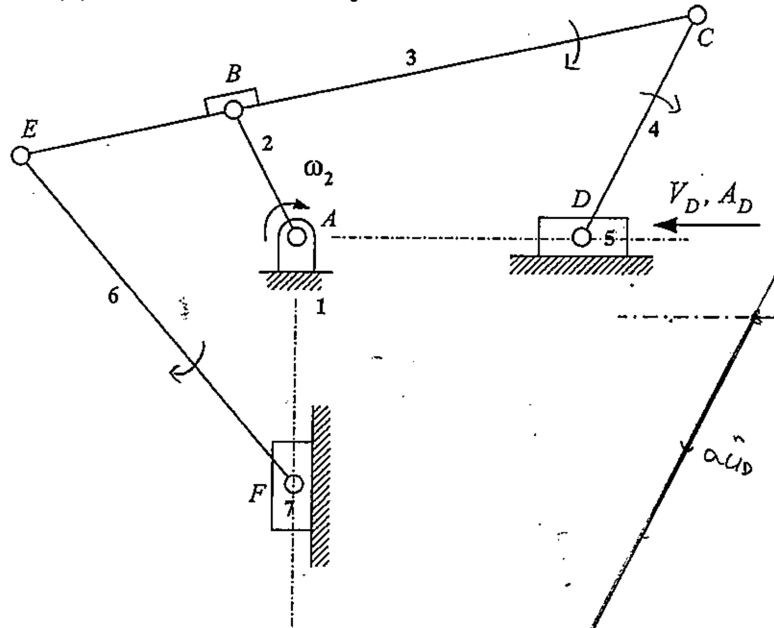
$V_F = 18$ mm/s ($\uparrow$) #

$\omega_3 = \frac{V_{BC}}{BC} = \frac{54}{66.42} = 0.813$

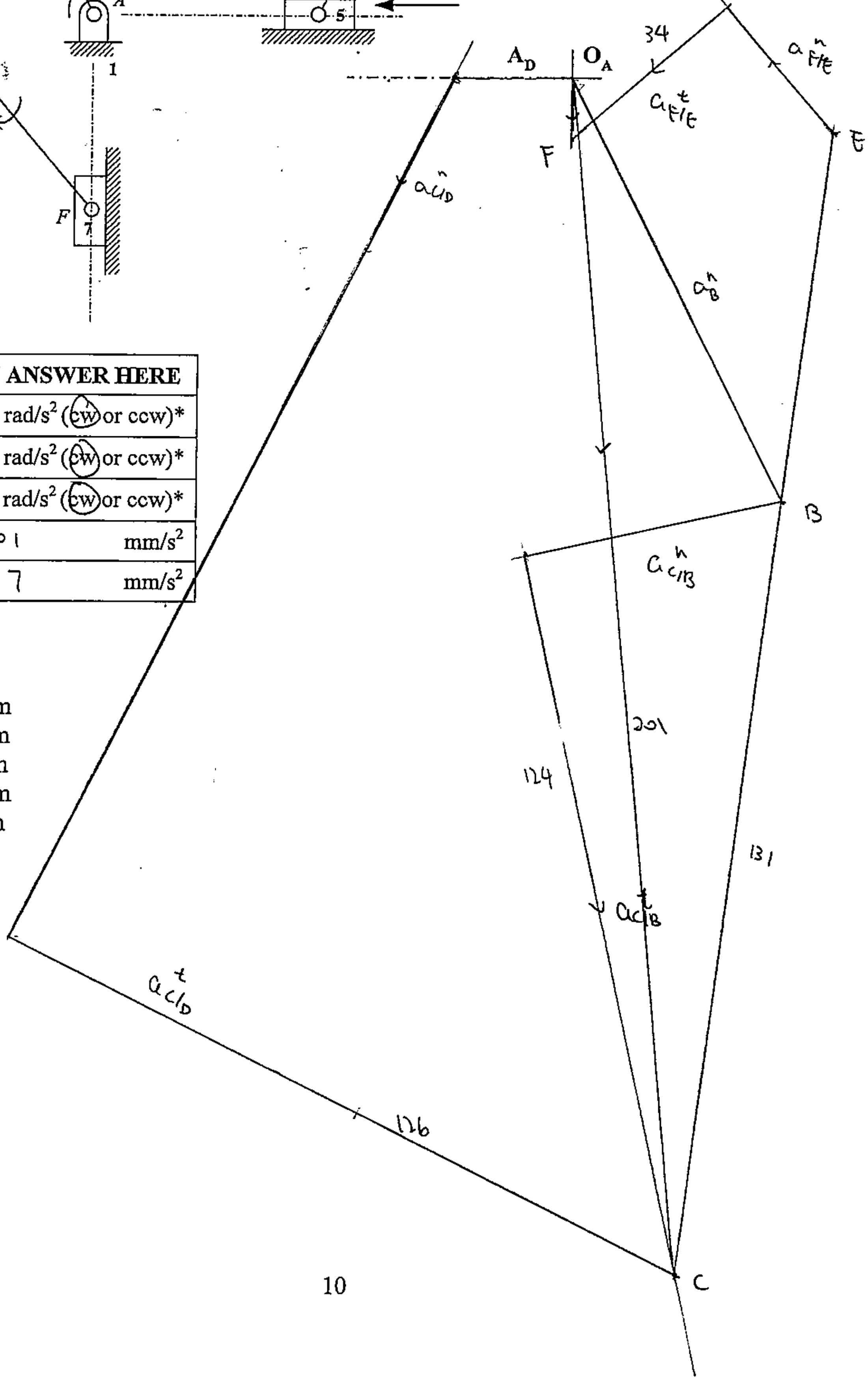
$\omega_4 = \frac{V_{CD}}{CD} = \frac{76}{35.66} = 2.131$

$\omega_6 = \frac{V_{FE}}{FE} = \frac{41}{60.48} = 0.678$

Q2(b) Acceleration Analysis



Acceleration polygon
Scale = 1 mm : 1 mm/s²



FILLING IN ANSWER HERE	
α_3	1.87 rad/s² (cw or ccw)*
α_4	3.53 rad/s² (cw or ccw)*
α_6	0.56 rad/s² (cw or ccw)*
A_C	201 mm/s²
A_F	7 mm/s²

Dimensions:

AB = 20.00 mm
BC = 66.42 mm
BE = 30.55 mm
CD = 35.66 mm
EF = 60.48 mm

Solution Q2(b) Acceleration Analysis (continued)

$a_B^t = 0$, $a_B^n = \omega_2^2 r = 2^2(20) = 80\ mm/s^2$ (↘)
$a_D = 20\ mm/s^2$ (←)

$a_C = a_{C/D}^t + a_{C/D}^n + \vec{a}_D = a_{C/B}^t + a_{C/B}^n + \vec{a}_B$

$a_{C/D}^n = \omega_4^2 r = 2.131^2\,(35.66) = 161.94$

$a_{C/B}^n = \omega_3^2 r = 0.813^2\,(66.42) = 43.9$

$a_{F/E}^n = \omega_6^2 r = 0.678^2\,(60.48) = 27.8$

$a_c = 201\ mm/s^2$

$\frac{BE}{131} = \frac{30.53}{66.42} \Rightarrow BE = 60.25$

$a_F = a_{F/E}^n + a_{F/E}^t + \vec{a}_E$

$a_F = 7\ mm/s^2$ (↓)

$\alpha_3 = \frac{124}{66.42} = 1.87$

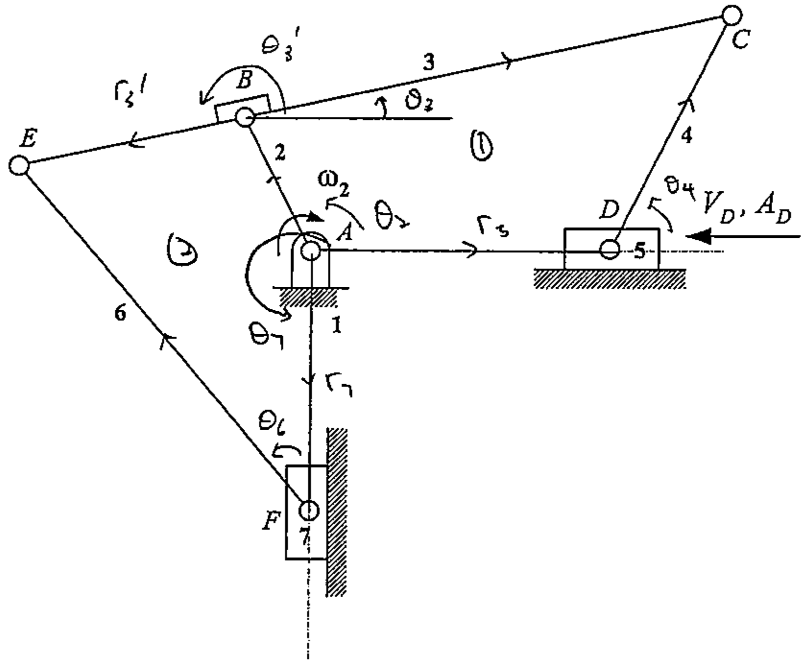
$\alpha_4 = \frac{126}{35.66} = 3.53$

$\alpha_6 = \frac{34}{60.48} = 0.56$

Q2(c) Vector loop equations

Write down the vector loop equations of the linkage for analytical position analysis and identify the unknowns in the equations. **(DO NOT solve the equations.)**

Solution (Indicate vectors clearly on the figure.)



loop ① $\quad \vec{r}_2 + \vec{r}_3 = \vec{r}_4 + \vec{r}_5$

$$i: \quad r_2 \cos\theta_2 + r_3 \cos\theta_3 = r_4 \cos\theta_4 + r_5$$

$$j: \quad r_2 \sin\theta_2 + r_3 \sin\theta_3 = r_4 \sin\theta_4$$

θ_3, θ_4 unknown

loop ② $\quad \vec{r}_2 + \vec{r}_3\,' = \vec{r}_7 + \vec{r}_6$

$$i: \quad r_2 \cos\theta_2 + r_3' \cos\theta_3' = r_6 \cos\theta_6$$

$$j: \quad r_2 \sin\theta_2 + r_3' \sin\theta_3' = -r_7 + r_6 \sin\theta_6$$

θ_6, θ_3' unknowns

Q3 (25 marks)

/25

(a) Figure 4 shows a cam with a flat-face follower that is a slider in a 6-bar mechanism. The follower dwells at zero lift for the first 20° of the motion cycle, rises 20 mm with a uniform motion for 120°, dwells for 60°, returns with a parabolic motion for 120°, and dwells for 40°. **Please write your solutions in the tables on the next two pages.**

(i) Write the mathematical expressions for the displacement (S) of the follower as a function of the cam rotation angle θ for $0° \leq \theta \leq 360°$.

(6 marks)

(ii) Calculate the displacement from 0° to 360°, at 20° intervals.

(6 marks)

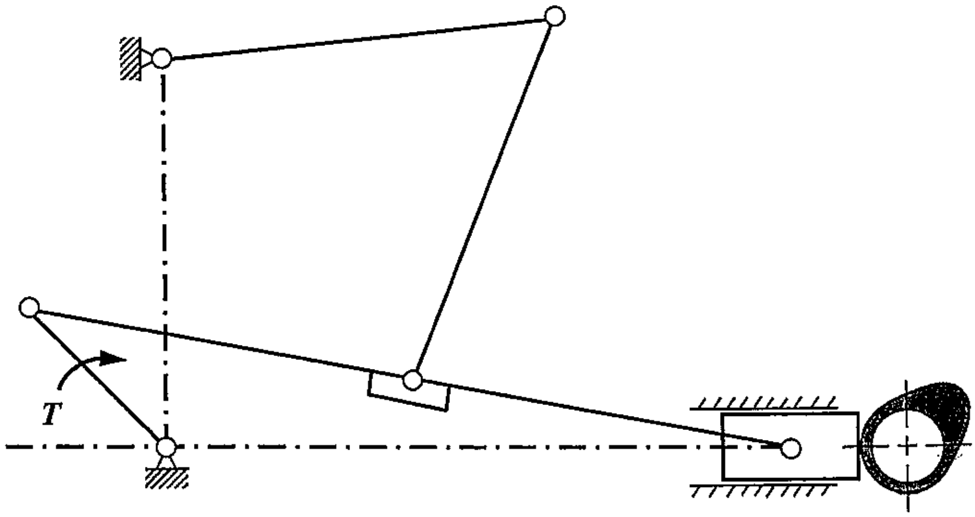


Figure 4

Solution Q3(a): Please write your solutions in the tables on this page and the next page.

Note: Round off all your answers to **two decimal places** for accuracy.

(i)

	θ	Function for follower displacement (S)
	$0° \le \theta \le 20°$	$S = 0$
uniform	$20° \le \theta \le 140°$	$S = \frac{h}{\beta}(\theta - \theta_s) = \frac{20}{120}(\theta - 20)$
	$140° \le \theta \le 200°$	$S = 20$
	$200° \le \theta \le 260°$	$S = -h + \frac{4h}{\beta}(\theta_e - \theta) - \frac{2h}{\beta^2}(\theta_e - \theta)^2 = -20 + \frac{4(20)}{120}(320 - \theta) - \frac{2(20)}{120^2}(320 - \theta)^2$
	$260° \le \theta \le 320°$	$S = \frac{2h}{\beta^2}(\theta_e - \theta)^2 = \frac{2(20)}{120^2}(320 - \theta)^2$
	$320° \le \theta \le 360°$	$S = 0$

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Solution Q3(a) (continued)

Note: Round off all your answers to **two decimal places** for accuracy.

(ii)

θ	S (mm)
0°	0
20°	0
40°	3.33
60°	6.67
80°	10
100°	13.33
120°	16.67
140°	20
160°	20
180°	20
200°	20
220°	18.89
240°	15.56
260°	10
280°	4.44
300°	1.11
320°	0
340°	0
360°	0

Q3 (continued)

(b) Figure 5 shows a 6-bar planar mechanism in static equilibrium with P = 50 N. Neglecting frictional and gravitational forces and link masses, calculate

(i) the joint force F_{Dx} at D, (2 marks)

(ii) the joint forces F_{Ex} and F_{Ey} at E, and (8 marks)

(iii) the input torque T. (3 marks)

<u>Note: Please write your solutions in the table below on this page.</u>

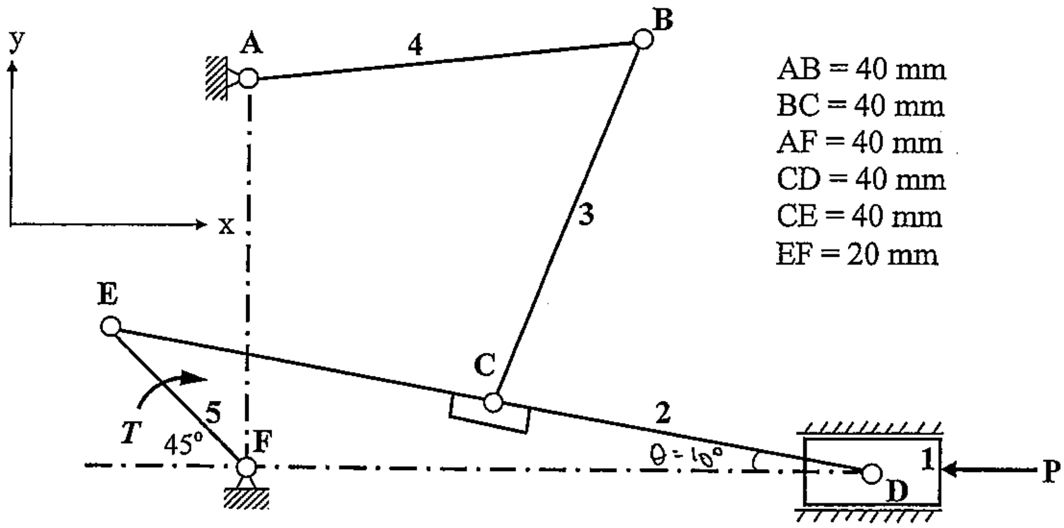

Figure 5

Solutions:

FILLING IN ANSWERS HERE		
F_{Dx}	50	N
F_{Ex}	50	N
F_{Ey}	8.82	N
T	582.37	Nmm

(Note: Round off all your answers to **<u>two decimal places</u>** for accuracy.)

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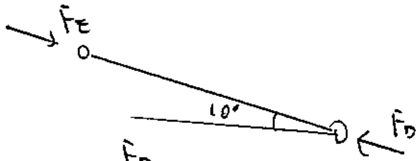
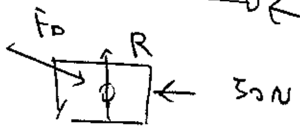
Solution Q3(b) (continued)

Due to freedom of motion of linkage ABC,
we assume that point C is forceless (ignore them)
ED becomes 2-bar linkage

②


①


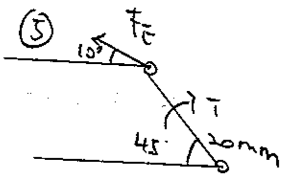
i: $F_D \cos\theta = P$

$F_D = \frac{50}{\cos 10} = 50.77$ N

$F_E = F_D = 50.77$

$F_E^y = 50.77 \sin 10 = 8.82$ N

$F_E^x = 50$ N

⑤


$\Sigma M = 0$

$T = F_E^x \, 20 \sin 45 - F_E^y \, 20 \cos 45$

$= 582.37$ N

Q4 (25 marks) /25

Figure 6 shows a 6-bar planar mechanism with P = 20 N. The various link parameters are given as follows:

Link	Mass	Location of CG	Moment of inertia about CG	Acceleration at CG	Angular acceleration
1	m_1	D	0	$a_{1x}\mathbf{i}$	0
2	m_2	C	I_2	$a_{2x}\mathbf{i} - a_{2y}\mathbf{j}$	α_2 (cw)
3	massless	~	~	~	~
4	massless	~	~	~	~
5	m_5	F	I_5	0	α_5 (cw)

Note: "~" means that this parameter is either not applicable or not necessary for the solution.

