NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2013-2014

MA2701 – FLIGHT PERFORMANCE

November/December 2013 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises of **THREE (3)** pages.
2. Answer **ALL** questions. Write down each step clearly for full credit.
3. All questions carry equal marks.
4. One double-sided A4 reference sheet is allowed.
5. This is a **RESTRICTED OPEN-BOOK examination.**

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1. A twin-engine jet aircraft has the following characteristics at sea level:

$$\text{Weight} = 50,000 \text{ lb}$$
$$\text{Wing area} = 1500 \text{ ft}^2$$
$$\text{Maximum lift coefficient} = 1.5$$
$$\text{Drag polar: } C_D = 0.02 + 0.05 C_L^2$$
$$\text{Sea level density} = 0.002377 \text{ slugs/ft}^3$$

Assume that the take-off portion of the thrust for each engine can be approximated as

$$T = 5000 - 3.28 V_\infty$$

(a) Show that the ground run lift coefficient $C_L = 0.5\mu_r / K$, where μ_r is the rolling friction coefficient, maximizes the net force during ground roll.

(8 marks)

(b) Use the optimal ground-roll lift coefficient described in (a) to calculate the total ground roll distance on a smooth, dry concrete runway ($\mu_r = 0.02$) at sea level for no-wind conditions.

(10 marks)

(c) Determine the takeoff flare distance and the total take-off distance.

(7 marks)

2. An aircraft has the following characteristics:

$$\text{Weight} = 36{,}000 \text{ lb}$$
$$\text{Wing area} = 450 \text{ ft}^2$$
$$\text{Sea level thrust} = 6{,}000 \text{ lb}$$
$$C_D = 0.014 + 0.05 C_L^2$$

If the thrust is assumed to be constant and the aircraft climbs at a speed of 300 ft/sec at a shallow angle, calculate

(a) Rate of climb. (5 marks)

(b) Angle of climb. (5 marks)

(c) Maximum rate of climb. (10 marks)

(d) Maximum angle of climb. (5 marks)

3. A light, single-engine, propeller-driven aircraft has the following characteristics:

Wing span = 35.8 ft
Wing area = 174 ft^2
Fuel empty weight = 1703 lb
Fuel weight = 367 lb
Power plant = one-piston engine of 230 hp at sea level
Specific fuel consumption = $2.27 \times 10^{-7}\ \text{ft}^{-1}$
Parasite drag coefficient $C_{D,0} = 0.025$
Oswald efficiency factor $e = 0.8$
Propeller efficiency = 0.8

(a) Show that at sea level $V_{\max} = 266$ ft/s. (10 marks)

(b) Determine the $(C_L / C_D)_{\max}$ and $(C_L^{3/2} / C_D)_{\max}$ values. (7 marks)

(c) Find the maximum range and the maximum endurance at sea level. (8 marks)

4. An aircraft in a steady level coordinated turn at sea level has the following characteristics:

Weight $= 2,000$ lb

Wing area $= 125$ ft^2

Maximum lift coefficient $= 1.5$

Maximum load factor $= 4$

Thrust $= 300$ lb for all flight speeds at sea level (turbojet)

$C_D = 0.02 + 0.05 C_L^2$

Sea level density $= 0.002377$ slugs/ft^3

(a) Is it possible to perform a constant altitude 180^o turn at the corner speed? Why?

(7 marks)

(b) Calculate the turn speed, the load factor, and the bank angle needed to perform this turn in minimum time, and determine the minimum time required.

(8 marks)

(c) Use the drag equation as $D = T_A$ to derive an equation of the form $n^2 = f(V)$ and then express the time-to-turn in terms of speed. Find the turn speed that minimizes the time-to-turn and compare your result to that obtained in part (b).

(10 marks)

Conversion Factors:
1 lb = 4.448 N
1 ft = 0.3048 m
1 slug = 14.59 kg

END OF PAPER

General comments: This paper is mainly calculation-based. Two questions are slightly more challenging. They are Q1a and Q4d. Be well-rested and alert and you should be able to do this easily. Do note that my numerical answers may not be correct due to carelessness. However, I ensure that my method of solving the questions is right for as far as I believe.

(Q1a) Note that $m\frac{dV_\infty}{dt} = T - D - \mu_r(W-L)$.

Let $F_H = m\frac{dV_\infty}{dt}$, then $F_H = T - D - \mu_r(W-L)$. — (*)

To maximise F_H wrt C_L implies $\frac{dF_H}{dC_L} = 0$.

Note that aircraft has two engines, i.e. thrust equation is

$$T = 2(5000 - 3.28V_\infty). \text{— (1)}$$

$D = q_\infty S C_D$

$= q_\infty S(C_{D0} + KC_L^2)$ — (2)

$L = q_\infty S C_L$ — (3)

Putting (1), (2) and (3) into (*), we obtain:

$2(5000 - 3.28V_\infty) - q_\infty S(C_{D0} + KC_L^2) - \mu_r(W - q_\infty S C_L) = F_H$ — (4)

Differentiating F_H wrt C_L, and equate the result to zero, we get:

$-2q_\infty SKC_L + q_\infty S\mu_r = 0$

$\Rightarrow 2KC_L = \mu_r$

$\Rightarrow C_L = \frac{\mu_r}{2K}$

$= 0.5\frac{\mu_r}{K}$

Comments: Many of my peers were unable to solve this. I think that many of us were being conditioned to differentiate anything wrt V_∞ throughout the course and in the heat of the exam, we were unable to think that differentiating F_H wrt C_L is the way to proceed.

(Q1b) $K = 0.02$, $\mu_r = 0.02$. Therefore, $C_L = \frac{1}{2}\frac{\mu_r}{K}$

$= 0.5$.

Formula: $S_{g0} = \frac{1.21(W/S)}{[g\rho_\infty C_{Lmax}((\frac{T}{W}) - (\frac{D}{W}) - \mu_r(1-(L/W))]_{0.7V_{LO}}}$

$\frac{W}{S} = \frac{50000}{1500}$

$= \frac{100}{3}$

$g = 32.2\ ft/s^2$

Comments: VERY IMPORTANT to know that $g = 32.2 ft/s^2$ instead of the usual $g = 9.81 m/s^2$ due to the change to imperial units.

