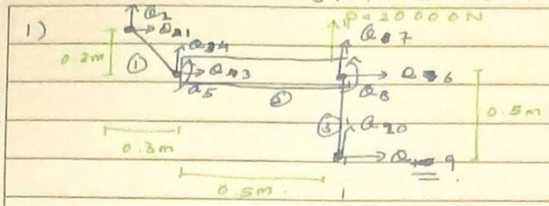
MA 4702: Aircraft Structure II : Sem 1 2021/2022.

1)

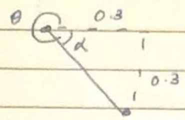


$EA = 1 \times 10^6$ N

$EI = 1 \times 10^5$ N.

a) For Truss ①

$\alpha = 45°$, $\theta = 360 - 45 = 315°$



$l = \cos 315 = 0.707$ $\quad m = \sin 315 = -0.707$

$l_e = \sqrt{0.3^2 + 0.3^2} = 0.424$, $l^2 = 0.500$, $m^2 = 0.500$

$lm = -0.500$

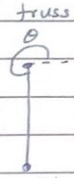
$$k^{①} = \frac{EA}{l_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ & m^2 & -lm & -m^2 \\ \text{Sym} & & l^2 & lm \\ & & & m^2 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1.18 & -1.18 & -1.18 & 1.18 \\ & 1.18 & 1.18 & -1.18 \\ \text{Sym} & & 1.18 & -1.18 \\ & & & 1.18 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \times 10^6$$

For Beam ②

$$k^{②} = \frac{EI}{l^2}\begin{bmatrix} 12 & 6l & -12 & 6l \\ & 4l^2 & -6l & 2l^2 \\ & & 12 & -6l \\ & & & 4l^2 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} 4 & 5 & 7 & 8 \end{array} \\ \begin{bmatrix} 96 & 24 & -96 & 24 \\ & 8 & -24 & 4 \\ & & 96 & -24 \\ & & & 8 \end{bmatrix} \end{array} \begin{array}{c} 4 \\ 5 \\ 7 \\ 8 \end{array} \times 10^5$$

$l = 0.5$m

For truss ③, $\theta = 270°$.



$l = \cos 270 = 0$, $m = \sin 270 = -1$, $l_e = 0.5$m

$l^2 = 0$, $m^2 = 1$, $lm = \cancel{-1}\ 0$

$$k^{③} = \frac{EA}{l_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ & m^2 & -lm & -m^2 \\ \text{Sym} & & l^2 & lm \\ & & & m^2 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} 6 & 7 & 9 & 10 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ \text{Sym} & & 0 & 0 \\ & & & 1 \end{bmatrix} \end{array} \begin{array}{c} 6 \\ 7 \\ 9 \\ 10 \end{array} \times 10^6$$

Assembling the Global stiffness matrix [10 x 10]

$$K = \begin{bmatrix} 1.18 & -1.18 & -1.18 & 1.18 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 1.18 & 1.18 & -1.18 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 1.18 & -1.18 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 10.78 & 2.4 & 0 & -9.6 & 2.4 & 0 & 0 \\ & & & & 0.8 & 0 & -2.4 & 0.4 & 0 & 0 \\ & & & & & 0 & 0 & 0.1 & 0 & 0 \\ & & & 0 & & & 10.6 & -2.4 & 0 & -1 \\ & & & & & & & 0.8 & 0 & 0 \\ & & & & & & & & 0 & 0 \\ & & & & & & & & & 1 \end{bmatrix} \times 10^6$$

b) By using $\{k\}\{x\} = \{F\}$

$$\{F\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 10\,000 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ R_9 \\ R_{10} \end{Bmatrix}.$$

By using Boundary Conditions

$Q_1 = Q_2 = Q_3 = Q_9 = Q_{10}$ (BC).

$Q_6 = Q_8 = 0$ (Symmetry).

$$10^6 \times \begin{bmatrix} 10.78 & 2.4 & -9.6 \\ 2.4 & 0.8 & -2.4 \\ -9.6 & -2.4 & 10.6 \end{bmatrix} \begin{Bmatrix} Q_4 \\ Q_5 \\ Q_7 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 10\,000 \end{Bmatrix}$$

Solving this Simultaneous Equation

$Q_4 = 0.00374\text{m}$ $\quad Q_5 = 0.00552\text{Nm}$ $\quad Q_7 = 0.00858\text{m}$ #

c).