Neglecting frictional and gravitational forces,

(a) Draw the free-body diagram (FBD) of each link (except the ground link) with inertial forces and inertial moments shown in FBDs using D'lambert Principle. (10 marks)

(b) Calculate the joint forces F_D at D if m_1 =0.5 kg, a_{1x}=6 m/s^2. (2 marks)

(c) Calculate the joint forces F_{Ex} and F_{Ey} at E if m_2 = 1.5 kg, a_{2x} = 3 m/s^2, a_{2y} = 2 m/s^2, I_2 = 3 kgm^2, α_2 = 5 rad/s^2. (9 marks)

(d) Calculate the required input torque T applied to link 5 if I_5 = 3 kgm^2, α_5 = 6 rad/s^2. (4 marks)

Note: Please write your solutions in the table on the next page and draw FBDs using the links given in the next pages.

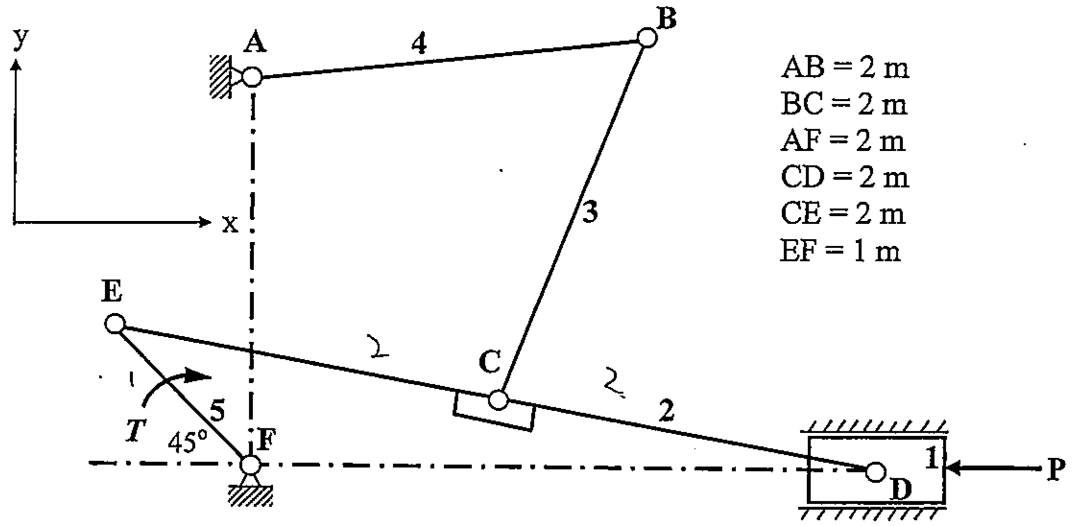


Figure 6

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Solution:

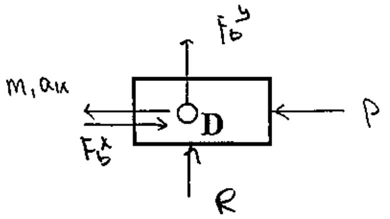
FILLING IN ANSWERS HERE		
F_{Dx}	23	N
F_{Ex}	27.5	N
F_{Ey}	-2.144	N
T	35.93	Nm (cw or ccw)*

* **Circle the correct one.**

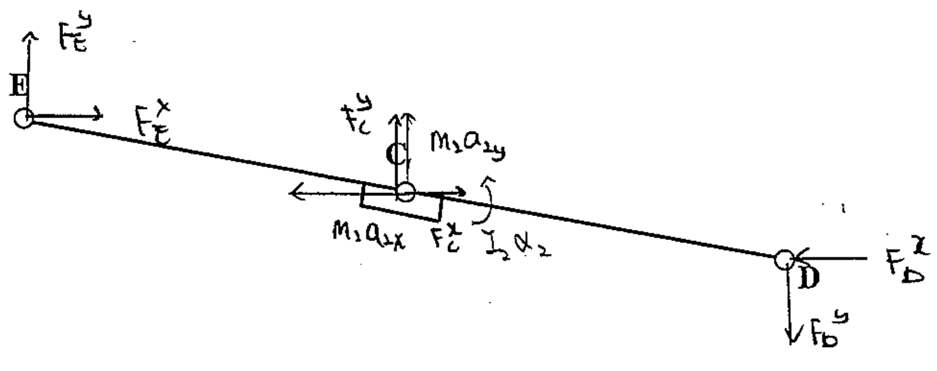
(Note: Round off all your answers to **two decimal places** for accuracy.)

Draw the free-body diagram of each link below

Link
1

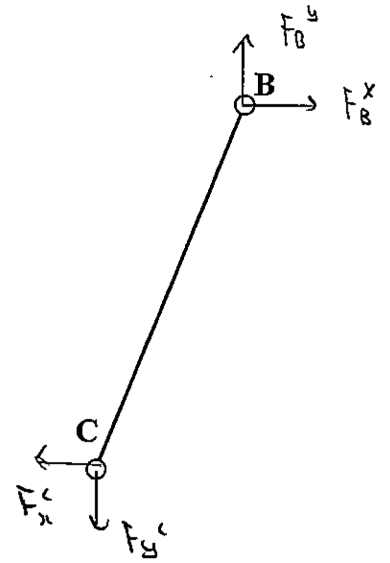


Link
2

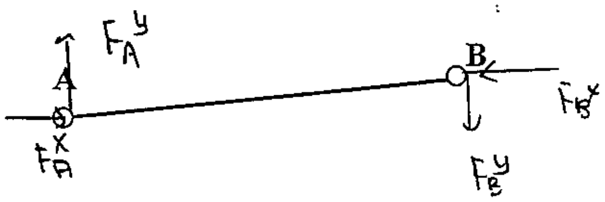


Draw the free-body diagram of each link below

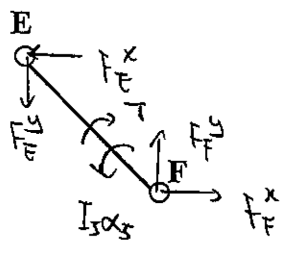
Link
3



Link
4



Link
5



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Solution Q4 (continued)

a) Link ①

$\Sigma F_x = 0$

$P + m_1 a_{1x} = F_D^x$

$F_D^x = 20 + 0.5 \times 6$

$= 23$ N

Link ②

b) Take pivot G, again we assume force due to point C is zero

$\Sigma M = 0$

$I_2 \alpha_2 + m_2 a_{2y} 2\cos 10 = m_2 a_{2x} \cdot 2\sin 10 + F_D^x 4 \sin 10 + F_D^y 4 \cos 10$

$3 \times 5 + 1.5 \times 2 \times 2 \cos 10 = 1.5 \times 3 \times 2 \sin 10 + 23 \times 4 \sin 10 + F_D^y 4 \cos 10$

$F_D^y = 0.856$ N

$\Sigma F_y = 0, \quad F_E^y + m_2 a_{2y} = F_D^y$

$F_E^y = 0.856 - 1.5 \times 2 = -2.144$ N ($\downarrow$)

$\Sigma F_x = 0, \quad F_E^x = m_2 a_{2x} + F_D^x$

$= 1.5 \times 3 + 23$

$= 27.5$ N ($\rightarrow$)

c) Link ⑤

$\Sigma M_F = 0$

$T = F_E^x \sin 45 + F_E^y \cos 45 + I_5 \alpha_5$

$= 27.5 \sin 45 - 2.144 \cos 45 + 3 \times 6$

$= 35.93$ Nm (cw)

END OF PAPER

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SEMESTER 1 EXAMINATION 2017-2018

MA2002 – THEORY OF MECHANISMS

November/December 2017 Time Allowed: $2^1/_2$ hours

SEAT NUMBER:

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Marks					

96

Q1 (25 marks)

/25

(a) A linkage mechanism is shown in Figure 1. Link 1 is the fixed link. Link 2, Link 3 and link 4 slide on Link 1 respectively. Link 2 and link 3, Link 3 and link 4 are connected through pin-in-a-slot joint respectively.

(i) Identify the types of links in the linkage.

(ii) Determine its degrees of freedom.

(4 marks)

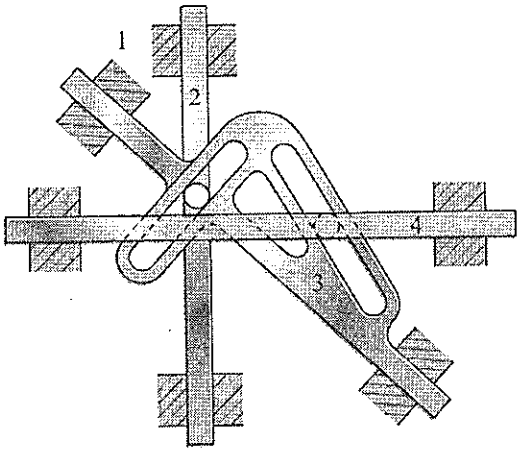

Figure 1

Solution

i) Binary links: 3 4 2

Ternary links: 1

Quaternary links:

Note: 1 is linked to 2, 3, 4
2 is linked to 3, 1
3 is linked to 2, 1
4 is linked to 1, 3

ii) DOF = $3(n_L - 1) - 2n'J - n''J$

$= 3(4-1) - 2(3) - 2$

$= 1$ *

Q1 (continued)

(b) A set of links are given with the following dimensions:

$L_1 = 120$ mm, $L_2 = 70$ mm, $L_3 = 35$ mm, $L_4 = 85$ mm, $L_5 = 55$ mm

(i) Determine tne set of links that can form **non-Grashof** 4-bar linkages.

(ii) Determine the range of unknown link length L_6, if L_1, L_2, L_3 and L_6 are used to form a **non-Grashof** 4-bar linkage. Note that L_1, L_2, L_3 and L_6 do not represent the locations of the links in the 4-bar linkage.

(6 marks)

1b i) For - Nongrasof condition

→ $l_{min} + l_{max} > l_a + l_b$

• Set 1, $l_{min} = l_3 = 35$, $l_{max} = l_1 = 120$

$l_a = l_2 = 70$, $l_b = l_4 = 85$

• $l_{min} + l_{max} = l_a + l_b$ (Reject)

• Set 2, $l_{min} = 35 = l_3$, $l_{max} = l_1 = 120$

$l_a = l_2 = 70$, $l_b = l_5 = 55$

• $l_{min} + l_{max} > l_a + l_b$ (Accept)

• Set 3, $l_{min} = l_3 = 35$, $l_{max} = l_1 = 120$

$l_a = l_4 = 85$, $l_b = l_5 = 55$

• $l_{min} + l_{max} > l_a + l_b$ (Reject)

• Set 4, $l_{max} = l_1 = 120$, $l_{min} = l_5 = 55$

$l_a = l_2 = 70$, $l_b = l_4 = 85$

• $l_{max} + l_{min} > l_a + l_b$ (Accept)

• Set 5, $l_{max} = l_4 = 85$, $l_{min} = l_3 = 35$

• $l_a = 70 = l_2$, $l_b = l_5 = 55$

$l_{max} + l_{min} < l_a + l_b$ (Reject)

Accept: Set 2 and set 4.

bii) $l_1 = 120$

$l_2 = 70$

$l_3 = 35$

$l_6 = ?$

Case 1: let l_6 be l_{min}

$l_{min} + l_{max} > l_a + l_b$

$l_6 + l_1 > l_2 + l_3$

$l_6 > l_2 + l_3 - l_1$

$l_6 > -15$ (impossible)

→ $(l_6 > 0)$ and $l_6 \leq 35$ (l_{min})

Case 2: let l_6 be l_{max}

$l_3 + l_6 > l_1 + l_2$

$35 + l_6 > 120 + 70$

$l_6 > 155$ and $l_6 \geq 120$ (l_{max})

→ $l_6 > 155$

Case 3: let l_6 be l_a / l_b

$l_1 + l_3 > l_2 + l_6$

$120 + 35 > 70 + l_6$

$85 > l_6$ — (1)

$l_6 \neq l_{max}$

$l_6 \neq l_{min}$

$l_{max} > l_6 > l_{min}$

$120 > l_6 > 35$ — (2)

∴ $120 > l_6 > 36$

The combination: $35 \geq l_6 > 0$

$l_6 > 155$

$120 > l_6 > 35$

27

Q1 (continued)

(c) A compound planetary gear train system is shown in Figure 2. Gear 1 is the input A running at 100 rpm (cw). Gears 1, 2, 3, and 4 form a reverted gear train. Gear 5 is a ring gear connected to gear 4. Sun gear 7 is fixed to the ground. Planet gears 6 and 9 are connected and supported by carrier 13. Sun gear 8 is output B. Gear 10 is a ring gear rigidly connected to Gear 11. Gear 12 is the output C. The tooth numbers are given as:

$n_8 = 1.$

$N_1 = 20$, $N_2 = 40$, $N_3 = 30$, $N_4 = 50$, $N_5 = 72$, $N_6 = 20$, $N_7 = 32$, $N_9 = 30$, $N_{10} = 100$, $N_{11} = 28$, $N_{12} = 44$.

All the gears are standard full-depth spur gears with a pressure angle of 20°. All gears except gears 1 and 2 have the same module of 3 mm.

(i) Determine the module of gears 1 and 2. (3 marks)

(ii) Determine the speed of gear 4. (3 marks)

(iii) Determine the speed of the output A at carrier 13. (3 marks)

(iv) Determine the speed and the number of teeth of gear 8 (N_8). (3 marks)

(v) Determine the speed of the output *C* at gear 12. (3 marks)

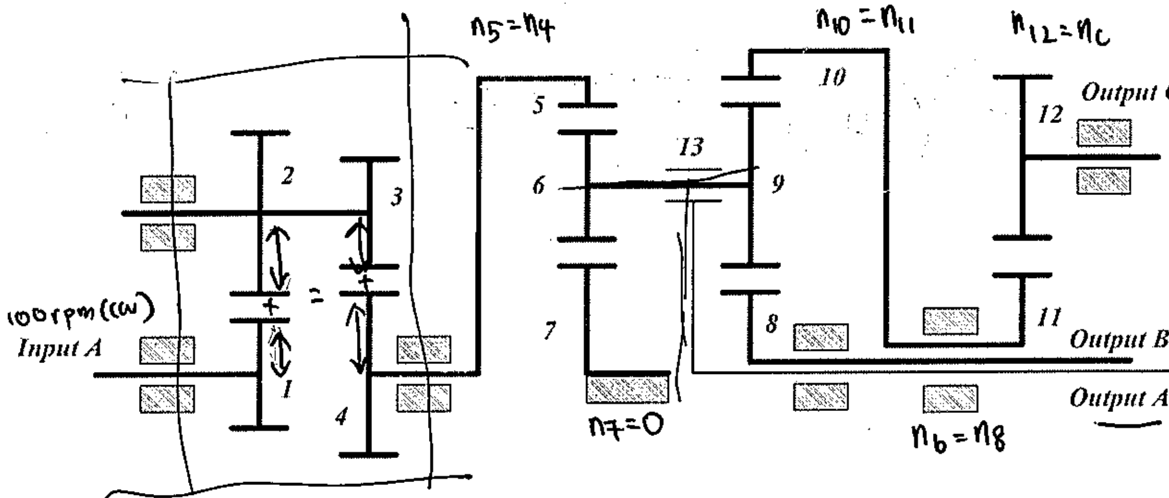

Figure 2

Solution:

i) Radius of 1 + Radius of 2 = Radius 3 + Radius 4

$$\frac{m_1 N_1}{2} + \frac{m_2 N_2}{2} = \frac{m_3 N_3}{2} + \frac{m_4 N_4}{2}$$

$$\frac{m_1(20)}{2} + \frac{m_1(40)}{2} = \frac{3(30)}{2} + \frac{3(50)}{2}$$

$$30 m_1 = 120$$

$$m_1 = 4\text{mm}$$

ii) $$\frac{n_4}{n_1} = \left(\frac{-N_1}{N_2}\right)\left(\frac{-N_3}{N_4}\right)$$

$$\frac{n_4}{-100} = 0.3$$

$$n_4 = -30$$

$$= 30 \text{ rpm (cw)}$$

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Solution Q1(c)

iii) $n_4 = n_5$

$= -30\text{rpm}$

$$\frac{n_7 - n_{13}}{n_5 - n_{13}} = \left(\frac{N_5}{N_6}\right)\left(-\frac{N_6}{N_7}\right)$$

$$= \left(\frac{72}{20}\right)\left(-\frac{20}{32}\right)$$

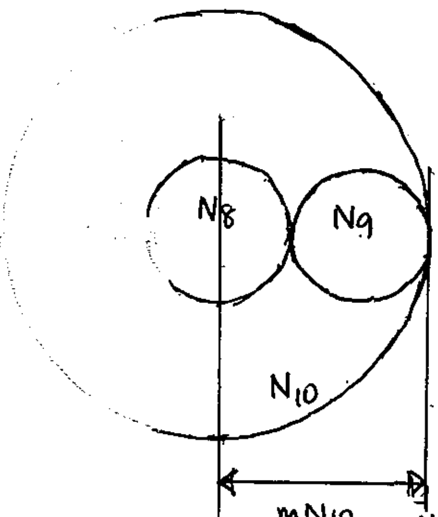
$$= -2.25$$

$$\frac{0 - n_{13}}{-30 - n_{13}} = -2.25$$

$\cancel{n_{13}} \rightarrow n_{13} = -20.8\text{rpm}$

$= 20.8\text{rpm (cw)}$

iv)



$$\frac{mN_{10}}{2} = \frac{mN_8}{2} + mN_9$$

$$r_{10} = r_8 + d_9$$

$$\frac{\cancel{m}N_8}{2} + \cancel{m}N_9 = \cancel{m}N_{10}/2$$

$$\frac{N_8}{2} + N_9 = \frac{N_{10}}{2}$$

$$\frac{N_8}{2} + 30 = 50$$

$$N_8 = 40$$

$$\frac{N_8 - N_{13}}{N_7 - N_{13}} = \left(-\frac{N_7}{N_6}\right)\left(-\frac{N_9}{N_8}\right)$$

$$\frac{N_8 - (-20.8)}{0 - (-20.8)} = \left(-\frac{32}{20}\right)\left(-\frac{30}{40}\right)$$

$$= 1.2$$

$$n_8 = 4.16 \text{ (ccw)}$$

v)

$$\frac{n_{10} - n_{13}}{n_7 - n_{13}} = \left(-\frac{N_7}{N_6}\right)\left(\frac{N_9}{N_{10}}\right)$$

$$\frac{n_{10} - (-20.8)}{0 - (-20.8)} = \left(-\frac{32}{20}\right)\left(\frac{30}{100}\right)$$

$$n_{10} = -30.8\text{rpm}$$

$$\rightarrow n_{10} = n_{11} = -30.8\text{rpm}$$

$$\frac{n_{12}}{n_{11}} = \left(-\frac{N_{11}}{N_{12}}\right)$$

$$n_{12} = \left(-\frac{28}{44}\right)(-30.8)$$

$$= 19.6\text{rpm (ccw)}$$

Q2 (25 marks)

/25

An eight-bar 1-DOF-linkage mechanism is shown in Figure 3. Joints B, C, and D are in a straight line on link 3. Joints E, F, and G are in a straight line on link 6. The input is slider 4 with a velocity V_D of 60 mm/s and an acceleration A_D of 100 mm/s^2.