$C_{Lmax} = 1.5$

$V_{LO} = 1.1 V_{stall}$

$= 1.1\sqrt{\frac{2}{\rho_\infty}\left(\frac{W}{S}\right)\frac{1}{C_{Lmax}}}$

$= 1.1\sqrt{\frac{2}{0.002377}\left(\frac{100}{3}\right)\left(\frac{1}{1.5}\right)}$

$V_{LO} = 150.4135\ ft/s.$

$0.7V_{LO} = 105.28945\ ft/s.$

$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$

$= \frac{1}{2}(0.002377)(105.28945)^2 (1500)(0.5)$

$= 9881.665839\ lb.$

$D = \frac{1}{2}\rho_\infty V_\infty^2 S (C_{D0} + KC_L^2)$

$= \frac{1}{2}(0.002377)(105.28945)^2(1500)(0.02 + 0.05(0.5)^2)$

$= 642.3082796\ lb.$

$T = 2(5000 - 3.28(105.28945))$

$= 9309.301208\ lb.$

$$S_{ga} = \frac{1.21\left(\frac{100}{3}\right)}{\left[32.2 \times 0.002377 \times 1.5 \times \left(\left(\frac{9309.3}{50000}\right) - \left(\frac{642.3082796}{50000}\right) - 0.02\left(1 - \frac{9881.665839}{50000}\right)\right)\right]}$$

$= 2233.422827\ ft.$

Comments: Avoid careless mistakes like forgetting the S in $D = \frac{1}{2}\rho_\infty V_\infty^2 S C_D$, V_∞^2 (squared) instead of just V_∞ in calculating L or D and general calculation mistakes. This applies in general.

Since W = 50 000 lb, it implies the aircraft is very big. Therefore, $t_R = 3s.$

$S_{gr} = 1.1 t_R \sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\frac{1}{C_{Lmax}}}$

$= (1.1)(3)\sqrt{\frac{2}{0.002377}\left(\frac{100}{3}\right)\left(\frac{1}{1.5}\right)}$

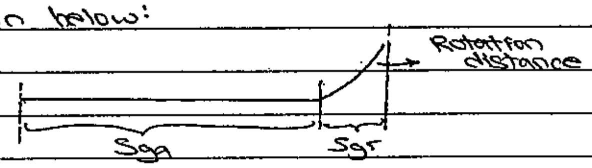
$= 451.2405\ ft.$

$S_g = S_{gr} + S_{ga}$

$= 2233.422827 + 451.2405$

$= 2684.663327\ ft.$

Comments: Do not forget about S_{gr}, graphically total ground-roll distance comprises of 2 parts, S_{ga} and S_{gr}, shown below:



Q1c) $R = \frac{6.69 V_{stall}^2}{g}$ [V_{stall} is found using $\sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\frac{1}{C_{Lmax}}}$].

$= \frac{6.69(136.73955)^2}{32.2}$

$= 3884.709487\ ft.$

$\gamma_{tr} = \cos^{-1}\left(1 - \frac{30}{3884.709487}\right)$ [30ft is the height of obstacle]

$= 7.1252297°$

$S_{tr} = R\sin\gamma_{tr}$

$= 3884.709487 \sin 7.1252297°$

$= 481.853265\ ft.$

$S_{TO} = S_{tf} + S_g$

$= 481.853265 + 2684.6633$

$= 3166.52 ft.$

Q2a) Constant thrust airplane is most probably a jet airplane.

$R/C = \frac{V_\infty(T-D)}{W}$.

Procedure: Find C_L, D and find R/C. Given $V_\infty = 300$ ft/s.

$C_L = \frac{W}{q_\infty S}$

$= \frac{36000}{\frac{1}{2}(0.002377)(450)(300)^2}$

$= 0.74790819$

$D = \frac{1}{2}\rho_\infty V_\infty^2 S(C_{D0} + KC_L^2)$ $[\frac{1}{2}(0.002377)(300)^2(450)(0.014+0.05(0.7479)^2)]$

$= 2020.114234$ lb

$R/C = \frac{(300)(6000-2020.114234)}{36000}$

$= 33.1657 ft/s.$

Q2b) $R/C = V_\infty \sin\gamma$

$\gamma = \sin^{-1}\left(\frac{R/C}{V_\infty}\right)$

$= 6.34716°$

Q2c) $\left(\frac{L}{D}\right)_{max} = \sqrt{\frac{1}{4KC_{D0}}}$

$= \sqrt{\frac{1}{4(0.014)(0.05)}}$

$= 18.89822365$

$\frac{T}{W} = \frac{6000}{36000}$

$= \frac{1}{6}$

$Z = 1 + \sqrt{1 + \frac{3}{(L/D)_{max}^2 (T/W)^2}}$

$= 2.141227409$

$(R/C)_{max} = \left[\frac{(\frac{W}{S})Z}{3\rho_\infty C_{D0}}\right]^{\frac{1}{2}} \left(\frac{T}{W}\right)^{\frac{3}{2}} \left[1 - \frac{Z}{6} - \frac{3}{2(\frac{T}{W})^2(\frac{L}{D})^2_{max} Z}\right]$

$= 51.02667 ft/s.$

Q2d) $\sin\gamma_{max} \simeq (T/W) - \frac{1}{(L/D)_{max}}$

$\gamma_{max} = \sin^{-1}\left(\frac{6000}{36000} - \frac{1}{18.89822365}\right)$ $[(\frac{L}{D})_{max}$ found in Q2c$]$

$= 6.5316268°$

Comments: Do not be careless.

Q3a) Recall that for propeller-driven aircraft, the maximum V_∞ ~~corresponds to the~~ occurs when power required is equal to power available, the latter being a constant. Also, note that 1 horsepower = 550 lbft/s.

$P_A = 0.8(230)(550)$

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. The MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

$P_A = 101200 \text{ lbft/s}$.