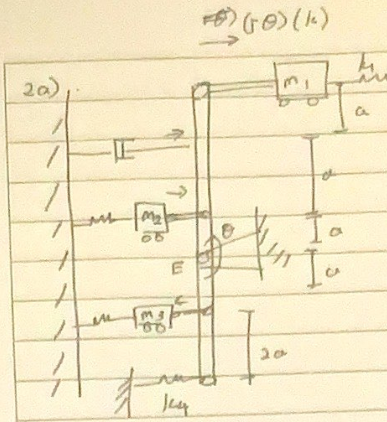
$$\begin{bmatrix} 10.78 & 2.4 & -9.6 \\ 2.4 & 0.8 & -2.4 \\ -9.6 & -2.4 & 10.6 \end{bmatrix} \begin{Bmatrix} Q_4 \\ Q_5 \\ 0.01 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ P/2 \end{Bmatrix}.$$

Solving this Simultaneous Equation

$10^{-6} \times P/2 = 0.0179.$

$\therefore P = 35\,800\text{N}$ #

2a)



Not Drawn to Scale (ACW +ve)

$k = 2\times10^4$ N/m $\quad k_4 = 1\times10^4$ N/m

Taking moment at E.

$\Sigma M_o = -9k_1\theta - k_2\theta - k_3\theta - 9k_4\theta$

$= (m_1)(3)^2\ddot\theta + (m_2)\ddot\theta + m_3\ddot\theta = \Sigma J_o\ddot\theta$

$47\ddot\theta = -310\,000\,\theta$

~~310 000~~ $47\ddot\theta + 310\,000\,\theta = 0$

$W_n = \sqrt{\frac{K}{M}}$

$= \sqrt{\frac{310000}{47}} = 81.21$ rad/s #

b) when $c = 50$ N·s/m.

Taking moment at E,

$\Sigma M_o = -9k_1\theta_4 - 4c_1\dot\theta - k_2\theta - k_3\theta - 9k_4\theta = 47\ddot\theta$

$\therefore 47\ddot\theta + 200\dot\theta + 310\,000\theta = 0 \quad (M\ddot\theta + c\dot\theta + K\theta = 0)$.

$\zeta = \frac{c}{2\sqrt{mk}} = \frac{200}{2\sqrt{47\times310000}} = 0.0262$

By using $x(t) = Xe^{-\zeta W_n t}\sin\left[\sqrt{1-\zeta^2}\,W_n t + \phi\right]$.

$W_d = \sqrt{1-\zeta^2}\,W_n = \sqrt{1-0.0262^2}\,(81.21)$

$= 81.18$ rad/s

$\Theta = \sqrt{\Theta_o^2 + \left(\frac{\dot\Theta_o + \zeta W_n\Theta_o}{W_d}\right)^2} = \sqrt{0.5^2 + \left[\frac{0.1+(0.0262)(81.21)(0.5)}{81.18}\right]^2}$

$= 0.500$.

$\phi = \tan^{-1}\left[\frac{\sqrt{1-\zeta^2}\,W_n\Theta_o}{\dot\Theta_o + \zeta W_n\Theta_o}\right] = \tan^{-1}\left[\frac{\sqrt{1-0.0262^2}\,(81.21)(0.5)}{0.1+(0.0262)(81.21)(0.5)}\right]$

$= 88.36^\circ$.

$\Theta(t) = 0.500e^{-2.13t}\sin\left[81.18t + 88.36^\circ\right]$ #.

c) Since $\Theta_o = \frac{F_o l}{k} = x_o \cdot l = 0.5\times3 = 1.5$ rad.

$\Theta = \Theta_o\frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = 1.5\frac{1}{\sqrt{(1-0.615^2)^2+(2(0.0262)r)^2}}$ $\quad r = \frac{\omega}{\omega_n} = \frac{50}{81.21}$

$= 0.615$.

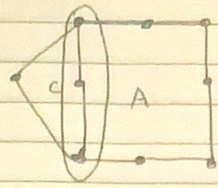
$= 3.88$

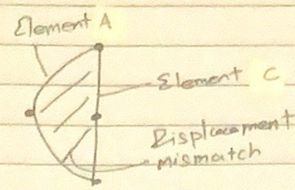
$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$

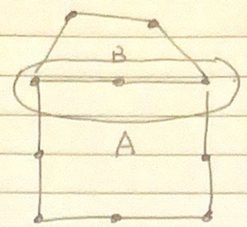
$= \tan^{-1}\left(\frac{2\times0.0262\times0.615}{1-0.615^2}\right) = 1.85^\circ$.

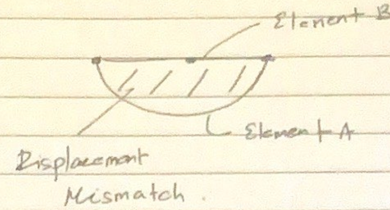
$\therefore \Theta(t) = 3.88\sin(50t - 1.85^\circ)$ #.