(a) Complete the velocity polygon according to the drawing scale given, and find the angular velocities ω_2, ω_3, ω_5, ω_6, and ω_7 of links 2, 3, 5, 6 and 7, and the velocity V_H of joint H.

(10 marks)

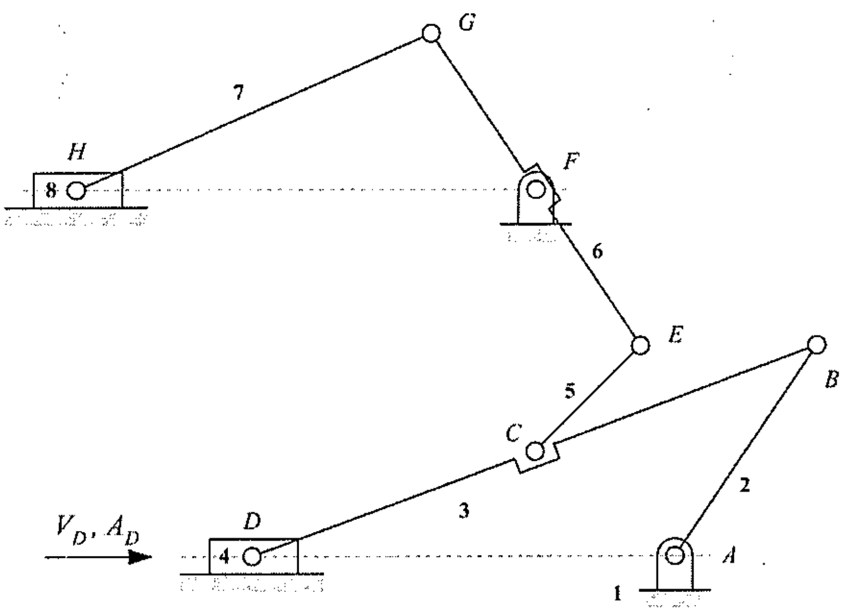
(b) Complete the acceleration polygon according to the drawing scale given, and find the angular accelerations α_2, α_3, α_5, α_6 and α_7 of links 2, 3, 5, 6 and 7, and the acceleration A_H of joint H.

(10 marks)

(c) The position of slider 4 is given as input. Write down the vector loop equations of the linkage for analytical position analysis and identify the unknowns in the equations. (DO NOT solve the equations.)

(5 marks)

Note: Round off all your answers to **two decimal places** for accuracy.



Dimensions:

AB = 36.00 mm
BC = CD = 42.72 mm
CE = 21.21 mm
EF = FG = 27.04 mm
GH = 54.70 mm

Figure 3

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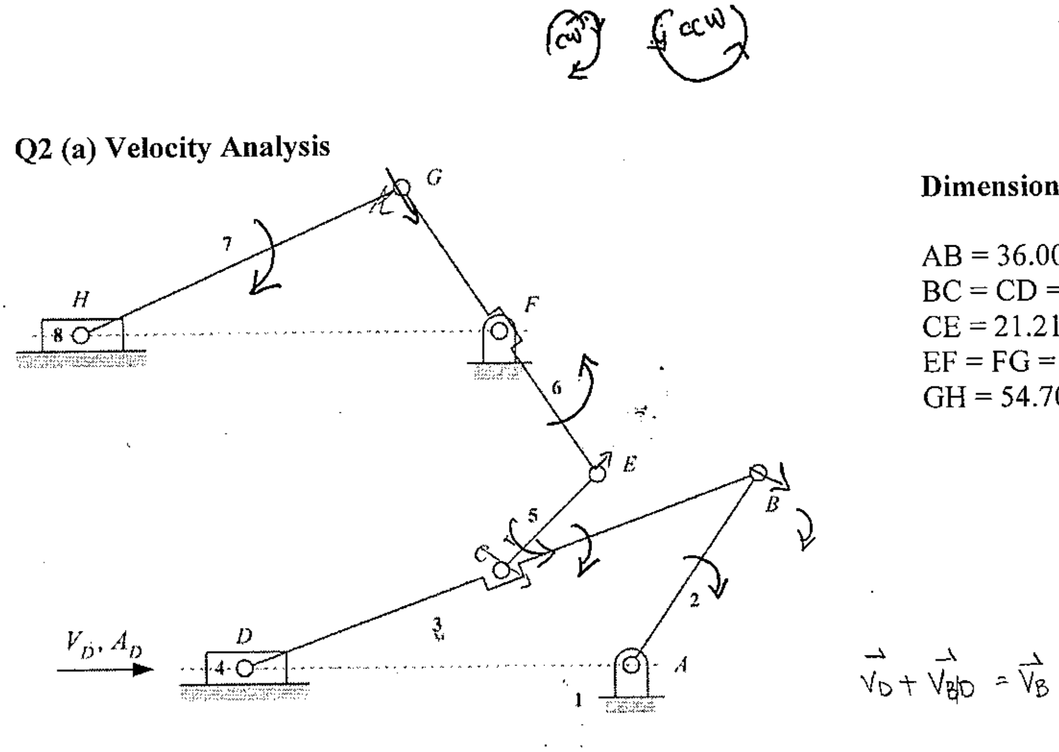
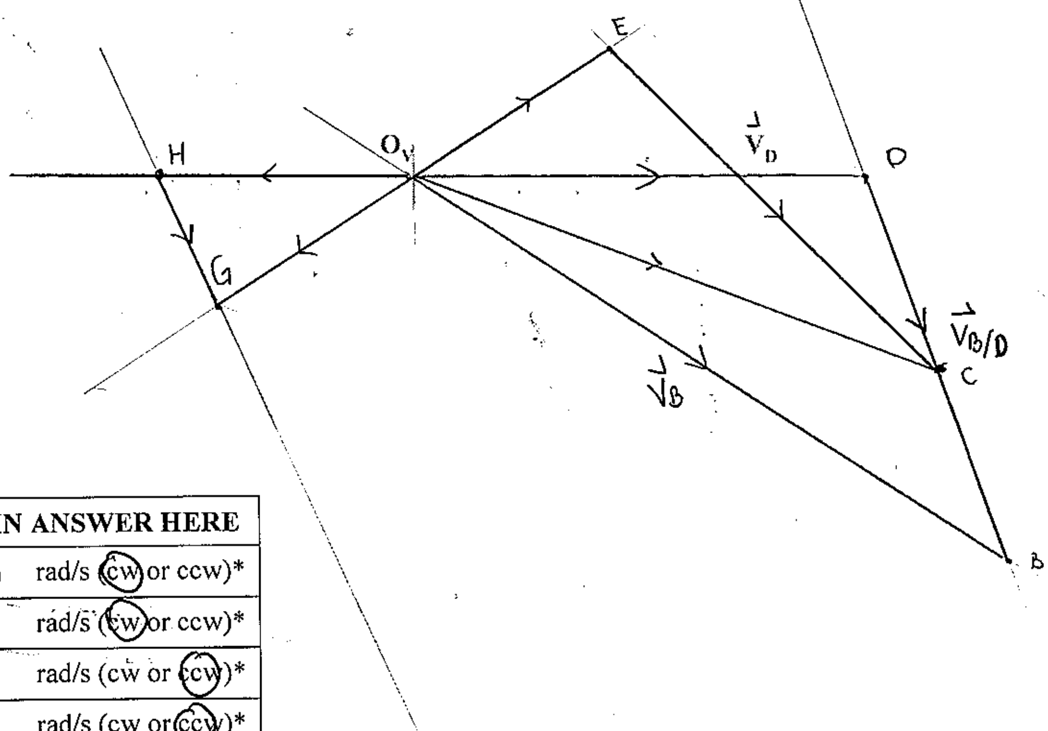
Q2 (a) Velocity Analysis



Dimensions:

AB = 36.00 mm
BC = CD = 42.72 mm
CE = 21.21 mm
EF = FG = 27.04 mm
GH = 54.70 mm

Velocity polygon
Scale = 1 mm : 1 mm/s



FILLING IN ANSWER HERE	
ω_2	2.60 rad/s (cw or ccw)*
ω_3	0.63 rad/s (cw or ccw)*
ω_5	2.83 rad/s (cw or ccw)*
ω_6	1.15 rad/s (cw or ccw)*
ω_7	0.35 rad/s (cw or ccw)*
V_H	~~34~~ $\angle -180°$ mm/s

34.00

* Circle the correct one.

Solution Q2(a) Velocity Analysis (continued)

$$\overset{\surd\surd}{\vec{V}_D} + \overset{\surd\times}{\vec{V}_{B/D}} = \overset{\surd\times}{\vec{V}_B}$$

$$\overset{\surd\times}{\vec{V}_E} + \overset{\surd\times}{\vec{V}_{C/E}} = \overset{\surd\surd}{\vec{V}_C}$$

$$\omega_G = \omega_E$$
$$r_G = r_E$$
$$V_G = V_E$$

$$\overset{\surd\times}{V_H} + \overset{\surd\times}{V_{G/H}} = \overset{\surd\surd}{V_G}$$

$$\omega_2 = \frac{V_B}{AB} = 2.597\,rad/s \;(cw)$$

$$\omega_3 = \frac{V_{BD}}{BD} = \frac{54}{85.44} = 0.632\,rad/s \;(cw)$$

$$\omega_5 = \frac{V_{CE}}{CE} = \frac{60}{21.21} = 2.83\,rad/s \;(ccw)$$

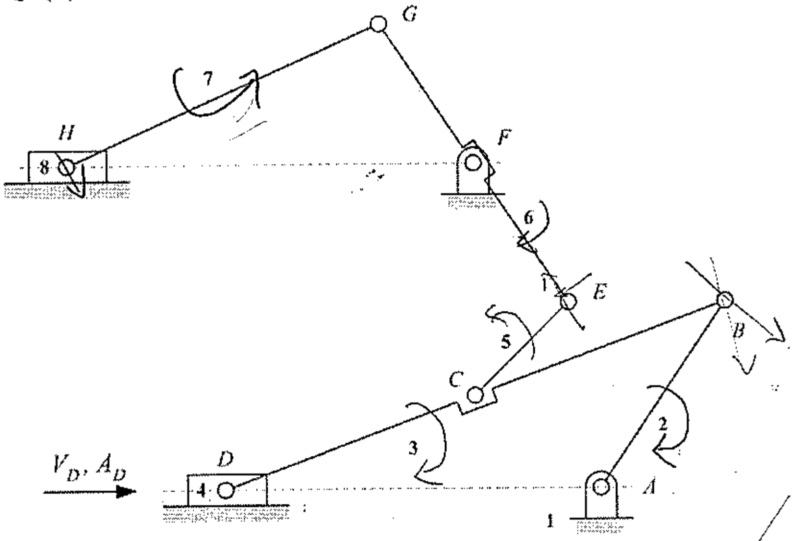
$$\omega_6 = \frac{V_E}{EF} = \frac{31}{27.04} = 1.15\,rad/s \;(ccw)$$

$$\omega_7 = \frac{V_{GH}}{GH} = \frac{19}{54.7} = 0.35\,rad/s \;(cw)$$

100

cw ccw

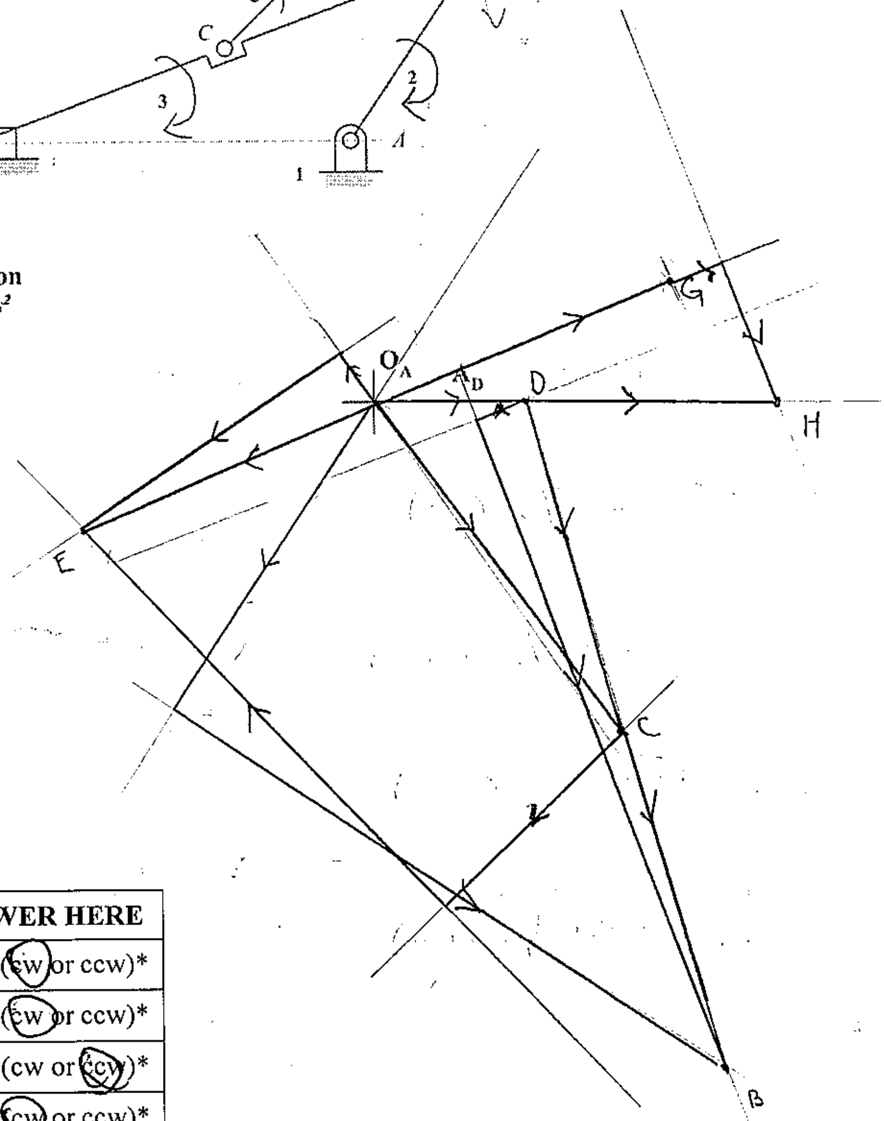
Q2(b) Acceleration Analysis



Dimensions:

AB = 36.00 mm
BC = CD = 42.72 mm
CE = 21.21 mm
EF = FG = 27.04 mm
GH = 54.70 mm

Acceleration polygon
Scale = 1 mm : 5 mm/s²



FILLING IN ANSWER HERE	
α_2	12.08 rad/s² (cw or ccw)*
α_3	5.38 rad/s² (cw or ccw)*
α_5	16.27 rad/s² (cw or ccw)*
α_6	7.58 rad/s² (cw or ccw)*
α_7	1.83 rad/s² (cw or ccw)*
A_H	270 ∠0° mm/s²

* Circle the correct one.