$AR = \frac{b^2}{S}$

$= \frac{35.8^2}{174}$

$= 7.365747$.

$K = \frac{1}{\pi e (AR)}$

$= \frac{1}{\pi (0.8)(7.365747)}$

$= 0.0540186$.

$\therefore \; C_D = 0.025 + 0.0540186 C_L^2$.

$P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}}$

$= \sqrt{\frac{2(2070)^3}{(174)(0.002377)}} \left(\frac{0.025 + 0.0540186 C_L^2}{C_L^{\frac{3}{2}}} \right)$

$P_A = P_R \Rightarrow P_R = 101200$

$\Rightarrow 0.025 + 0.0540186 C_L^2 = 0.048865147 21 C_L^{\frac{3}{2}}$

$\Rightarrow (6.25 \times 10^{-4}) + 0.00270093 C_L^2 + 0.0029180091 C_L^4 - 0.002387800612 C_L^3 = 0.$

Solving for C_L, $C_L = 0.1417756975$.

$V_{max} = \sqrt{\frac{2W}{\rho_\infty S C_L}}$

$= \sqrt{\frac{2(2070)}{(0.002377)(174)(0.1417756975)}}$

$= 265.7 \text{ ft/s}$

$\approx 266 \text{ ft/s}$ (shown).

Comments: Use GC to solve the fourth order polynomial. The weight of the aircraft will be the full gross weight because it is supposed to be flying with fuel.

Q3b) $\left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4 C_{D0} K}}$

$= 13.60959332$

$\left(\frac{C_L^{3/2}}{C_D}\right)_{max} = \frac{1}{4} \left(\frac{3}{K C_{D0}^{\frac{1}{3}}} \right)^{\frac{3}{4}}$

$= 12.79050745$.

Comments: Ensure that in your cheatsheet, especially those typed ones, the power is clearly displayed. Formulae should be BIG. In this case, ensure you will see in your cheatsheet that the power of C_{D0} is $\frac{1}{3}$ while for the bracket its $\frac{3}{4}$

Q3c) $R_M = \frac{\eta_{pr}}{c} \left(\frac{L}{D}\right)_{max} \ln \left(\frac{W_0}{W_1}\right)$ $[W_0 = 2070 \text{ lb}, W_1 = 1703 \text{ lb}]$

$= 9357871.092 \text{ ft}$

$E_M = \frac{\eta_{pr}}{c} \sqrt{2 \rho_\infty S} \left(\frac{C_L^{3/2}}{C_D}\right)_{max} (W_1^{-\frac{1}{2}} - W_0^{-\frac{1}{2}})$

$= 92360.7453 \text{ s}$

$\approx 25.7 \text{ hr}$.

Q4a) Procedure: Find corner velocity. Find n_{max} corresponding to it.

Constraint due to maximum thrust,

$$n_{max} = \left[\frac{\frac{1}{2}\rho_\infty V_\infty^2}{K\left(\frac{W}{S}\right)}\left(\left(\frac{T}{W}\right)_{MAX} - \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{D0}}{W/S}\right)\right]^{\frac{1}{2}}$$

~~$(2.2284375\times10^{-4} V_\infty^2 - 2.20708164\times10^{-9} V_\infty^4)^{\frac{1}{2}}$~~

~~Constraint due to stall~~

~~$n_{max} = \frac{1}{2}\rho_\infty V_\infty \frac{C_{LMAX}}{W/S}$~~

~~$= 1.7774378$~~

$$= \left[\frac{\frac{1}{2}(0.002377)V_\infty^2}{(0.05)(2000/125)}\left(\left(\frac{300}{2000}\right) - \frac{1}{2}(0.002377)V_\infty^2\left(\frac{0.02}{2000/125}\right)\right)\right]^{\frac{1}{2}}$$

$$= (2.2284375\times10^{-4} V_\infty^2 - 2.20708164\times10^{-9} V_\infty^4)^{\frac{1}{2}}$$

Constraint due to stall, $n_{max} = \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{Lmax}}{W/S}$

$$= \frac{1}{2}(0.002377)(V_\infty^2)\left(\frac{1.5}{2000/125}\right)$$

Equating ~~the~~ both n_{max}, solving for V_∞, which gives:

$V_{corner} = 123.4512$ ft/s.

Using any n_{max} (thrust or stall),

e.g. $n_{max} = \frac{1}{2}(0.002377)(123.4512)^2 \frac{1.5(125)}{2000}$ [n_{max} for stall]

$= 1.698$

$\because n_{max} = 1.698 < 4 \Rightarrow$ the turn is possible.

Q4b) $\omega_{max} = g\sqrt{\frac{\rho_\infty}{W/S}\left(\frac{(T/W)}{2K} - \left(\frac{C_{D0}}{K}\right)^{\frac{1}{2}}\right)}$

$$= 32.2\sqrt{\frac{(0.002377)}{(2000/125)}\left(\frac{(300/2000)}{2(0.05)} - \left(\frac{0.02}{0.05}\right)^{\frac{1}{2}}\right)}$$

$= 0.365558$ rad s^{-1}.

$n_{\omega max} = \sqrt{\frac{(T/W)}{\sqrt{KC_{D0}}} - 1}$

$$= \sqrt{\frac{(300/2000)}{\sqrt{(0.05)(0.02)}} - 1}$$

$= 1.934791$

$\cos\phi = \frac{1}{n_{max}}$

$\phi = \cos^{-1}\left(\frac{1}{n_{max}}\right)$

$= \cos^{-1}\left(\frac{1}{1.934791}\right)$

$= 58.878696°$

$t_{min} = \frac{\theta}{\omega}$

$= \frac{(\pi/2)}{0.365558}$

$= 4.29698$ s.

$(V_\infty)_{\omega max} =$ ~~[illegible]~~ $\left(\frac{2(W/S)}{\rho_\infty}\right)^{\frac{1}{2}}\left(\frac{K}{C_{D0}}\right)^{\frac{1}{4}}$

$$= \left(\frac{2(2000/125)}{0.002377}\right)^{\frac{1}{2}}\left(\frac{0.05}{0.02}\right)^{\frac{1}{4}}$$

$= 145.896677$ ft/s.