3a)









b) Jacobian Matrix.

$x_1 = 1.1 \qquad x_2 = 8.8 \qquad x_3 = 3.3 \qquad x_4 = 1.10$

$y_1 = 8.8 \qquad y_2 = 8.8 \qquad y_3 = 11.0 \qquad y_4 = 16.5$

$J_{11} = \frac{1}{4}\left[-(1-\eta)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+\eta)x_4\right]$.

$= \frac{1}{4}\left[-1.1 + 8.8 + 3.3 - 1.1 + 1.1\eta - 8.8\eta + 3.3\eta - 1.1\eta\right]$.

$= \frac{1}{4}\left[9.9 - 5.5\eta\right]$.

$J_{12} = \frac{1}{4}\left[-(1-\eta)y_1 + (1-\eta)y_2 + (1+\eta)y_3 - (1+\eta)y_4\right]$.

$= \frac{1}{4}\left[\cancel{-8.8} + \cancel{8.8} + 11.0 - 16.5 + \cancel{8.8\eta} - \cancel{8.8\eta} + 11.0\eta - 16.5\eta\right]$

$= \frac{1}{4}\left[-5.5 - 5.5\eta\right]$.

$J_{21} = \frac{1}{4}\left[-(1-\xi)x_1 - (1+\xi)x_2 + (1+\xi)x_3 + (1-\xi)x_4\right]$.

$= \frac{1}{4}\left[\cancel{-1.1} - 8.8 + 3.3 + \cancel{1.1} + \cancel{1.1\xi} - 8.8\xi + 3.3\xi - \cancel{1.1\xi}\right]$

$= \frac{1}{4}\left[-5.5 - 5.5\xi\right]$.

$J_{22} = \frac{1}{4}\left[-(1-\xi)y_1 - (1+\xi)y_2 + (1+\xi)y_3 + (1-\xi)y_4\right]$.

$= \frac{1}{4}\left[-8.8 - 8.8 + 11.0 + 16.5 + 8.8\xi - 8.8\xi + 11.0\xi - 16.5\xi\right]$.

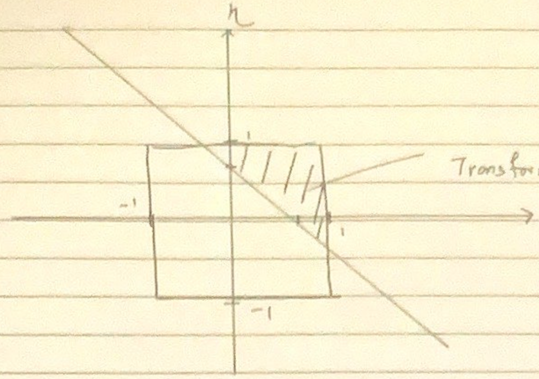
$= \frac{1}{4}\left[9.9 - 5.5\xi\right]$.

$$[J] = \begin{bmatrix} \frac{1}{4}[9.9 - 5.5\eta] & \frac{1}{4}[-5.5 - 5.5\eta] \\ \frac{1}{4}[-5.5 - 5.5\xi] & \frac{1}{4}[9.9 - 5.5\xi] \end{bmatrix}.$$

$\det[J] = \frac{1}{16}\left[(9.9 - 5.5\eta)(9.9 - 5.5\xi) - (5.5 + 5.5\eta)(5.5 + 5.5\xi)\right]$.

$= \frac{1}{16}\left[98.01 - 54.45\eta - 54.45\xi - 30.25 - 30.25\eta - 30.25\xi\right]$.

$= 4.235 - 5.297\eta - 5.297\xi$ #.

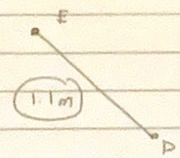


when $\eta = 0$, $\xi = 0.801$

$\xi = 0$, $\eta = 0.801$

Transformation not valid as det [J] is not positive as indicated in the shaded Region

c)

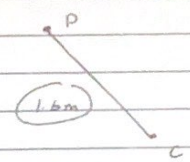


$t = 0.005m$

$T_{x1} = 0$, $T_{y1} = -225\ N/m^2$

$= T_{x2}$ $\quad T_{y2} = -225\ N/m^2$

$$f_e^{(1)} = \frac{t_e L_{ED}}{6}\begin{Bmatrix} 2T_{x1} + T_{x2} \\ 2T_{y1} + T_{y2} \\ T_{x1} + 2T_{x2} \\ T_{y1} + 2T_{y2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -0.61875 \\ 0 \\ -0.61875 \end{Bmatrix} N$$