Solution Q2(b) Acceleration Analysis (continued)

$$\vec{A_D} + \vec{A_{B/D}} = \vec{A_B}$$

$$\vec{A_D} + \vec{A^n_{B/D}} + \vec{A^t_{B/D}} = \vec{A^n_B} + \vec{A^t_B}$$

$$|\vec{A^n_{B/D}}| = r\omega_3^2 = 85.44(0.63)^2 = 33.91\ mm/s^2$$

$$|\vec{A^n_B}| = r\omega_1^2 = 36(2.6)^2 = 243.63$$

$$\alpha_2 = \frac{A^t_B}{AB} = \frac{435}{36} = 12.08\ rad/s^2$$

$$\alpha_3 = \frac{A^t_{B/D}}{BD} = \frac{460}{85.44} = 5.38\ rad/s^2$$

$$\vec{A_{E/C}} + \vec{A_C} = \vec{A_E}$$

$$\vec{A^n_{E/C}} + \vec{A^t_{E/C}} + \vec{A_C} = A^t_E + A^n_E$$

$$|\vec{A^n_{E/C}}| = r\omega_5^2 = (21.21)(2.83)^2 = 169.87\ mm/s^2$$

$$\alpha_5 = \frac{A^t_{EC}}{CE} = \frac{345}{21.21} = 16.27\ rad/s^2$$

$$|A^n_E| = r\omega_6^2 = 35.75\ mm/s^2$$

$$\alpha_6 = \frac{A^t_E}{EF} = \frac{205}{27.04} = 7.58\ rad/s^2$$

$$\vec{A_G} + \vec{A_{H/G}} = \vec{A_H}$$

$$A_G + A^t_{H/G} + A^n_{H/G} = A_H$$

$$|A^n_{H/G}| = r\omega_7^2 = 6.7\ mm/s^2$$

$$\alpha_7 = \frac{A^t_{H/G}}{GH} = \frac{100}{54.7} = 1.83\ rad/s^2$$

$\alpha =$

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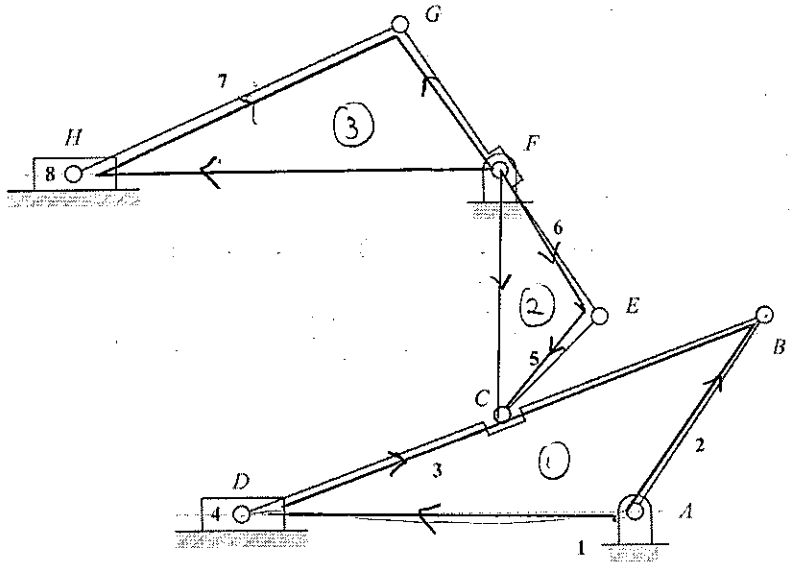
Q2(c) Vector loop equations

Write down the vector loop equations of the linkage for analytical position analysis and identify the unknowns in the equations. **(DO NOT solve the equations.)**

Solution (Indicate vectors clearly on the figure.)



$$\vec{r}_{D/A} + \vec{r}_{B/D} = \vec{r}_{B/A} \quad \text{— } 1$$

$$\vec{r}_{E/F} + \vec{r}_{C/E} = \vec{r}_{C/F} \quad \text{— } 2$$

$$\vec{r}_{H/F} + \vec{r}_{G/H} = \vec{r}_{G/F} \quad \text{— } 3$$

Q3 (25 marks)

/25

(a) A cam with a roller follower rotates at a constant speed in a precision machine. The follower dwells at zero lift for the first 60° of the motion cycle, rises 60 mm with a cycloidal motion for 150°, dwells for 60°, and returns with a harmonic motion for 90°. **Please write your solutions in the tables on this page and the next page.**

(i) Write the mathematical expressions for the displacement (S) of the follower as a function of the cam rotation angle θ **with the θ unit being degree (°) only (not radian)**, for $60° \le \theta \le 210°$ and $270° \le \theta \le 360°$.

(2 marks)

(ii) Calculate the displacement from 210° to 360°, at 30° intervals.

(2 marks)

(iii) Calculate the constant rotation speed (ω) of the cam, if the jerk is 529.37 mm/sec^3 when $\theta = 90°$.

$\dot{\omega} = 0$

(5 marks)

Solution Q3(a): Please write your solutions in the tables on this page and the next page.

Note: Round off all your answers to **two decimal places** for accuracy.

(i)

θ	Follower displacement (S) as a function of the cam rotation angle θ **with the θ unit being degree (°) only (not radian)**
$60° \le \theta \le 210°$	~~$\theta_i < 60 < \theta <$~~ $S = h\left(\frac{\theta-\theta_i}{\beta}\right) - \frac{h}{2\pi}\sin\left[2\pi\left(\frac{\theta-\theta_i}{\beta}\right)\right] = 60\left(\frac{\theta-60}{150}\right) - \frac{60}{2\pi}\sin\left[360\left(\frac{\theta-60}{150}\right)\right]$
$270° \le \theta \le 360°$	$S = \frac{h}{2}\left[1-\cos\left(\frac{\pi(\theta_e-\theta)}{\beta}\right)\right] = \frac{60}{2}\left[1-\cos\left(\frac{180(360-\theta)}{90}\right)\right]$

$$= 30\left[1-\cos\left(\frac{180(360-\theta)}{90}\right)\right]$$

iii) At $\theta = 90°$

$$\ddot{S} = \cancel{\frac{h}{\beta}\ddot{\theta}} - \cancel{\frac{h}{\beta}\ddot{\theta}\cos\left[2\pi\left(\frac{\theta-\theta_i}{\beta}\right)\right]} + \frac{2\pi h\dot{\theta}^2}{\beta^2}\sin\left[2\pi\left(\frac{\theta-\theta_i}{\beta}\right)\right]$$

$\ddot{\theta} = 0$

$$\ddot{S} = \frac{2\pi h\dot{\theta}^2}{\beta^2}\sin\left[2\pi\left(\frac{\theta-\theta_i}{\beta}\right)\right] = \frac{2\pi h\dot{\theta}\dot{\theta}}{\beta^2}\sin\left[2\pi\frac{(\theta-\theta_i)}{\beta}\right]$$

$$\dddot{S} = \frac{4\pi^2}{\beta^2}h\dot{\theta}^2\cos\left(\frac{2\pi(\theta-\theta_i)}{\beta}\right) \quad (1)$$

$$\dddot{S} = \cancel{\frac{2\pi h\ddot{\theta}\dot{\theta}}{\beta}\sin\left[2\pi\left(\frac{\theta-\theta_i}{\beta}\right)\right]} + \cancel{\frac{2\pi h\dot{\theta}\ddot{\theta}}{\beta}\sin\left[2\pi\frac{(\theta-\theta_i)}{\beta}\right]} + \frac{2\pi h\dot{\theta}^2}{\beta}\cos\left[2\pi\frac{(\theta-\theta_i)}{\beta}\right]\left(\frac{2\pi}{\beta}\right)$$

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Solution Q3(a) (continued)

(ii)

θ	S (mm)
210°	60
240°	60
270°	60
300°	45
330°	15
360°	0

Note: Round off all your answers to **two decimal places** for accuracy.

(iii) $\ddot{s} = \frac{4\pi^2}{\beta^2} h \dot{\theta}^2 \cos\left(\frac{2\pi}{\beta}(\theta - \theta_i)\right)$ —— (1)

$529.37 = \frac{4\pi^2}{2.618^2}(60)\dot{\theta}^2 \cos\left(\frac{2\pi}{2.618}\left(\frac{\pi}{2} - \frac{\pi}{3}\right)\right)$

$= 345.6\,\dot{\theta}^2 (0.309)$

$= 106.8\,\dot{\theta}^2$

$\dot{\theta} = 2.23 \text{ rad/s}$

ω (rad/s)
2.23

Note: Round off all your answers to **two decimal places** for accuracy.

Q3 (continued)

(b) Figure 4 shows a 6-bar planar mechanism in static equilibrium with Q = 2.6 N and T_5 = 15 Nm. Neglecting frictional and gravitational forces, calculate

(i) the joint force F_B at B, (2 marks)

(ii) the joint force F_{Cx} at C, (2 marks)

(iii) the joint force F_{Cy} at C, (5 marks)

(iv) the joint forces F_{Dx} and F_{Dy} at D, and (4 marks)

(v) the required input torque T_1 applied to link 1. (3 marks)

Note: Please write your solutions in the table below on this page.

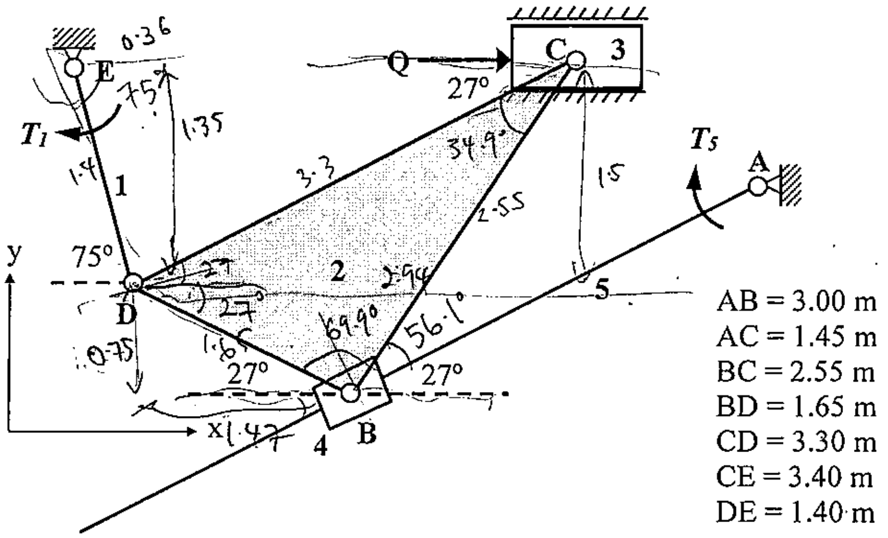


Figure 4

$$\frac{54^\circ}{2.55} = \frac{\theta_{32}}{1.65} = \frac{\theta_{42}}{3.3}$$

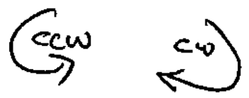
$\theta_{32} = 34.94^\circ$ $\theta_{42} = 69.9^\circ$

Solutions:

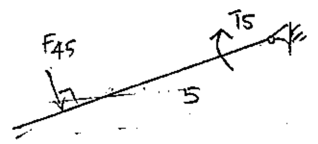
FILLING IN ANSWERS HERE		
F_B	~~5.0~~ 5.00	N
F_{Cx}	~~2.30~~ 2.30	N
F_{Cy}	~~-0.32~~ -0.32	N
F_{Dx}	0.33	N
F_{Dy}	4.13	N
T_1	1.93	Nm

(Note: Round off all your answers to **two decimal places** for accuracy.)

'03



Solution Q3(b) (continued)

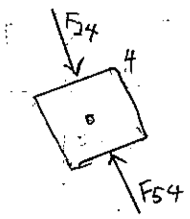


$F_{45}(3) - T_5 = 0$

$F_{45}(3) - 15 = 0$

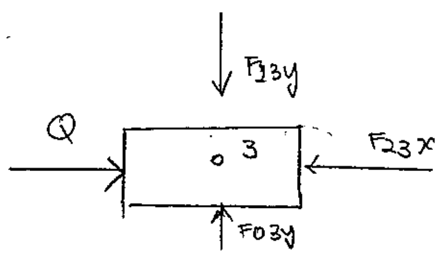
$F_{45} = 5N$

$F_B = F_{45} = 5N$



$F_{54} - F_{24} = 0$

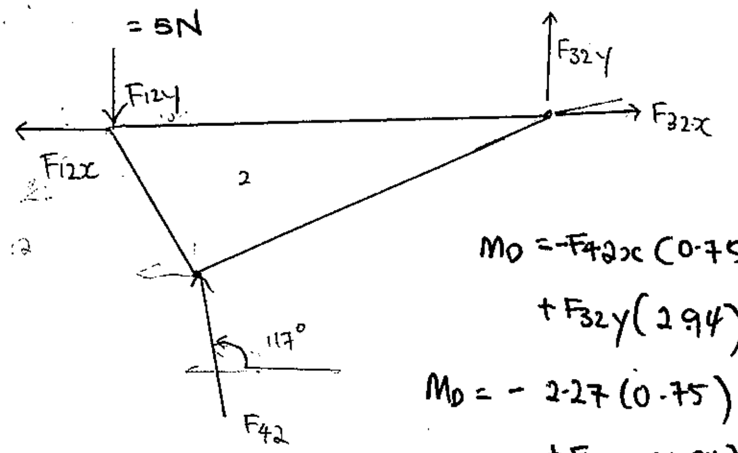
$F_{24} = F_{54}$
$= 5N$



$\Sigma F_x = 0 \quad Q - F_{23x} = 2.6 - F_{23x} = 0$

$F_{23x} = 2.6N$

$\Sigma F_y = 0 \quad F_{23y} - F_{03y} = 0 \quad - (1)$



$M_0 = -F_{42x}(0.75) + F_{42y}(1.47) - F_{32x}(1.5) + F_{32y}(2.94) = 0$

$M_0 = -2.27(0.75) + 4.45(1.47) - 2.6(1.5) + F_{32y}(2.94) = 0$

$F_{32y} = -0.32N$

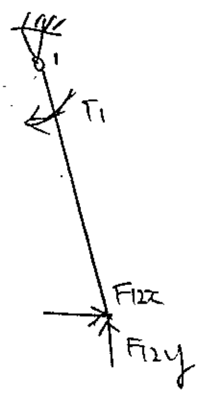
$F_{Cy} = 0.32N \downarrow$

$\vec{F}_{42} = 5N \angle 117°$

~~F_{42x}~~

$F_{42x} = 2.27$

$F_{42y} = 4.45$



$\Sigma F_y = -F_{12y} + F_{42y} + F_{32y} = 0 \quad F_{12y} = 4.45 + (-0.32) = 4.13N$

$F_{Dy} = 4.13N$

$\Sigma F_x = F_{32x} - F_{12x} - F_{42x}$

$= 2.6 - F_{12x} - 2.27 = 0 \quad F_{12x} = 0.33N \quad F_{Dx} = 0.33N \rightarrow$

$\Sigma M_1 = F_{12y}(0.36) + F_{12x}(1.35) - T_1 = 0 \rightarrow 4.13(0.36) + 0.33(1.35) - T_1 = 0 \quad T_1 = 1.93Nm$

Q4 (25 marks) /25

Figure 5 shows a 6-bar planar mechanism with Q = 13 N and T_5 = 180 Nmm. The various link parameters are given as follows:

Link	Mass	Location of CG	Moment of inertia about CG	Acceleration at CG	Angular acceleration
1	m_1	E	I_1	0	α_1 (cw)
2	m_2	D	I_2	$-a_{2x}\mathbf{i}-a_{2y}\mathbf{j}$	α_2 (cw)
3	m_3	C	0	$-a_{3x}\mathbf{i}$	0
4	massless	~	~	~	~
5	massless	~	~	~	~

Note: "~" means that this parameter is either not applicable or not necessary for the solution.

Neglecting frictional and gravitational forces,

(a) Draw the free-body diagram (FBD) of each link (except the ground link) with inertial forces and inertial moments shown in FBDs using D'lambert Principle. (10 marks)

(b) Calculate the joint force F_B from link 5 at B. (2 marks)

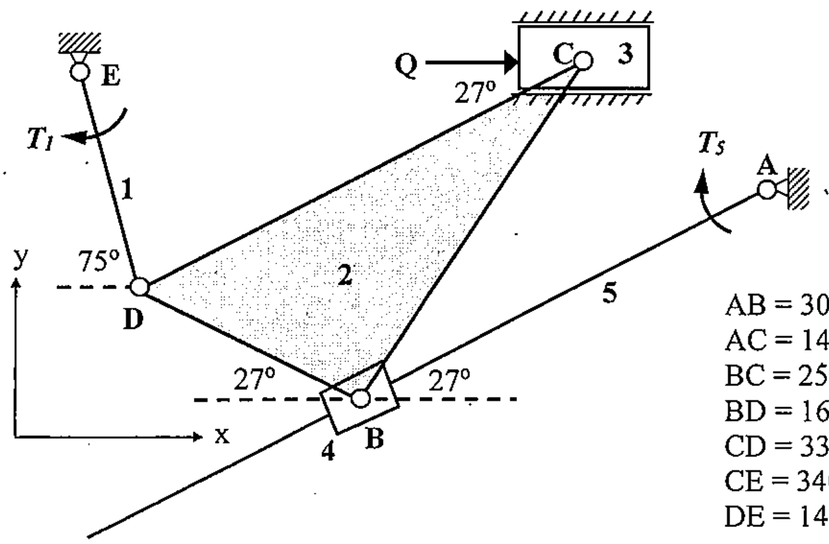
(c) Calculate the joint force F_{Cx} at C if m_3 = 2 kg, a_{3x} = 3 m/s^2. (2 marks)

(d) Calculate the joint force F_{Cy} at C if I_2 = 2 kgm^2, α_2 = 3 rad/s^2. (5 marks)

(e) Calculate the joint forces F_{Dx} and F_{Dy} at D if m_2 = 3 kg, a_{2x} = 2.5 m/s^2, a_{2y} = 4 m/s^2. (4 marks)

(f) Calculate the required input torque T_1 applied to link 1 if I_1 = 0.5 kgm^2, α_1 = 2 rad/s^2. (2 marks)

Note: Please write your solutions in the table on the next page and draw FBDs using the links given in the next pages.



AB = 300 mm
AC = 145 mm
BC = 255 mm
BD = 165 mm
CD = 330 mm
CE = 340 mm
DE = 140 mm

Figure 5

'04

+CW +CCW

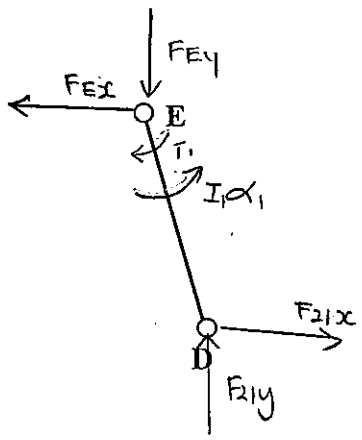
Solution:

FILLING IN ANSWERS HERE		
F_B	0.6	N
F_{Cx}	19N	N
F_{Cy}	7.46	N
F_{Dx}	26.23N	N
F_{Dy}	19.99N	N
T_1	43.61	Nm

(Note: Round off all your answers to **two decimal places** for accuracy.)