Q4c)

Comments: The workings are going to be SUPER LONG!

$L = nW \Rightarrow q_\infty S C_L = nW$

$\Rightarrow C_L = \frac{nW}{q_\infty S}$

$C_D = C_{D0} + KC_L^2$

$= C_{D0} + K\left(\frac{nW}{q_\infty S}\right)^2$

$D = q_\infty S C_D$

$= q_\infty S C_{D0} + \frac{Kn^2W^2}{q_\infty S}$

$D = T_A \Rightarrow q_\infty S C_{D0} + \frac{Kn^2W^2}{q_\infty S} = T_A$

$\frac{Kn^2W^2}{q_\infty S} = T_A - q_\infty S C_{D0}$

$Kn^2W^2 = q_\infty S T_A - (q_\infty S)^2 C_{D0}$

$n^2 = \frac{1}{KW^2}\left[q_\infty S T_A - (q_\infty S)^2 C_{D0}\right]$

$\because q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$, $n^2 = f(V_\infty)$ (shown).

Comments: If you are not happy with the expressing n^2 as $f(q_\infty)$, you can replace all q_∞ by $\frac{1}{2}\rho_\infty V_\infty^2$.

$\omega = \frac{1}{V_\infty} g\sqrt{n^2-1} \Rightarrow \frac{\omega V_\infty}{g} = \sqrt{n^2-1}$

$\Rightarrow n^2 = \left(\frac{\omega V_\infty}{g}\right)^2 + 1$

$\left(\frac{\omega V_\infty}{g}\right)^2 + 1 = \frac{1}{KW^2}\left(q_\infty S T_A - (q_\infty S)^2 C_{D0}\right)$

$$\omega = \frac{g}{V_\infty}\sqrt{\frac{q_\infty S T_A - (q_\infty S)^2 C_{D0}}{KW^2} - 1}$$

$t_T = \frac{(\pi/2)}{\omega}$

$$= \left(\frac{\pi}{2}\right)\left\{\frac{V_\infty}{g}\sqrt{\frac{KW^2}{q_\infty S T_A - (q_\infty S)^2 C_{D0} - KW^2}}\right\}$$

$$\frac{dt_T}{dq_\infty} = \left[\frac{1}{g\rho_\infty V_\infty}\sqrt{\frac{KW^2}{q_\infty S T_A - (q_\infty S)^2 C_{D0} - KW^2}} - \frac{1}{2}\left\{\frac{\sqrt{KW^2}\,(ST_A - 2q_\infty S^2 C_{D0})}{\left[q_\infty S T_A - (q_\infty S)^2 C_{D0} - KW^2\right]^{3/2}}\right\}\right]\left(\frac{\pi}{2}\right)$$

$$\frac{dt_T}{dq_\infty} = 0 \Rightarrow \frac{1}{\rho_\infty V_\infty g}\sqrt{\frac{KW^2}{q_\infty S T_A - (q_\infty S)^2 C_{D0} - KW^2}} = \frac{V_\infty}{2g}\sqrt{KW^2}\left(\frac{T_A S - 2q_\infty S^2 C_{D0}}{(q_\infty S T_A - (q_\infty S)^2 C_{D0} - KW^2)^{3/2}}\right)$$

Comments: To get $\frac{dt_T}{dV_\infty}$, you first use Product Rule, $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$, where x is q_∞, v is $\frac{V_\infty}{g}$ and u is everything inside that $\sqrt{\ }$. Then use Chain Rule, $\frac{df(q_\infty)}{dV_\infty} = \frac{df}{dq_\infty}\frac{dq_\infty}{dV_\infty}$, $\frac{dq_\infty}{dV_\infty} = \rho_\infty V_\infty$, $\frac{dV_\infty}{dq_\infty} = \frac{1}{\rho_\infty V_\infty}$. It is easier to differentiate t_T wrt q_∞ instead of V_∞, therefore I differentiated it wrt q_∞ then multiply whatever that involves V_∞ by $\frac{dV_\infty}{dq_\infty} = \frac{1}{\rho_\infty V_\infty}$ using the power Chain Rule. Notice that everything on the LHS will be cancelled by the RHS to give:

$$1 = \frac{q_\infty(ST_A - 2q_\infty S^2 C_{D0})}{q_\infty S T_A - (q_\infty S)^2 C_{D0} - KW^2}$$

$q_\infty S T_A - (q_\infty S)^2 C_{D0} - KW^2 = q_\infty S T_A - 2q_\infty^2 S^2 C_{D0}$

$q_\infty^2 = \frac{KW^2}{S^2 C_{D0}}$

$\frac{1}{4}\rho_\infty^2 V_\infty^4 = \frac{KW^2}{S^2 C_{D0}}$

Solving for V_∞, $V_\infty = 145.8966$ ft/s.

Closing comments: Find your friends to do cheatsheet together if you want the most information in the shortest period of time. Though many of my peers who did very well by doing their own cheatsheet as a form of revision, I felt that time could be better spent elsewhere. Thank you, good luck and all the best! JL

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NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

MA2701 – FLIGHT PERFORMANCE

November/December 2015 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL** questions. Write down each step clearly for full credit. Illustrate your answers wherever possible with sketches.
3. All questions carry equal marks.
4. One double A4 reference sheet is allowed as reference material.
5. This is a **RESTRICTED OPEN-BOOK** Examination.

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1. The drag polar, engine thrust and other data for a turbojet aircraft are given below:

Drag polar $C_D = 0.015 + 0.0468\ C_L^2$,
Engine thrust $F = 200\ \sigma$ kN,
Weight = 600 kN,
Wing area $S = 150\ m^2$ and
$C_{Lmax} = 1.1$.

(a) Calculate the following flight speeds at 10 km altitude. Density at 10 km in ISA is $0.4127\ kg/m^3$.

(i) Maximum and minimum steady level flight speeds (V_{max} and V_{min}) with full throttle setting of the engine.

(ii) Stall speed (V_{Stall}).

(iii) Minimum operating speed for the aircraft.