$t = 0.005m$

$$f_e^{(2)} = \frac{t_e L_{DC}}{6}\begin{Bmatrix} 0 \\ 2T_{y1} + T_{y2} \\ 0 \\ T_{y1} + 2T_{y2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -0.9 \\ 0 \\ -0.9 \end{Bmatrix} N$$

$$f_T = \begin{Bmatrix} 0 \\ -0.61875 \\ 0 \\ -1.51875 \\ 0 \\ -0.9 \end{Bmatrix} N.$$

$\therefore F_D = 1.51875\ N$

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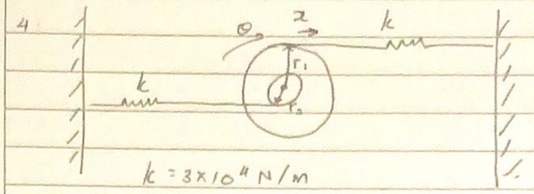
4.



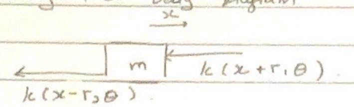
Assume $x > r_1\theta$ & $r_2\theta$.

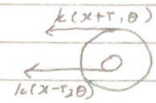
$k = 3\times10^4$ N/m

$r_1 = 0.3$m, $r_2 = 0.2$m

$J_0 = 1.0$kgm^2, $M_0 = 20$kg.

By Free Body Diagram





$m\ddot{x} = -k(x+r_1\theta) - k(x-r_2\theta)$ — (1) $J_0\ddot{\theta} = -r_1 k(x+r_1\theta) + r_2 k(x-r_2\theta)$ — (2)

From (1)

$m\ddot{x} + 2kx - r_1k\theta + r_2k\theta = 0$

$20\ddot{x} + 60\,000x + 3000\,\theta = 0$.

From (2).

$J_0\ddot{\theta} + r_1kx + r_1^2k\theta - r_2kx + r_2^2k\theta = 0$

$\ddot{\theta} + 9000x + 2700\theta - 6000x + 1200\theta = 0$.

$\ddot{\theta} + 3000x + 3900\theta = 0$.

$$\begin{bmatrix} 20 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 60\,000 & 3000 \\ 3000 & 3900 \end{bmatrix}\begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}.$$

$$\begin{bmatrix} 60\,000 - 20\omega^2 & 3000 \\ 3000 & 3900 - \omega^2 \end{bmatrix}\begin{bmatrix} X \\ \Theta \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$(60\,000 - 20\omega^2)(3900 - \omega^2) - (3000)(3000) = 0$

$2.34\times10^8 - 138\,000\omega^2 + 20\omega^4 - 9\times10^6 = 0$.

$20\omega^4 - 138\,000\omega^2 + 2.25\times10^8 = 0$..

Solving this Simultaneous Equation

$\omega_1^2 = 4257.8$ $\omega_2^2 = 2642.2$

$\omega_1 = 65.25$ rad/s ## $\omega_2 = 51.4$ rad/s ##

4b) From previously Defined system

$$\begin{bmatrix} 20 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 60000 & 3000 \\ 3000 & 3900 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 3 \sin 30t \end{Bmatrix}$$

$$x = X_2 \sin 30t \longrightarrow \ddot{x} = -900 X_2 \sin 30t$$

$$\theta = \Theta_2 \sin 30t \longrightarrow \ddot{\theta} = -900 \Theta_2 \sin 30t$$

By Cramer Rule.

$$\Delta = \det \begin{bmatrix} 60\,000 - 20(900) & 3000 \\ 3000 & 3900 - 900 \end{bmatrix}$$

$$= \det \begin{bmatrix} 42000 & 3000 \\ 3000 & 3000 \end{bmatrix} = 1.17 \times 10^8$$

$$\Delta_1 = \det \begin{bmatrix} 0 & 3000 \\ 3 & 3000 \end{bmatrix} = -9000$$

$$\Delta_2 = \det \begin{bmatrix} 42\,000 & 0 \\ 3000 & 3 \end{bmatrix} = 126000.$$

$$X_2 = \frac{\Delta 1}{\Delta} = -7.69 \times 10^{-5}\ \text{m}$$

$$\cancel{X_2}\ \Theta = \frac{\Delta 2}{\Delta} = 0.0011\ \text{rad}.$$

$$\therefore\ x_2 = -7.69 \times 10^{-5} \sin 30t\ (\text{m}) \qquad \theta = 0.0011 \sin 30t\ (\text{rad})_{\#}$$

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