Draw the free-body diagram of each link below

Link 1

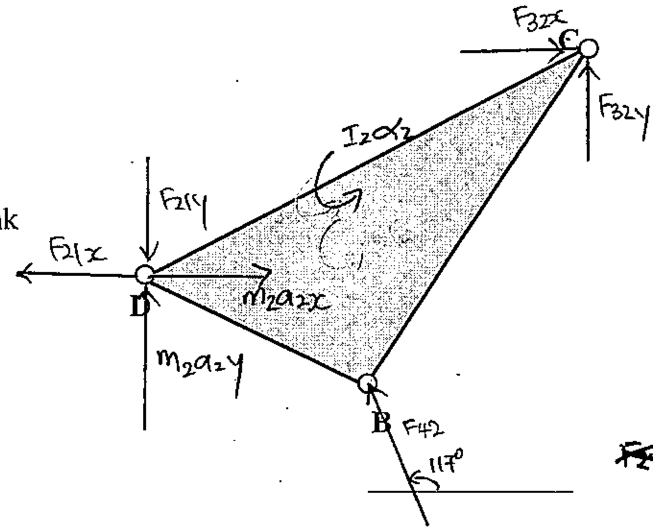


$$M_E = I_1\alpha_1 - T_1 + F_{21x}(1.35) + F_{21y}(0.36) = 0$$
$$= 0.5(2) - T_1 + 26.23(1.35) + 20(0.36) = 0$$
$$T_1 = 43.61\text{Nm}$$

+ CW   + CCW

Draw the free-body diagram of each link below

Link 2



~~F24 = 0.6N~~

$F_{42} = 0.6N$

$$\Sigma M_D = I_2\alpha_2 + F_{32y}(2.94) - F_{32x}(1.5) + F_{42y}(1.47) - F_{42x}(0.75)$$
$$= 0$$

$$\Sigma M_D = 2(3) + F_{32y}(2.94) - 19(1.5) + 0.53(1.47) - 0.27(0.75)$$
$$= 0$$

$F_{32y} = 7.46N$ *

$F_{cy} = 7.46N$

$$\Sigma F_x = m_2a_{2x} - F_{21x} - F_{42x} + F_{32x} = 0$$

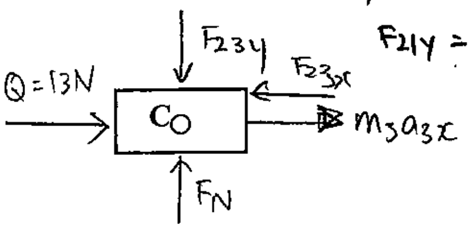
$$= 3(2.5) - F_{21x} - 0.27 + 19 = 0$$

$F_{21x} = 26.23N$ * $F_{Dx} = 26.23N$

$$\Sigma F_y = m_2a_{2y} - F_{21y} + F_{32y} + F_{42y} = 0$$
$$= 3(4) - F_{21y} + 7.46 + 0.53 = 0$$

$F_{21y} = 19.99N$ * $F_{Dy} = 19.99N$

Link 3



$$\Sigma F_x = 13 - F_{23x} + 2(3) = 0$$

$F_{23x} = 19N$

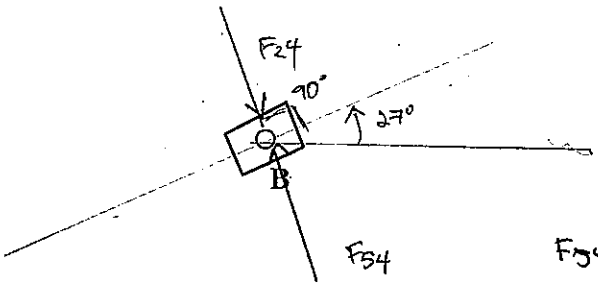
$F_{cx} = 19N$

105

+cw +ccw

Draw the free-body diagram of each link below

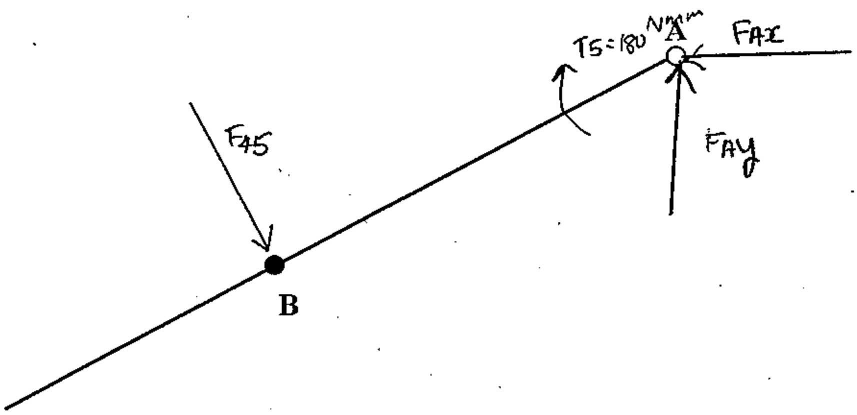
Link
4



$F_{24} = F_{54} = 0.6N$ (2force member)

$F_{54} = F_B = 0.6N$

Link
5



$F_{45}(360) - T_5 = 0$

$F_{45} = 0.6N$

$F_B = 0.6N$

END OF PAPER

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2018-2019

MA2002 – THEORY OF MECHANISM

November/December 2018

Time Allowed: $2^1/_2$ hours

SEAT NUMBER:

MATRICULATION NUMBER:

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **TWENTY-ONE (21)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is an **OPEN-BOOK** examination.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

For examiners:

Questions	1 (25)	2 (25)	3 (25)	4 (25)	**Total** (100)
Marks					

'06

Q1 (25 marks) /25

(a) A linkage mechanism for the digging equipment is shown in Figure 1. Note that points O_3ABC are all on the same link 3; Points $CDEFH$ are all on the same link 6.

(i) Identify only the binary, ternary and quaternary links in the linkage.

(ii) Determine its degrees of freedom.

(5 marks)

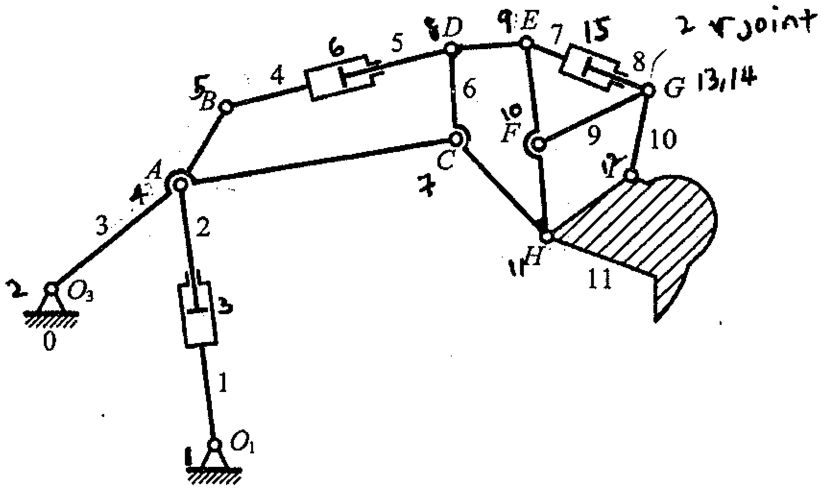


Figure 1

Solution

Binary links: 1, 2, 4, 5, 7, 8, 9, 10, 11

Ternary links: 0

Quaternary links: 3, 6

(ii) $nL = 12$

$nJ = 15$

$n''J = 0$

$$Dof = 3(nL-1) - 2(nJ) - n''J$$

$$= 3$$

Q1 (continued)

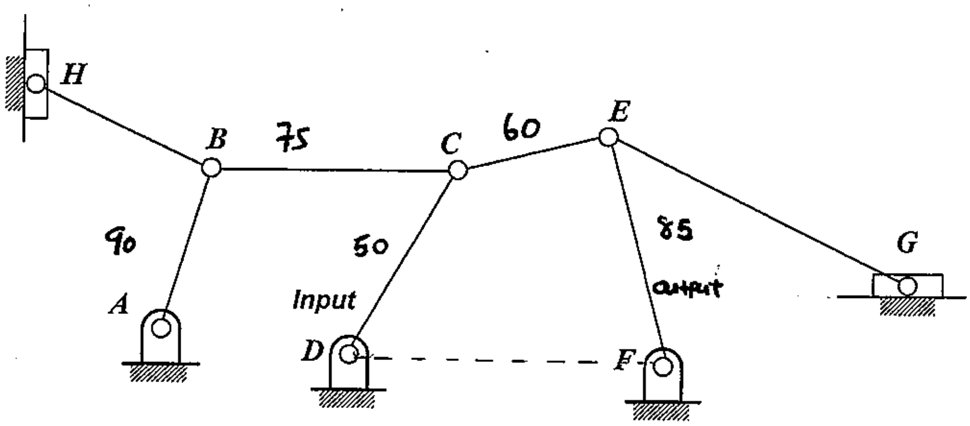
(b) A 10-bar linkage is shown in Figure 2. Link DC is the input and can make full 360° rotations. Link $DCEF$ and $ABCD$ form two 4-bar sub-linkages. The dimensions of the links are as follows.

AB = 90 mm, BC = 75 mm, DC = 50 mm, CE = 60 mm, EF = 85 mm, EG = 90 mm, BH = 100 mm

(i) Determine the range of length of Link DF in order to make Link EF full 360° rotations.

(ii) Determine the range of length of Link AD in order to make Link AB oscillatory movement without full rotation. Note that link dimensions are not drawn to scale.

(5 marks)



$L_{min} + L_{max} < L_a + L_B$

Figure 2

(i) DCEF is drag link mechanism. Fixed link shortest link.

DF + 85 < 50+60

DF < 25 #

(ii) ABCD is crank rocker,

if AB is Lmax,

50 + 90 < 75 + AD

AD > 65

if AD is Lmax,

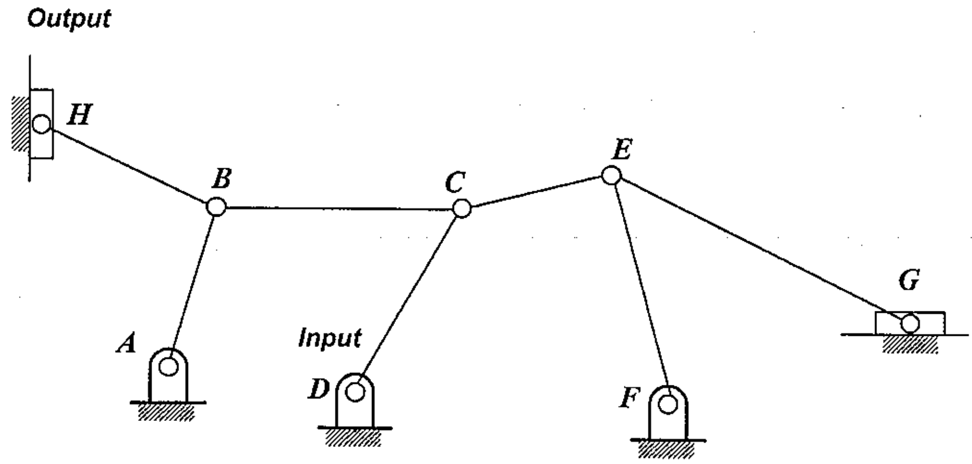
AD + 50 < 90+75

AD < 115.

Range: 65 < AD < 115 #

'07

Solution Q1(b)



Q1 (continued)

(c) A compound planetary gear train system is shown in Figure 3. Gear 1 is the input running at 200 rpm (cw) and Gear 10 is fixed. There are two carriers C1 and C2. The carrier C1 rotates at 100 rpm (ccw). Gear 4 and 5 are connected together. Gear 6 and 7 are connected together. The tooth numbers of the gears are given as:

$N_1 = 20$, $N_2 = 32$, $N_3 = 28$, $N_5 = 18$, $N_6 = 50$, $N_7 = 44$, $N_8 = 18$, $N_9 = 22$, $N_{11} = 40$.

All the gears are standard full-depth spur gears with a pressure angle of 20°. All gears have the same module of 5 mm.

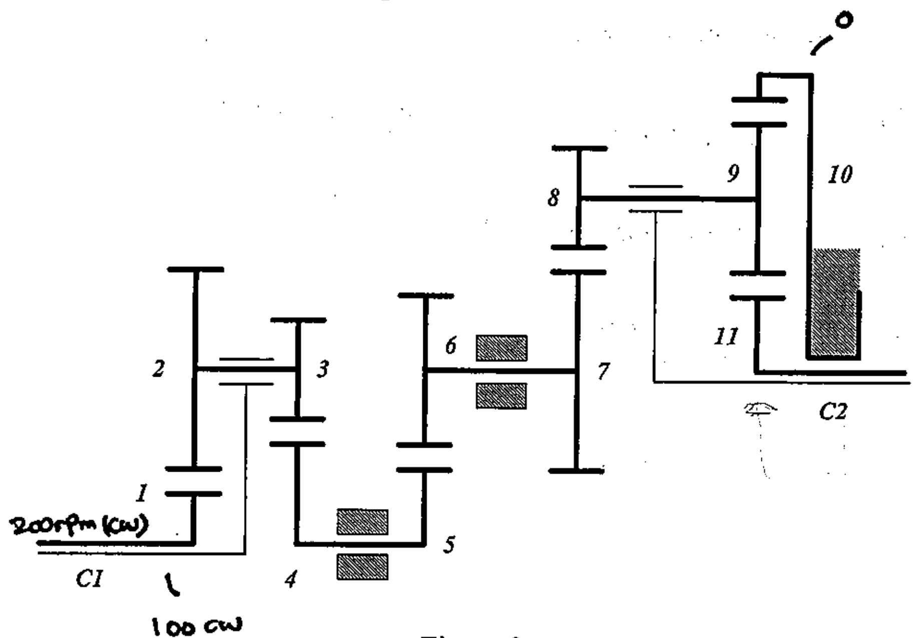
(i) Determine the speed and the number of teeth of gear 4 (N_4). (3 marks)

(ii) Determine the speed of gear 6. (3 marks)

(iii) Determine the number of teeth of gear 10 (N_{10}). (3 marks)

(iv) Determine the speed of Carrier C2. (3 marks)

(v) Determine the speed of gear 11. (3 marks)



ccw as +ve
cw as -ve.

Figure 3

Solution:

(i) $\frac{n_2 - 100}{-200 - 100} = -\frac{N_1}{N_2}$

$n_2 = 287.5\text{rpm. (ccw)}$

$n_2 = n_3 = 287.5\text{rpm (ccw)}$

input 1 has same axis of rotation as output 4.

$\therefore r_1 + r_2 = r_3 + r_4.$

$\cancel{\tfrac{1}{2}}\cancel{(5)}(20+32) = \cancel{\tfrac{1}{2}}\cancel{(5)}(28+N_4)$

$N_4 = 24$ #

$\frac{n_4 - 100}{287.5 - 100} = \frac{-28}{24}$, $n_4 = -118.75\text{rpm (cw)}$ #

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Solution Q1(c)

(ii) $n_4 = n_5 = -118.75\text{rpm}$

$$\frac{n_6}{n_5} = \frac{-N_5}{N_6}, \quad n_6 = -\left(\frac{18}{50}\right)(-118.75)$$
$$= 42.75\text{ rpm (ccw)} \#$$

(iii) PGT property: $N_R = N_S + 2N_P$

$$N_{10} = N_{11} + 2N_9$$
$$N_{10} = 40 + 2(22)$$
$$= 84 \#$$

(iv) $n_6 = n_7 = 42.75$ rpm (ccw)

$$\frac{n_8}{n_7} = -\frac{N_7}{N_8}, \quad n_8 = -104.5\text{rpm. (cw)}, \quad n_8 = n_9$$

taking N_{10} as input, N_9 as output,

$$\frac{-104.5 - n_{c2}}{0 - n_{c2}} = +\frac{84}{22}$$
$$-104.5 = -2.818\, n_{c2}$$
$$n_{c2} = 37.08\text{ rpm (ccw)} \#$$

(v) $$\frac{n_{11} - 37.08}{-104.5 - 37.08} = -\frac{22}{40}$$ using N_9 as input, N_{11} as output,

$$n_{11} = 114.95\text{rpm (ccw)} \#$$

Q2 (25 marks)

/25

A seven-bar 2-DOF-linkage mechanism is shown in Figure 4. The input slider 2 has a velocity V_A of 50 mm/s and acceleration A_A of 40 mm/s^2. The input slider 7 has a constant velocity V_E of 60 mm/s.

(a) Complete the velocity polygon according to the drawing scale given, and find the angular velocities ω_3, ω_4, ω_5, and ω_6 of links 3, 4, 5, and 6, and the velocities V_D of joint D and V_F of point F.

(10 marks)

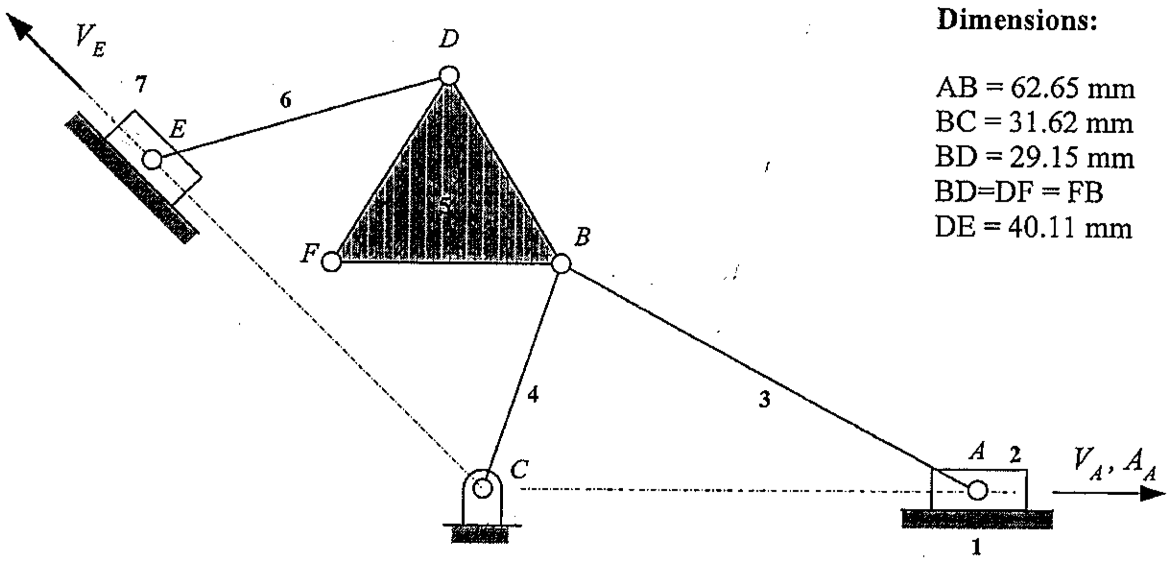
(b) Complete the acceleration polygon according to the drawing scale given, and find the angular accelerations α_3, α_4, α_5 and α_6 of links 3, 4, 5 and 6, and the accelerations A_D of joint D and A_F of point F.