(8 marks)

(b) Calculate $(L/D)_{max}$, minimum drag (D_{min}) and the corresponding speed (V_{Dmin}) for this aircraft at sea level.

(6 marks)

(c) Obtain absolute ceiling of this aircraft and level flight speed possible at absolute ceiling with the engine set at full throttle in ISA. The relationship between the density ratio ($\sigma = \rho/\rho_0$) and the altitude (h) is given by $h = 44.3(1 - \sigma^{0.235})$ km.

(6 marks)

(d) Calculate climb rate for this aircraft at minimum drag (V_{Dmin}) speed at sea level. If the available Specific Excess Power (SEP) at this speed is used for level acceleration what is the maximum possible acceleration at sea level?

(5 marks)

2. The sea-level power required curve of a turboprop aircraft at a weight of 690,000 N is given in Figure 1. The stall limit is also indicated in Figure 1. The aircraft's wing span is 40 m and its wing planform area is 162 m^2. The power available from aircraft's engine can be assumed constant with flight speed, but varies proportionally with the air density. Its power specific fuel consumption (PSFC) is independent of both the flight speed and the altitude. At sea level, the maximum power available is 17,000 kW and its PSFC is 0.2605 kg/kW/hr. Assume its propeller efficiency is 0.85.

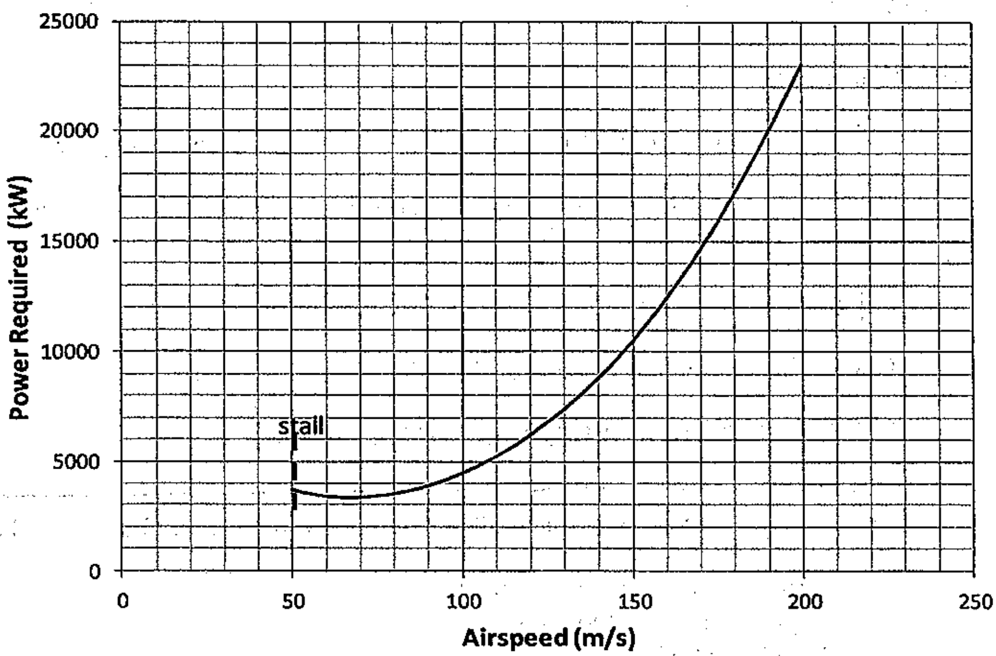


Figure 1: Power required curve at sea level

Assume sea level flight condition in answering the questions below, except in part (c) when it is indicated otherwise.

(a) Determine the maximum steady level speed and the lift-to-drag ratio (L/D) to fly at this maximum speed.

(4 marks)

(b) Determine the maximum rate of climb and its associated climb angle.

(5 marks)

(c) Calculate the maximum lift coefficient of the aircraft and its stall speed at an altitude of 5 km (Air density at 5 km altitude is 0.7361 kg/m^3).

(5 marks)

(d) The maximum range of the turboprop aircraft is achieved when flying at the maximum lift-to-drag ratio. Without resorting to the drag polar of the aircraft, determine its maximum lift-to-drag ratio (or minimum drag condition).

(5 marks)

(e) What should be the flight speed of the aircraft at sea level to achieve maximum range? If the aircraft is flying against a headwind of 30 m/s, estimate the amount of airspeed adjustment that the pilot needs to make to get the maximum range (indicate clearly whether the adjustment involves a speed increase or decrease).

(6 marks)

3 (a) The following data is given for estimation of takeoff distance for a jet trainer aircraft at sea level (ρ= 1.225 kg/m^3):

Aircraft Weight and Wing Area: W = 68000 N and S= 25 m^2
Maximum sea level thrust F_0= 40800 N
Drag Polar for clean aircraft: $C_D = 0.025 + 0.07C_L^2$
Maximum lift coefficient for take off $C_{LmaxTO} = 1.8$
Rolling friction coefficient $\mu = 0.02$.
For ground run: C_L= 0.15 and $\Delta C_D = 0.01$ (accounting for flap & landing gear drag).
Aircraft rotation speed is $1.1V_{stall}$
Takeoff speed at the end of rotation and flare is $1.2V_{stall}$.
Time for transition - rotation and part of flare : 3 seconds,
For the climb segment incremental $\Delta C_D = 0.0125$ and $C_L = 0.9C_{LmaxTO}$.

Calculate the following three segments of takeoff and the total takeoff distance:

(i) Ground run distance to reach rotation speed with appropriate average acceleration.

(ii) Transition distance covering rotation phase and part of the flare.

(iii) Climb segment at $1.2V_{stall}$ for clearing an obstacle of 10.67m with a climb angle not exceeding 12^0, if necessary by throttling down the engine.

(9 marks)

(b) If the runway has a 3 degree down slope, what is the ground run distance to reach the rotation speed of 1.1 V_{stall}?

(3 marks)

(c) What is corner velocity of the above aircraft at sea level given that the maximum load factor of the aircraft is 8? Calculate the turn rate at this corner speed.

(8 marks)

(d) Is this turn rate at the corner speed sustainable in a level turn with engine set at full throttle? If not what is the thrust deficit and sink angle?

(5 marks)

4 (a) A baseline mission profile of a transport aircraft consists of 5 flight segments including takeoff, climb, cruise, descent and landing. Sometimes the Air Traffic Controller intervenes during its decent flight and commands the aircraft to hold on in flight (loiter around) for certain duration varying from 30 minutes to 1 hour at an assigned altitude of 3 km, before it is allowed to continue its descent and land. In such a scenario, the aircraft mission profile will have 7 flight segments including loiter at 3 km and the second descent from 3 km to sea level before landing.

Draw the mission profile for such an aircraft identifying these 7 flight segments sequentially.

(4 marks)

(b) Weight fractions (ratio of final weight to initial weight) for various flight segments other than that for Cruise (at 11km altitude) and Loiter (at 3km altitude) are given below:

Take Off	$W_1/W_0 = 0.995$
Climb	$W_2/W_1 = 0.98$
Descent 1	$W_4/W_3 = 0.99$
Descent 2	$W_6/W_5 = 0.995$
Landing	$W_7/W_6 = 0.995$

Calculate the weight fractions W_3/W_2 for the cruise segment with constant speed and W_5/W_4 for the loiter segment using the following data:
Range covered during cruise segment is 12000 km. The cruise Mach number at 11km altitude is 0.85 (speed of sound at 11 km = 295m/s). (L/D) for cruise segment is given to be 16. The thrust specific fuel consumption (TSFC) at 11 km is given to be 25 kg/kN/hr. For the loiter segment of 1 hour duration at 3 km altitude the TSFC is 30 kg/kN/hr and L/D is 16.5.

(8 marks)

(c) Given takeoff weight (W_0) of the aircraft to be 2500 kN, obtain landing weight (W_7) without any fuel allowance. Calculate the fuel to be carried for this mission (takeoff to landing) allowing for 10% extra fuel.

(5 marks)

(d) Calculate the weight fraction W_3/W_2 for cruise segment if there is a tail wind of 50 m/s experienced by the aircraft flying at 11 km altitude. Assuming weight fractions remain the same for all the other segments including the loiter segment, what is the fuel actually saved or burnt in excess due to tail wind in the cruise segment.

(8 marks)

End of Paper

1 (a) (i) using formula:

$$V^2 = \frac{\left(\frac{T_{Amax}}{W}\right)\left(\frac{W}{S}\right) \pm \left(\frac{W}{S}\right)\sqrt{\left(\frac{T_{Amax}}{W}\right)^2 - 4C_{D_0}k}}{\rho_\infty C_{D_0}}$$

$$T_{Amax} = T_{ASL}\,\sigma$$

$$= 200000\left(\frac{\rho_\infty}{\rho_{SL}}\right)$$

$$= 200000\left(\frac{0.4127}{1.225}\right)$$ ← pls remember $\rho_{SL} = 1.225$

$$= 67379.6\ N$$

Given
$C_D = 0.015 + 0.0468\,C_L^2$
$C_D = C_{D_0} + k\,C_L^2$
$W = 600\,000\ N$
$S = 150\ m^2$
$C_{Lmax} = 1.1$

$$V^2 = \frac{\left(\frac{67379.6}{600\,000}\right)\left(\frac{600\,000}{150}\right) \pm \left(\frac{600\,000}{150}\right)\sqrt{\left(\frac{67379.6}{600\,000}\right)^2 - 4(0.015)(0.0468)}}{(0.4127)(0.015)}$$

$= 136537.4364\ ;\ 8587.19$

$V = 369.51\ m/s\ ;\ 92.667\ m/s$

$\quad\ = V_{max} \qquad\quad = V_{min}$

(ii)

$$V_{stall} = \sqrt{\frac{2W}{\rho_\infty S\,C_{Lmax}}}$$

$$= \sqrt{\frac{2(600\,000)}{0.4127 \times 150 \times 1.1}}$$

$$= 132.749\ m/s$$

b)

$\left(\frac{L}{D}\right)_{max} = \frac{1}{2\sqrt{kC_{D_0}}}$

$= \frac{1}{2\sqrt{0.0468 \times 0.015}}$

$= 18.87$

$D_{min} = 2W\sqrt{kC_{D_0}}$

$= 2(600000)\sqrt{0.0468(0.015)}$

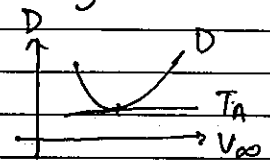
$= 31794.339\ N$

$V_{D_{min}} = \sqrt{\frac{2W}{\rho S\sqrt{\frac{C_{D_0}}{k}}}}$

$= \sqrt{\frac{2(600\,000)}{(1.225)(150)\sqrt{\frac{0.015}{0.0468}}}}$

$= 107.403\ m/s.$

c) At absolute ceiling, $R/C = 0$ & $T_{A_{max}} = D_{min}$ for turbojet.



$F = T_A = 200\,000\,\sigma = D_{min} = 31794.339$

$\sigma = 0.15897$

Given, $h = 44.3(1-\sigma^{0.235})$

sub $\sigma = 0.15897$

$h = 44.3(1-0.15897^{0.235})$

$= 15.545\ km$

~~$C_{L_{max}}$~~ ? As D_{min}, $C_{D_0} = kC_L^2 \Rightarrow C_L = \sqrt{\frac{C_{D_0}}{k}} = 0.56614$