(10 marks)

(c) The positions of slider 2 and slider 7 are given as input. Write down the vector loop equations of the linkage for analytical position analysis and identify the unknowns in the equations. (DO NOT solve the equations.)

(5 marks)

Note: Round off all your answers to **two decimal places** for accuracy.



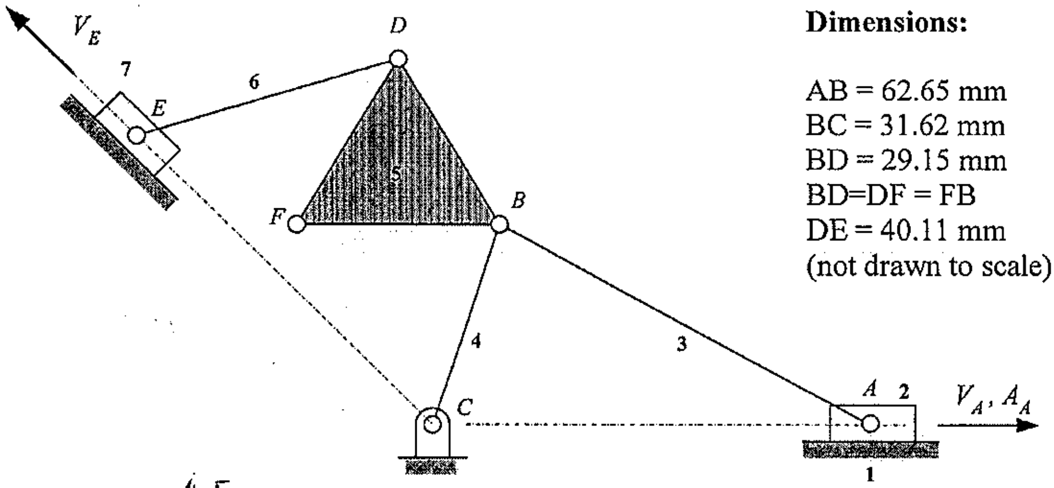
Dimensions:

AB = 62.65 mm
BC = 31.62 mm
BD = 29.15 mm
BD=DF = FB
DE = 40.11 mm

Figure 4

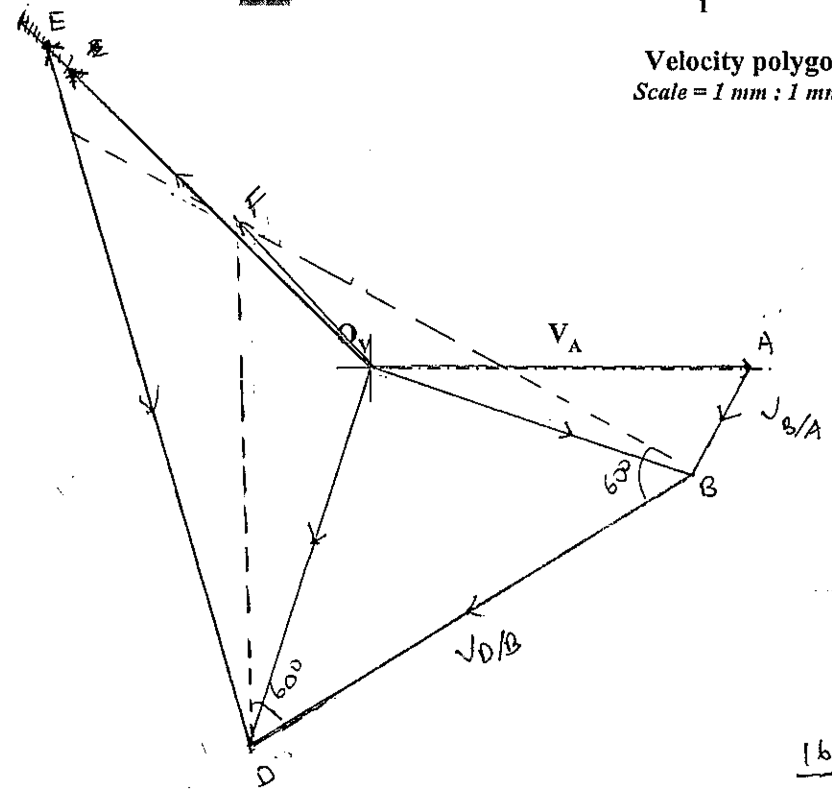
'09

Q2(a) Velocity Analysis



Dimensions:

AB = 62.65 mm
BC = 31.62 mm
BD = 29.15 mm
BD=DF = FB
DE = 40.11 mm
(not drawn to scale)



Velocity polygon
Scale = 1 mm : 1 mm/s

FILLING IN ANSWER HERE	
ω_3	0.255 rad/s (cw or ccw)*
ω_4	1.42 rad/s (cw or ccw)*
ω_5	2.37 rad/s (cw or ccw)*
ω_6	2.42 rad/s (cw or ccw)*
V_D	52 mm/s
V_F	26 mm/s

From graph, $\vec{V}_{B/A} = 16$ mm/s, $\omega_3 = \frac{16}{62.65} = 0.255$ rad/s (ccw)

$\vec{V}_B = 45$ mm/s, $\omega_4 = \frac{45}{31.62} = 1.42$ rad/s cw

$V_{D/B} = 69$ mm/s, $\omega_5 = \frac{69}{29.15} = 2.37$ rad/s (ccw)

$\vec{V}_{D/E} = 97$ mm/s, $\omega_6 = \frac{97}{40.11} = 2.42$ rad/s cw

$\vec{V}_D = 52$ mm/s

$\vec{V}_F = 26$ mm/s

Solution Q2(a) Velocity Analysis (continued)

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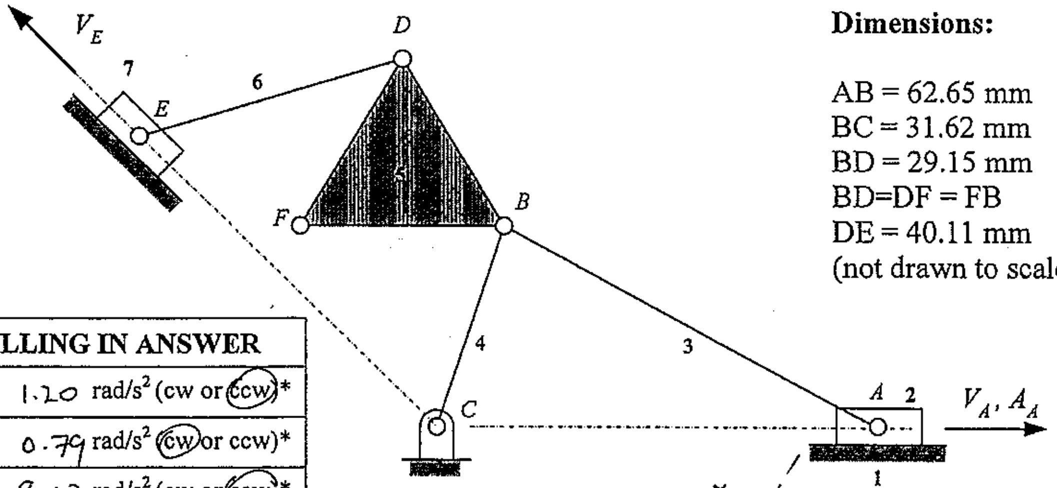
Q2(b) Acceleration Analysis

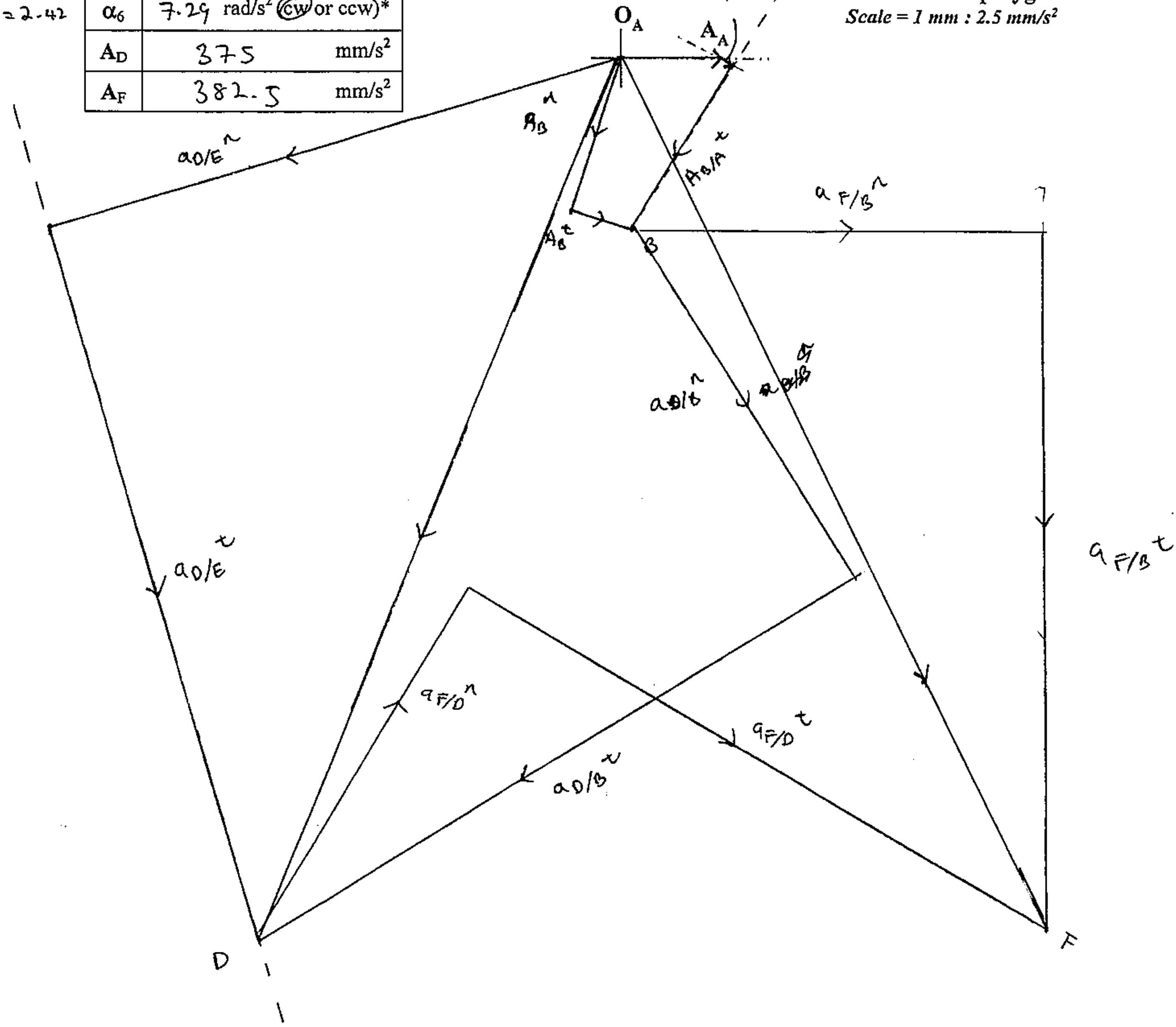


Dimensions:

AB = 62.65 mm
BC = 31.62 mm
BD = 29.15 mm
BD=DF = FB
DE = 40.11 mm
(not drawn to scale)

	FILLING IN ANSWER	
$\omega_3 = 0.255$	α_3	1.20 rad/s² (cw or ccw)*
$\omega_4 = 1.42$	α_4	0.79 rad/s² (cw or ccw)*
$\omega_5 = 2.37$	α_5	9.43 rad/s² (cw or ccw)*
$\omega_6 = 2.42$	α_6	7.29 rad/s² (cw or ccw)*
	A_D	375 mm/s²
	A_F	382.5 mm/s²

Acceleration polygon
Scale = 1 mm : 2.5 mm/s²



Solution Q2(b) Acceleration Analysis (continued)

$a_{B/A}^n = 0.255^2 \times 62.65 = 4.07\ mm/s^2$

From graph, $A_{B/A}^t = 75 mm/s^2$, $\alpha_3 = 1.20 rad/s^2$ (ccw)

$A_B^t = 25 mm/s^2$, $\alpha_4 = 0.79\ rad/s^2$ (cw)

$A_{D/B}^t = 275 mm/s^2$, $\alpha_5 = 9.43\ rad/s^2$ (ccw)

$A_{D/E}^t = 292.5 mm/s^2$, $\alpha_6 = 7.29\ rad/s^2$ (cw)

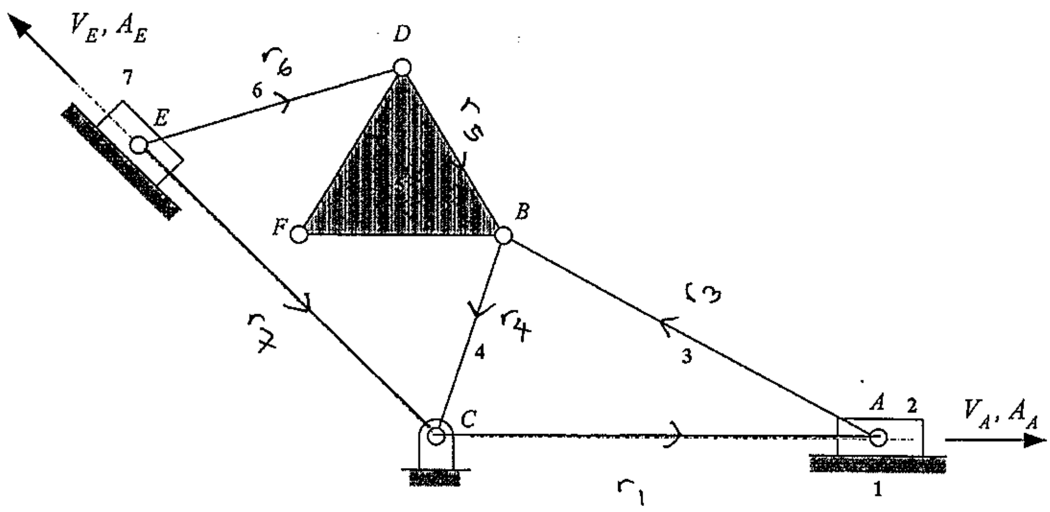
$A_D = 375\ mm/s^2$

$A_F = 382.5\ mm/s^2$.

Q2(c) Vector loop equations

Write down the vector loop equations of the linkage for analytical <u>position analysis</u> and identify the unknowns in the equations if the input positions of slider 2 and slide 7 are given. **(DO NOT solve the equations.)**

Solution (Indicate vectors clearly on the figure.)



① $r_1 + r_3 + r_4 = 0.$

unknowns : θ_3, θ_4.

② $r_6 + r_5 + r_4 = r_7.$

unknowns : $\theta_6, \theta_5, \theta_4$

Q3 (25 marks)

/25

(a) A cam with a flat-face follower rotates at a constant speed. The follower dwells at zero lift for the first 30° of the motion cycle, rises 40 mm with a parabolic motion for 180°, dwells for 30°, and returns with a cycloidal motion for 120°. **Please write your solutions in the tables on this page and the next page.**

(i) Write the mathematical expressions for the displacement (S) of the follower as a function of the cam rotation angle θ **with the θ unit being degree (°) only (not radian)**, for $30° \leq \theta \leq 210°$ and $240° \leq \theta \leq 360°$.

(3 marks)

(ii) Calculate the displacement for $60° \leq \theta \leq 180°$ and $270° \leq \theta \leq 330°$, at 30° intervals.

(3 marks)

(iii) A good sensor is used by an engineer to measure the acceleration of the follower. When the cam rotation angle is $61° < \theta < 119°$, the acceleration of the follower is measured to be 7.5 $\mu m/sec^2$ (0.0075 mm/sec^2). Calculate the constant rotation speed (ω) of the cam.

(4 marks)

Solution Q3(a): Please write your solutions in the tables on this page and the next page.

Note: Round off all your answers to **two decimal places** for accuracy.

(i)

θ	Follower displacement (S) as a function of the cam rotation angle θ **with the θ unit being degree (°) only (not radian)**
$30° \leq \theta \leq 120°$	$S = \frac{2(40)}{(180°)^2}(\theta - 30)^2$
$120° \leq \theta \leq 210°$	$S = -40 + \frac{4(40)}{180°}(\theta - 30) - \frac{2(40)}{(180)^2}(\theta - 30)^2$
$240° \leq \theta \leq 360°$	$S = 40\left(\frac{360 - \theta}{120}\right) - \frac{40}{2\pi}\sin\left(2\pi\left[\frac{360-\theta}{120}\right]\right)$

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Solution Q3(a) (continued)

(ii)

θ	S (mm)
60°	2.22
90°	8.88
120°	~~15~~ 20.00
150°	31.11
180°	37.78
270°	36.37
300°	20.00
330°	3.63

Note: Round off all your answers to **two decimal places** for accuracy.

(iii)

ω (rad/s)
1.23

Note: Round off all your answers to **two decimal places** for accuracy.

(iii) $S = \frac{80}{180^2}(\theta - 30)^2$

$\dot{S} = \frac{160}{180^2}(\theta - 30)\,\omega$

$\ddot{S} = \frac{160}{180^2}\omega^2 + \frac{160}{180^2}(\theta - 30)\,\alpha$

$= \frac{160}{180^2}\omega^2$, since $\alpha = 0$.

$0.0075 = \frac{160}{180^2}\omega^2$

$\omega = 1.2323$ rad/s

≈ 1.23 rad/s.