$\sigma = \frac{\rho_\infty}{\rho_{SL}} = 0.15897$

$\therefore \rho_\infty = (0.15897)(1.225)$

$= 0.1947\ kg/m^3$

$\therefore V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$

$= \sqrt{\frac{2(600\,000)}{(0.1947)(150)(0.56614)}}$

$= 269.40\ m/s$

1) (d)

$$R/C = \left(\frac{T_A - D_{min}}{W}\right) V_{D_{min}}$$

$$= \left(\frac{200\,000 - 31794.339}{600\,000}\right)(107.403)$$

$$= 30.11 \text{ m/s}^2$$

~~SEP =~~

~~$SEP = (T_A - D_{min}) V_{D_{min}}$~~

~~$= (30.11)(600\,000)$~~

~~$= 18\,066\,000$~~

Newton 2nd Law,

$$\Sigma F = m\vec{a}$$

$$T_A - D_{min} = \frac{W}{g} a$$

$$200\,000 - 31794.339 = \frac{600\,000}{9.81} a.$$

$$a = 2.75 \text{ m/s}^2$$

2) a) V_{max} occurs when $P_R = P_0$ (i.e. P_A)

$\therefore V_{max} = 180$ m/s $\qquad P_R = P_A \eta_p = 17000k \times 0.85$

$= 14450$ kW

$$\frac{L}{D} = \frac{W}{T} = \frac{W}{\frac{P_R}{V_\infty}} = \frac{690\,000}{\frac{14450 \times 10^3}{180}} = ~~8.595~~$$

b) $$(R/C)_{max} = \frac{P_A - P_R}{W} = \frac{17000k - 3300k}{690\,000} = ~~19.855 \text{ m/s}^2~~$$

$$(R/C)_{max} = V_{P_{min}} \cdot \sin \gamma_{max}$$

$$19.855 = (65)(\sin \gamma)$$

$$\gamma = 17.786°$$

c) V_{stall} at SL = 50 m/s from graph

relationship between V & density, ρ

$$V_2 = \frac{1}{\sqrt{\sigma}} V_1$$

$$V_{stall@5km} = \frac{1}{\sqrt{\frac{0.7361}{1.225}}} V_{stall@SL}$$

$$= 1.29(50)$$

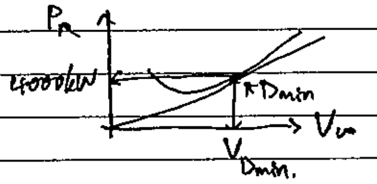
$$= 64.50 \text{ m/s}$$

$$V_{stall@5km} = \sqrt{\frac{2W}{\rho_{5km} S C_{Lmax}}}$$

$$64.50^2 = \sqrt{\frac{2(690000)}{(0.7361)(162)(C_{Lmax})}}$$

$$C_{Lmax} = 2.782$$

d) For D_{min} to occur,



P_R = 4000kW

V_{Dmin} = 90 m/s

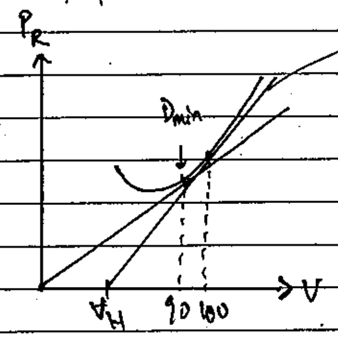
$$\left(\frac{L}{D}\right)_{max} = \frac{W}{D_{min}}$$

$$= \frac{690000}{\frac{4000k}{90}}$$

$$= 15.525$$

$$D_{min} = \frac{P_R}{V_{Dmin}} = \frac{4000k}{90}$$

e) For turboprop aircraft, max range occurs when P_{Rmin}, $\left(\frac{L}{D}\right)_{max}$ or D_{min}



~~draw a line that tangents to the curve & intercept the x-axis at V_H (head wind)~~

V_H = 30 m/s

V_∞ = 100 m/s for R_{max}

∴ airspeed adjustment = $V_\infty - V_H$

= 70 m/s

∴ V_{Rmax} = 90 m/s

*<u>**DISCLAIMER:**</u> The solutions are done by students who scored A or above in this subject. The MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

3(a) This question is using the Andeson Approach.

(i) $V_{stall} = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$

$= \sqrt{\frac{2(68000)}{(1.225)(25)(1.8)}}$

$= 49.67$ m/s

rotation speed or Lift off speed, $V_{LO} = 1.1 V_{stall}$

$= 54.637$ m/s

using: $S_{ga} = \frac{W V_{LO}^2}{2g} \frac{1}{[T - D - \mu_r(W-L)]_{0.707 V_{LO}}}$

$D = \frac{1}{2} \rho S V^2 C_D^*$, $C_D^* = C_D + \Delta C_D$ ~~= 0.015 + 0~~

$= 0.015 + 0.07(0.15)^2 + 0.01$

$= 0.036575$

$D = \frac{1}{2}(1.225)(25)(38.634)^2(0.036575)$

$= 835.93$ N

$L = \frac{1}{2} \rho S V^2 C_L$

$= \frac{1}{2}(1.225)(25)^2(38.634)^2(0.15)$

$=$ ~~85707.08~~ 3428.28

$T - D - \mu_r(W-L) = 40800 - 835.93 - 0.02(68000 - 3428.28)$

$= 38672.636$ N

$S_{ga} = \frac{68000(54.637)^2}{2 \times 9.81} \frac{1}{38672.636}$

$= 267.53$ m

(ii) $S_{gr} = t_R V_{LO}$

$= 3(54.637)$

$= 163.911$ m

(iii) $V_2 = 1.2 V_{stall}$

$= 1.2(49.67)$

$= 59.604$ m/s

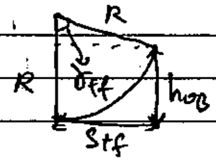
$n = \frac{L}{W} = \frac{\frac{1}{2}\rho_\infty (1.2)^2 V_{stall}^2 (S) C_{Lmax TO}}{\frac{1}{2} \rho_0 S C_{Lmax TO} V_{stall}^2}$

$n = 1.296$

assume pull-up maneuver

$R = \frac{V_2^2}{g(n-1)}$

$R = \frac{59.604^2}{(9.81)(1.296-1)} = 1223.46$ m



$\cos \gamma_{tf} = \frac{R - h_{OB}}{R}$

$= \frac{1223.46 - 10.67}{1223.46}$

$\gamma_{tf} = 7.573°$ $< 12°$ (satisfied)