Solution Q3(a) (continued)

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Q3 (continued)

(b) Figure 5 shows a planar mechanism in static equilibrium with P = 10 N. Neglecting frictional and gravitational forces, calculate

(i) the joint force F_{Ax} at A, (2 marks)

(ii) the joint force F_B at B, (3 marks)

(iii) the joint force F_C at C, and (6 marks)

(iv) the required input torque T_4 applied to link 4. (4 marks)

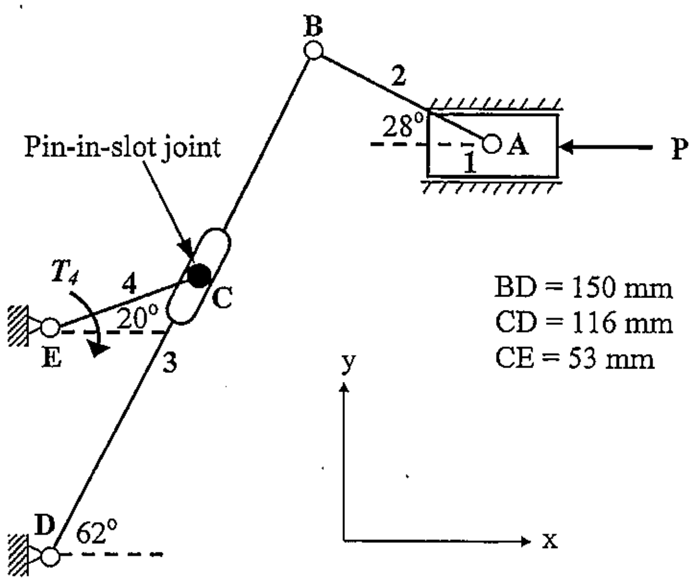


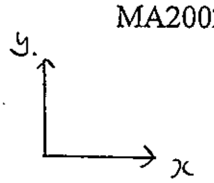
Figure 5

Solutions:

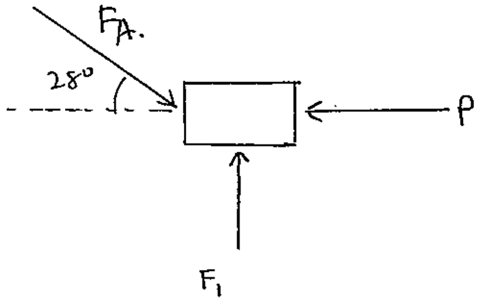
FILLING IN ANSWERS HERE		
F_{Ax}	10	N
F_B	11.3	N
F_C	12.11	N
T_4	641.8	Nmm

(Note: Round off all your answers to **two decimal places** for accuracy.)

Solution Q3(b) (continued)



FBD link 1.

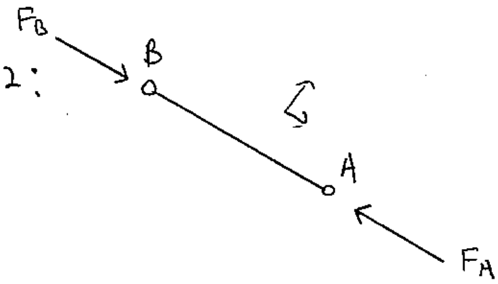


$\Sigma F_x = 0$

$F_{Ax} - P = 0$

$F_{Ax} = P = 10N.$

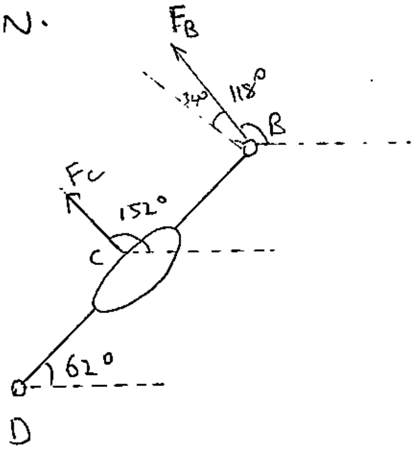
FBD link 2:

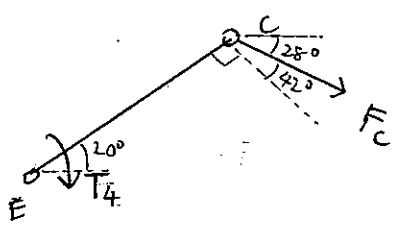


$F_A = \dfrac{F_{Ax}}{\cos 28^\circ} = 11.3N.$

$F_A = F_B = 11.3N.$

FBD link 3:





$\Sigma M_D = 0.$

$-F_C\, DC - F_B \cos 34^\circ\, DB = 0$

$F_C = -\left(\dfrac{F_B \cos 34^\circ\, DB}{DC}\right)$

$= -12.11\ N$

$T_4 + F_c \cos 42^\circ\, EC = 0$

$T_4 = 477\ Nmm$ #

'14

Q4 (25 marks) /25

Figure 6 shows a planar mechanism with P = 50 N. The various link parameters are given as follows:

Link	Mass	Location of CG	Moment of inertia about CG	Acceleration at CG	Angular acceleration
1	m_1	A	0	$a_{1x}i$	0
2	massless	~	~	~	~
3	m_3	D	I_3	0	α_3 (cw)
4	m_4	E	I_4	0	α_4 (cw)

Note: "~" means that this parameter is either not applicable or not necessary for the solution.

Neglecting frictional and gravitational forces,

(a) Draw the FBD (free-body diagram) of each link (except the ground link) with inertial forces and inertial moments shown in FBDs using D'lambert Principle. (10 marks)

(b) Calculate the joint force F_{Ax} at A if m_1 =3 kg, a_{1x}=1.5 m/s^2. (3 marks)

(c) Calculate the joint force F_B at B. (3 marks)

(d) Calculate the joint force F_C at C if I_3 = 4 kgm^2, α_3 = 2 rad/s^2. (5 marks)

(e) Calculate the required input torque T_4 applied to link 4 if I_4 = 0.75 kgm^2, α_4 = 3 rad/s^2. (4 marks)

Note: Please write your solutions in the table on the next page and draw FBDs using the links given in the next few pages.

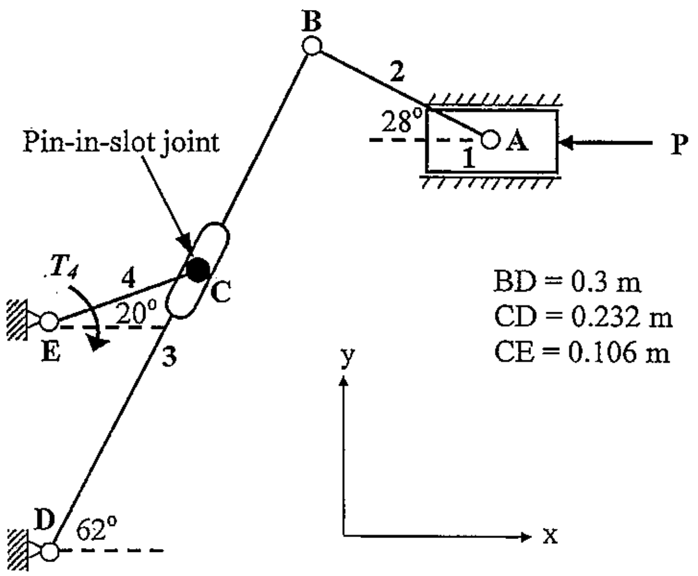


Figure 6

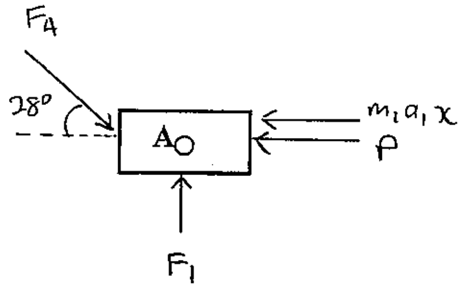
Solution:

FILLING IN ANSWERS HERE		
F_{Ax}	14.5	N
F_B	16.42	N
F_C		N
T_4		Nm

(Note: Round off all your answers to **two decimal places** for accuracy.)

Draw the free-body diagram of each link below

Link **1**



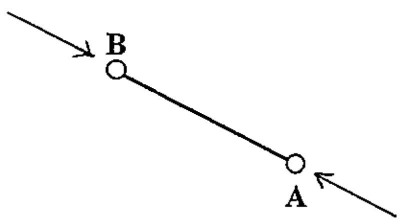
$\Sigma F_x = 0$

$F_{Ax} - P - m_1 a_{1x} = 0$

$F_{Ax} = 10 + 3(1.5)$

$= 14.5N.$

Link **2**



$$F_A = \frac{F_{Ax}}{\cos 28°} = 16.42N.$$

$F_A = F_B = 16.42N.$

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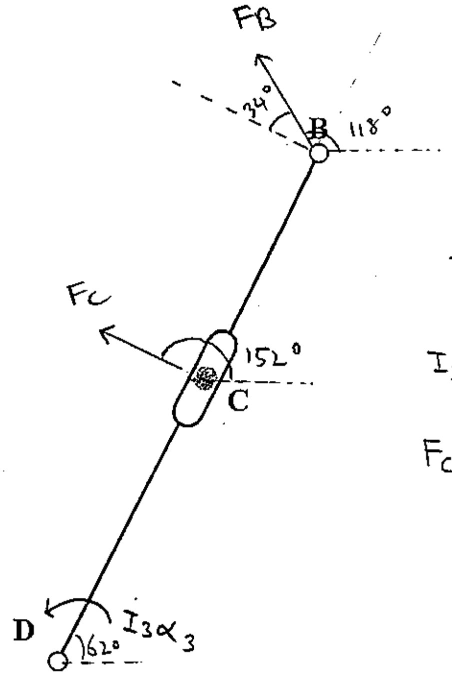
Draw the free-body diagram of each link below

Link 3



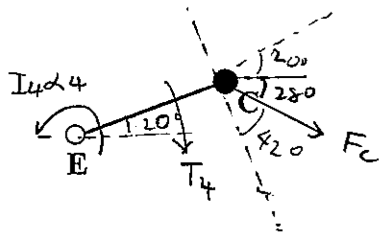
$$\Sigma M_D = 0$$

$$I_3\alpha_3 + F_c\,DC + F_B\cos34^\circ\,DB = 0$$

$$F_c = -\left(\frac{F_B\cos34^\circ\,DB - I_3\alpha_3}{DC}\right)$$

$$= 16.88\,N$$

Link 4



$$\Sigma M_E = 0.$$

$$T_4 + F_c\cos42^\circ\,EC = I_4\alpha_4$$

$$T_4 = 0.92\,Nm$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2019-2020

MA2002 – THEORY OF MECHANISM

November/December 2019 Time Allowed: $2^1/_2$ hours

SEAT NUMBER:

MATRICULATION NUMBER:

INSTRUCTIONS

1. This paper contains **FOUR** (4) questions and comprises **NINETEEN** (19) pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED-OPEN BOOK** examination. One double-sided A4 size reference sheet is allowed.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them all in at the end of the examination.

For examiners:

Questions	1 (25)	2 (25)	3 (25)	4 (25)	Total (100)
Marks					

Q1 (25 marks) /25

(a) A linkage mechanism is shown in Figure 1.

(i) Identify the types of links in the linkage.

(ii) Determine its degrees of freedom.

(6 marks)

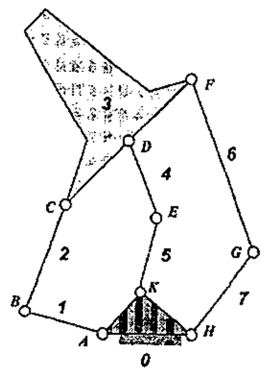


Figure 1

Solution

Binary links: 1, 2, 4, 5, 6, 7

Ternary links: 0, 3

Quaternary links:

ii. $DOF = 3(n-1) - 2n_J' - n_J''$

$= 3(8-1) - 2(9)$

$= 3$

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Q1 (continued)

(b) A compound planetary gear train system is shown in Figure 2. Gear 1 is the input running at 200 rpm (cw). Ring gears 5 and 10 are fixed. Gears 2 and 3 are connected together. Planetary gears 3, 4, 5 and 6, 7 are supported by carrier 11. Gears 7 and 8 are connected together. Planetary gears 8, 9, 10 are supported by carrier 12. The tooth numbers of the gears are given as:

$$N_1 = 20, N_2 = 80, N_3 = 40, N_4 = 16, N_6 = 24, N_7 = 32, N_8 = 20, N_9 = 30.$$

All the gears are standard full-depth spur gears with a pressure angle of 20°. All gears have the same module of 5 mm. Determine

ϕ

(i) the number of teeth of ring gear 5 (N_5); (3 marks)

(ii) the number of teeth of ring gear 10 (N_{10}); (3 marks)

$m = 5mm$

(iii) the speed of carrier 11 (output B); (3 marks)

(iv) the speed of gear 7; (3 marks)

(v) the speed of carrier 12 (output A); (3 marks)

(vi) the contact ratio between gear 3 and gear 4. (4 marks)

ccw (+)
cw (-)

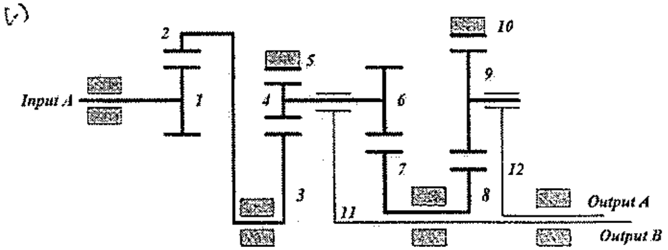


Figure 2

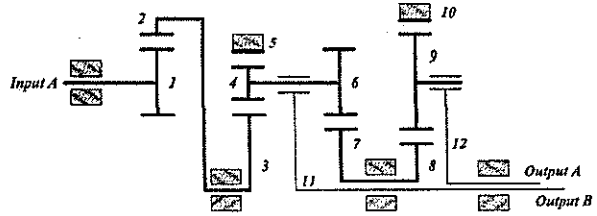
i. $N_5 = 2N_3 + N_4$
$= 2(40) + 16$
$= 96$

ii. $N_{10} = 2N_8 + N_9$
$= 2(20) + 30$
$= 70$

iii. $n_1 = 200rpm$ (cw)
$n_5 = 0rpm$
$\frac{n_5 - n_{11}}{n_1 - n_{11}} = \left(-\frac{N_1}{N_2}\right)\left(-\frac{N_3}{N_4}\right)\left(\frac{N_4}{N_5}\right)$
$\frac{0 - n_{11}}{-200 - n_{11}} = \left(-\frac{20}{80}\right)\left(-\frac{40}{16}\right)\left(\frac{16}{96}\right)$
$-\frac{8}{9}n_{11} = -\frac{200}{9}$
$n_{11} = 25rpm$ (ccw)

iv. $\frac{n_7 - n_{11}}{n_5 - n_{11}} = \left(-\frac{N_5}{N_4}\right)\left(-\frac{N_6}{N_7}\right)$
$\frac{n_7 - 25}{0 - 25} = \left(\frac{96}{16}\right)\left(-\frac{24}{32}\right)$
$n_7 = 137.5rpm$ (ccw)

Solution Q1(b)



V. $n_7 = n_8$

$\frac{n_8 - n_{12}}{n_{10} - n_{12}} = \left(-\frac{N_{10}}{N_9}\right)\left(-\frac{N_9}{N_8}\right)$

$\frac{137.5 - n_{12}}{0 - n_{12}} = \left(\frac{70}{30}\right)\left(-\frac{30}{20}\right)$

$n_{12} = 30.56rpm$

Vi. $C.R. = \frac{\sqrt{(r_4+a_4)^2 - r_4^2\cos^2\phi} - r_4\sin\phi}{P_b} + \frac{\sqrt{(r_3+a_3)^2 - r_3^2\cos^2\phi} - r_3\sin\phi}{P_b}$

$= \frac{\sqrt{(40+5)^2 - 40^2\cos^2 20°} - 40\sin 20°}{14.761} + \frac{\sqrt{(100+5)^2 - 100^2\cos^2 20°} - 100\sin 20°}{14.761}$

$= 0.749 + 0.857$

$= 1.606$

$P_b = m\pi\cos\phi$
$= 5(\pi)\cos(20°)$
$= 14.761$

$r_4 = \frac{5(16)}{2} = 40$

$r_3 = \frac{5(40)}{2} = 100$

$a_3 = a_4 = m = 5mm$

Q2 (25 marks)

/25

A six-bar 1-DOF Watt linkage mechanism is shown in Figure 3. The input link 2 rotates at angular speed ω_2 of 2 rad/s (cw) and angular acceleration α_2 of 2 rad/s^2 (cw).

(a) Complete the velocity polygon according to the drawing scale given, and find the angular velocities ω_3, ω_5, ω_6, of links 3, 5, and 6, and the velocities V_C, V_D, V_E of joint C, D, and E.

(10 marks)

(b) Complete the acceleration polygon according to the drawing scale given, and find the angular accelerations α_3, α_5, α_6 of links 3, 5, 6, and the accelerations A_C, A_D, A_E of joint C, D, and E.

(10 marks)

(c) Write down the vector loop equations of the linkage for analytical position analysis and identify the unknowns in the equations if the input position of link 2 is given. (DO NOT solve the equations.)

(5 marks)

Note: Round off all your answers to **two decimal places** for accuracy.

Dimensions:

AB = 50.00 mm
BC = 62.65 mm
BD = 30.00 mm
CD = 39.05 mm
DE = 24.22 mm
EF = 36.16 mm
d = 7.50 mm

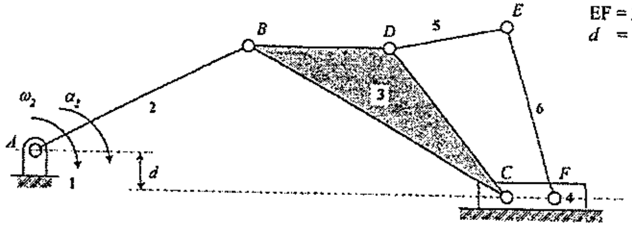
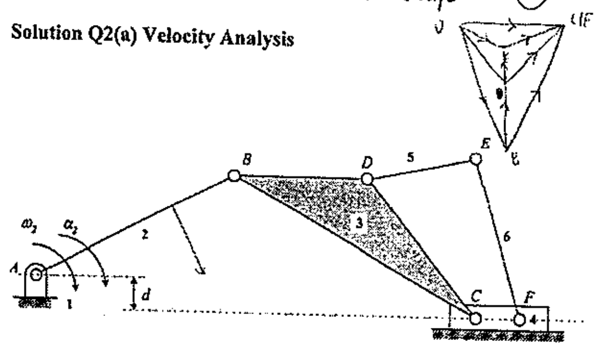


Figure 3

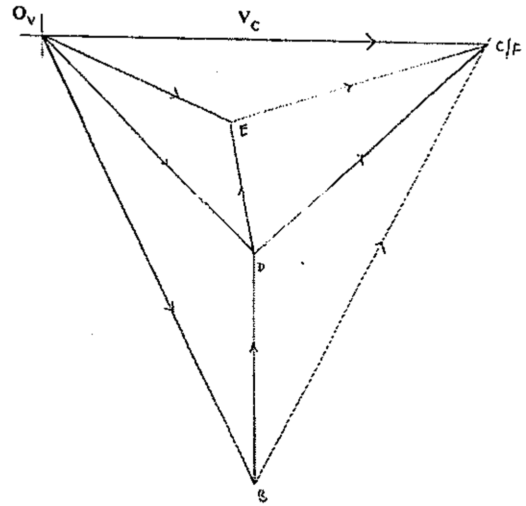
ω_2 = 2rad/s (Ans)

Solution Q2(a) Velocity Analysis



Dimensions:
AB = 50.00 mm
BC = 62.65 mm
BD = 30.00 mm
CD = 39.05 mm
DE = 24.22 mm
EF = 36.16 mm
d = 7.50 mm
(not drawn to scale)

Velocity polygon
Scale = 1 mm : 1 mm/s



FILLING IN ANSWER HERE		
ω_3	1.63	rad/s (cw or ccw)
ω_5	1.11	rad/s (cw or ccw)
ω_6	1.55	rad/s (cw or ccw)
V_C	93	mm/s
V_D	62	mm/s
V_E	43	mm/s

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$V = r\omega$

Solution Q2(a) Velocity Analysis (continued)

$V_B = 50(2)$
$= 100 \text{ mm/s}$

$V_{CB} = 102 \text{ mm/s}$

$\omega_3 = 102 \div 62.65$
$= 1.63 \text{ rad/s}$

$V_{ED} = 27 \text{ mm/s}$

$\omega_5 = 27 \div 24.22$
$= 1.11 \text{ rad/s}$

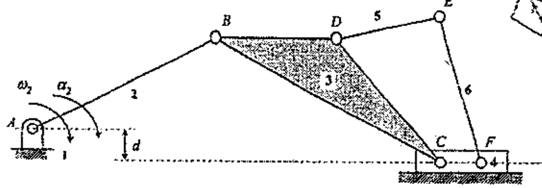
$V_{FE} = 56 \text{ mm/s}$

$\omega_6 = 56 \div 36.16$
$= 1.55 \text{ rad/s}$

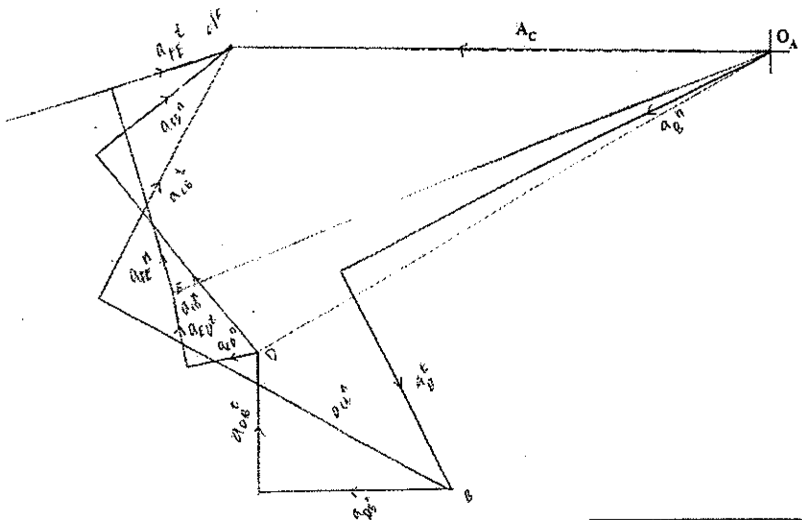
$\omega_2 = 2 \text{ rad/s}$ $\quad \alpha_2 = 2 \text{ rad/s}^2$ (not constant)

Solution Q2(b) Acceleration Analysis

Dimensions:
AB = 50.00 mm
BC = 62.65 mm
BD = 30.00 mm
CD = 39.05 mm
DE = 24.22 mm
EF = 36.16 mm
d = 7.50 mm
(not drawn to scale)



Acceleration polygon
Scale = 1 mm : 2 mm/s²



FILLING IN ANSWER	
α_3	1.85 rad/s² (cw or ccw)
α_5	1.32 rad/s² (cw or ccw)
α_6	1.49 rad/s² (cw or ccw)
A_C	224 mm/s²
A_D	246 mm/s²
A_E	268 mm/s²

$a^t = r\alpha \quad a^n = r\omega^2$

Solution Q2(b) Acceleration Analysis (continued)

$a_B^t = 50(2)$
$= 100\ mm/s^2\ (5)$

$a_B^n = 50(2)^2$
$= 200\ mm/s^2\ (10)$

$a_{CB}^n = 62.65(1.63)^2$
$= 166.45\ mm/s^2\ (2.3)$

$a_{DB}^n = 30(1.63)^2$
$= 79.71\ mm/s^2\ (4)$

$a_{DB}^t = 30(1.85)$
$= 55.5\ mm/s^2\ (2.8)$

$a_{CD}^n = 39.05(1.63)^2$
$= 103.75\ mm/s^2\ (5.2)$

$a_{CD}^t = 39.05(1.85)$
$= 72.24\ mm/s^2\ (3.6)$

$a_{ED}^n = 24.22(1.11)^2$
$= 29.84\ mm/s^2\ (1.5)$

$a_{FE}^n = 36.16(1.55)^2$
$= 86.87\ mm/s^2\ (4.3)$

$a_{CB}^t = 116\ mm/s^2$
$\alpha_3 = 116 \div 62.65$
$= 1.85\ rad/s^2$

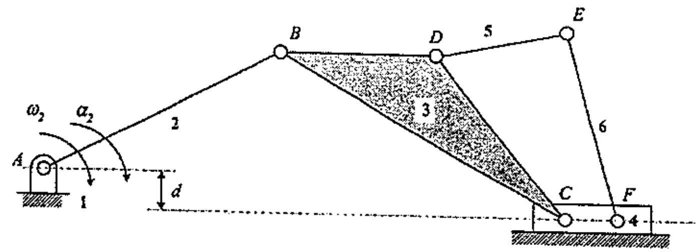
$a_{ED}^t = 32\ mm/s^2$
$\alpha_5 = 32 \div 24.22$
$= 1.32\ rad/s^2$

$a_{FE}^t = 54\ mm/s^2$
$\alpha_6 = 54 \div 36.16$
$= 1.49\ rad/s^2$

Q2(c) Vector loop equations

Write down the vector loop equations of the linkage for analytical <u>position analysis</u> and identify the unknowns in the equations if the input position of link 2 is given. **(DO NOT solve the equations.)**

Solution (Indicate vectors clearly on the figure.)



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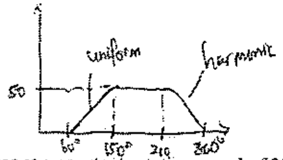

Q3 (25 marks)

/25

(a) A cam with a roller follower rotates at a constant rotation speed of 30 rpm. The follower dwells at zero lift for the first 60° of the motion cycle, rises 50 mm with a uniform motion for 90°, dwells for 60°, and returns with a harmonic motion for 150°. **Please write your solutions in the tables on this page and the next page.**

(i) Write the mathematical expressions for the displacement (S) of the follower as a function of the cam rotation angle θ **with the θ unit being degree (°) only (not radian)**, for $60° \leq \theta \leq 150°$ and $210° \leq \theta \leq 360°$.

(3 marks)

(ii) Calculate the displacement for $90° \leq \theta \leq 120°$ and $240° \leq \theta \leq 330°$, at 30° intervals.

(3 marks)

(iii) Calculate the maximum acceleration (a_{max}) of the follower in the return portion of the harmonic motion.

(4 marks)

Note: Round off all your answers to **two decimal places** for accuracy.

Solutions
(i)

$m = \frac{50-0}{150-60} = 5/9$

θ	Follower displacement (S) as a function of the cam rotation angle θ **with the θ unit being degree (°) only (not radian)**
$60° \leq \theta \leq 150°$	$S = \frac{5}{9}\theta - \frac{100}{3}$ formula ($y = mx + c$)
$210° \leq \theta \leq 360°$	$S = \frac{50}{2}\left(1 - \cos\frac{180°(360° - \theta)}{150°}\right)$

Solutions of Q3(a) (continued)
(ii)

θ	S (mm)
90°	~~[illegible]~~ 16.67
120°	33.33
240°	45.23
270°	32.73
300°	17.27
330°	4.77

Note: Round off all your answers to **two decimal places** for accuracy.

(iii)

a_{max} (mm/sec²)

Note: Round off all your answers to **two decimal places** for accuracy.

$S = \frac{50}{2}\left(1 - \cos\frac{180°(360° - \theta)}{150°}\right)$

$= 25 - 25\cos(432° - 1.2\theta)$

$\dot{S} = \frac{d}{d\theta}\left[25 - 25\cos(432° - 1.2\theta)\right]$

$= 25(-1.2)\omega\sin(432° - 1.2\theta)$

$= -30\omega\sin(432° - 1.2\theta)$

$\ddot{S} = -30(-1.2)\cos(432° - 1.2\theta)$

$= 36\omega^2\cos(432° - 1.2\theta)$

(Jerk) $\dddot{S} = -36(-1.2)\sin(432° - 1.2\theta)$

[highest point] $= 43.2\omega^3\sin(432° - 1.2\theta)$

Solutions of Q3(a) (continued)

Q3 (continued)

(b) Figure 4 shows a planar mechanism in static equilibrium with input torque $T_1 = 500$ Nmm applied to link 1. Link 3 is a ternary link with 2 revolute and 1 sliding joints, and links 2 and 5 are sliders. Neglecting frictional and gravitational forces, calculate

(i) joint force F_{21} at A, which is the force from link 2 to link 1. (5 marks)

(ii) joint force F_{43} at B, which is the force from link 4 to link 3. (7 marks)

(iii) the required force Q applied to link 5. (3 marks)

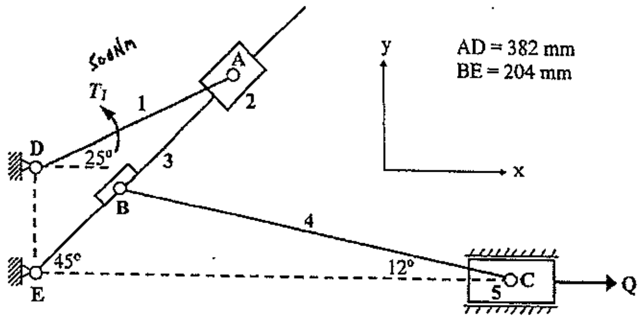

Figure 4

Solutions

FILLING IN ANSWERS HERE		
F_{21}	3.60	N
F_{43}	8.01	N
Q	7.83	N

(Note: Round off all your answers to **two decimal places** for accuracy.)

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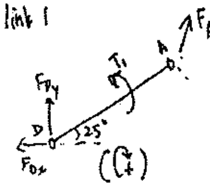
Solutions of Q3(b) (continued)

(i) link 1

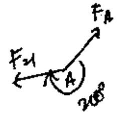


$\Sigma M_D = 0$.

$-T_1 - F_A \cos 110^\circ (382) = 0$

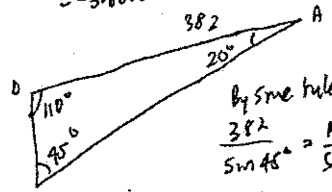
$= 500 - F_A \cos 110^\circ (382) = 0$

$F_A = -3.83N$



$F_{21} = F_A \cos 200^\circ$

$= -3.60N$

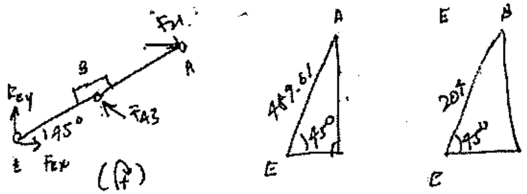


By sine rule,

$\frac{382}{\sin 45^\circ} = \frac{AE}{\sin 115^\circ}$

$AE = 489.61mm$

(ii) link 3



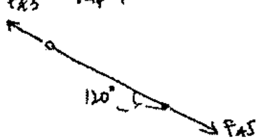
$\Sigma M_E = 0$

$(F_{21} \cos 25^\circ)(489.61 \sin 45^\circ) - (F_{43} \cos 12^\circ)(204 \sin 45^\circ) = 0$

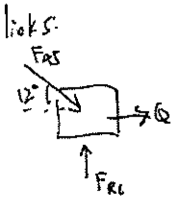
$F_{43} = 8.01N$

(iii) $F_{45} = F_{43} = 8.01N$

link 4



link 5



$\Sigma F_x = 0$

$F_{45} \cos 12^\circ + Q = 0$

$Q = -7.83N$

Q4 (25 marks)

/25

Figure 5 shows a planar mechanism with input torque $T_1 = 2.5$ Nm applied to link 1. Link 3 is a ternary link with 2 revolute and 1 sliding joints, and links 2 and 5 are sliders. The various link parameters are given as follows:

Link	Mass	Location of CG	Moment of inertia about CG	Acceleration at CG	Angular acceleration
1	m_1	D	I_1	0	α_1 (ccw)
2	massless	~	~	~	~
3	m_3	E	I_3	0	α_3 (ccw)
4	massless	~	~	~	~
5	m_5	C	0	$-a_{5x}i$	0

Note: "~" means that this parameter is either not applicable or not necessary for the solution.

Neglecting frictional and gravitational forces,

(a) Draw the FBD (free-body diagram) of each link (except the ground link) with inertial forces and inertial moments shown in FBDs using D'Alembert's Principle.
(10 marks)

(b) Calculate joint force F_{21} at A, which is the force from link 2 to link 1, if $I_1 = 0.5$ kgm^2, $\alpha_1 = 4$ rad/s^2.
(5 marks)

(c) Calculate joint force F_{43} at B, which is the force from link 4 to link 3, if $I_3 = 0.6$ kgm^2, $\alpha_3 = 0.4$ rad/s^2.
(7 marks)

(d) Calculate the required force Q applied to link 5, if $m_5 = 2$ kg, $a_{5x} = 2.14$ m/s^2.
(3 marks)

Note: Please write your solutions in the table on the next page and draw FBDs using the links given in the next pages.

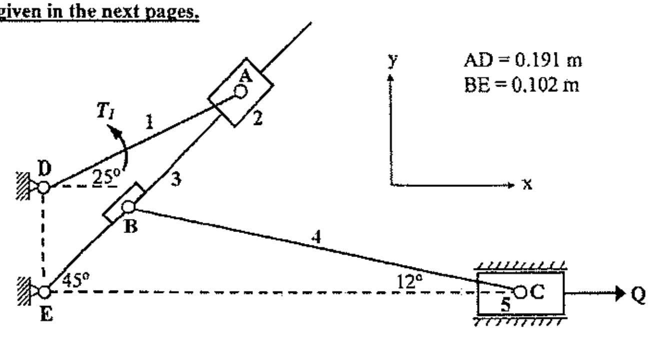


Figure 5

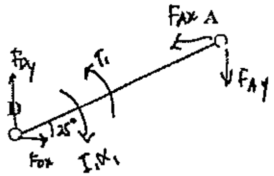
Solutions

FILLING IN ANSWERS HERE	
F_{21}	N
F_{43}	N
Q	N

(Note: Round off all your answers to **two decimal places** for accuracy.)

Draw the free-body diagram of each link below.

Link 1



$\sum M_D = 0$ (Q)

$I_1\alpha_1 - T_1 + F_{Ay} AD\cos 25^\circ - F_{Ax} AD\sin 25^\circ = 0$

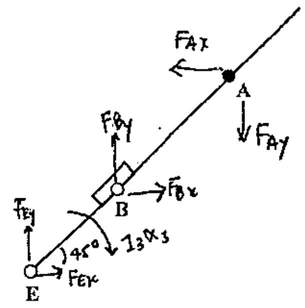
$2 - 2.5 + F_{Ay}(0.191\cos 25^\circ) - F_{Ax}(0.191\sin 25^\circ) = 0$

$0.17F_{Ay} - 0.08F_{Ax} = 0.5$ —(1)

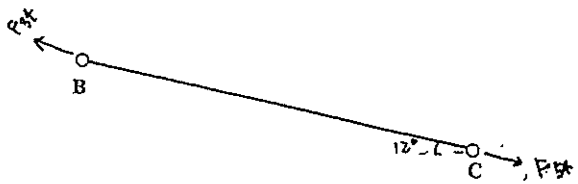
Link 2

Solutions of Q4 (continued)
Draw the free-body diagram of each link below.

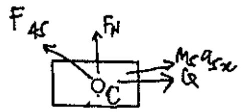
Link 3



Link 4



Link 5



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Solutions of Q4 (continued)

From link ③:

$\Sigma M_E = 0 \quad (\circlearrowleft)$

$I_3 \alpha_3 + F_{Bx} BE \sin 45° - F_{By} BE \cos 45° - F_{Ax} AE \sin 45° + F_{Ay} AE \cos 45° = 0$

$0.24 + F_{Bx}(0.102 \sin 45°) - F_{By}(0.102 \cos 45°) - F_{Ax}(0.245 \sin 45°) + F_{Ay}(0.245 \cos 45°) = 0$

$0.07 F_{Bx} - 0.07 F_{By} - 0.17 F_{Ax} + 0.17 F_{Ay} = -0.24 \quad -(7)$

From link ④:

$\Sigma F_x = 0$ $\qquad\qquad$ $\Sigma F_y = 0$

$-F_{Bx} - F_{Cx} = 0 \quad -(3)$ $\qquad$ $-F_{By} - F_{Cy} = 0 \quad -(4)$

From link ⑤:

$\Sigma F_x = 0$ $\qquad\qquad$ $\Sigma F_y = 0$

$-F_{Cx} + m_B a_{Bx} + Q = 0$ $\qquad$ $-F_{Cy} + F_N = 0 \quad -(6)$

$-F_{Cx} + 4.28 + Q = 0 \quad -(5)$

End of Paper