$\therefore S_{tf} = R \sin \gamma_{tf}$

$= 1223.46 \sin 7.573°$

$= 161.239$ m

~~b)~~ $S_{ga}' = S_{ga}$

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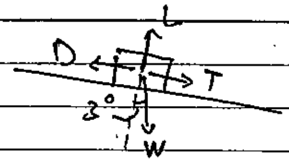
b)



$T + W\sin 3^\circ - D - \mu_r(W\cos 3^\circ - L)$

$= 40800 + (68000 \sin 3^\circ) - 835.93 - 0.02(68000\cos 3^\circ - 34328.28)$

$= 42233$

$S_{ga} = \dfrac{68000(54.637)^2}{2\times 9.81} \dfrac{1}{42283}$

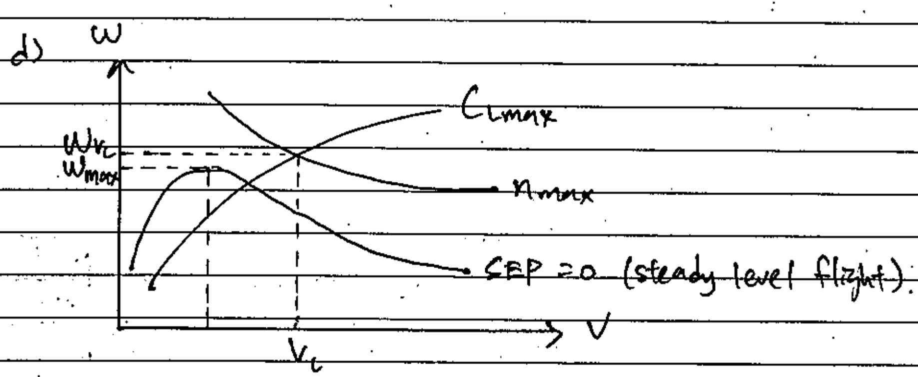
$= 244.98\,m.$

c) $V_c = \sqrt{\dfrac{2 W n_{max}}{\rho_0 S C_{Lmax}}} = \sqrt{\dfrac{2(68000)(8)}{(1.225)(25)(1.8)}}$

$= 140.488\ m/s$

$\omega = \dfrac{g\sqrt{n_{max}^2 - 1}}{V} = \dfrac{9.81\sqrt{8^2-1}}{140.488} = 0.5542\ rad/s.$

d)



$\omega_{max} = g\sqrt{\dfrac{\rho_0}{\left(\frac{W}{S}\right)}\left[\dfrac{\frac{T}{W}}{2k} - \sqrt{\left(\dfrac{C_{D0}}{k}\right)}\right]}$

$= 0.3998\ rad/s < \omega_{Vc}$

∴ so the aircraft can't sustainable in a level turn at corner speed.

e) $T = D = \frac{1}{2}\rho S V_c^2 C_D = \frac{1}{2}(1.225)(25)(140.488)^2\left[0.025 + 0.07(1.8)^2\right]$

$= 76099.23$

Thrust deficit, $T_d = T - F$

$= 76099.23 - 40800$

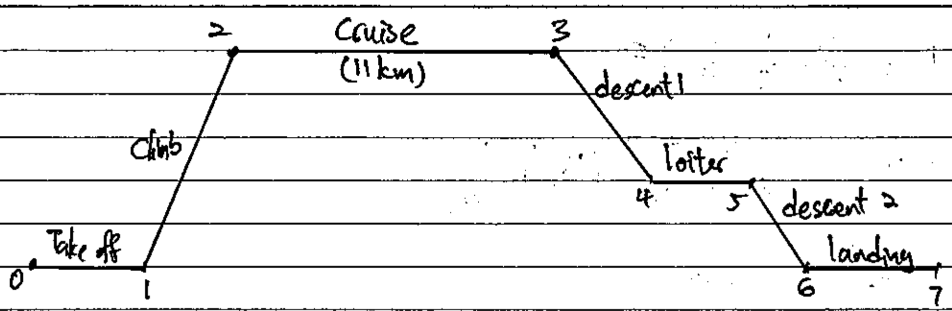
$= 35299.23\,N$

$\sin\gamma = \dfrac{1}{\frac{L}{D}} = \dfrac{1}{\frac{W}{T_d}}$

$= \dfrac{35299.23}{68000}$

$\therefore \gamma = 31.27^\circ$

4(a)



(b) $M = 0.85$

$= \frac{V_\infty}{c}$, $c = 295\,m/s$

$\therefore V_\infty = 250.75\,m/s$

$\left(\frac{L}{D}\right) = 16$

$TSFC = 25\,kg/kN\,hr$

$C_t = \frac{25 \times 9.81}{1000 \times 60 \times 60} = 6.8125 \times 10^{-5}$

$R = \frac{V_\infty}{C_t}\left(\frac{L}{D}\right)\ln\frac{W_2}{W_3}$

$12000 \times 10^3 = \frac{250.75}{6.8125 \times 10^{-5}}(16)\ln\frac{W_2}{W_3}$

$\frac{W_3}{W_2} = 0.816$ *

$E = \frac{1}{C_t}\left(\frac{L}{D}\right)\ln\frac{W_0}{W_1}$, $C_t = \frac{30 \times 9.81}{1000 \times 60 \times 60} = 8.175 \times 10^{-5}$

$60 \times 60 = \frac{1}{8.175 \times 10^{-5}}(16.5)\ln\frac{W_4}{W_5}$

$\frac{W_5}{W_4} = 0.982$ *

(c) W_7, without any fuel allowance;

$\left(\frac{W_1}{W_0}\right)\left(\frac{W_2}{W_1}\right)\left(\frac{W_3}{W_2}\right)\left(\frac{W_4}{W_3}\right)\left(\frac{W_5}{W_4}\right)\left(\frac{W_6}{W_5}\right)\left(\frac{W_7}{W_6}\right) = \left(\frac{W_7}{W_0}\right)$

$(0.995)(0.98)(0.816)(0.99)(0.982)(0.995)(0.995) = \frac{W_7}{2500 \times 10^3}$

$W_7 = 1914574.05\,N$

W_7', with 10% extra fuel allowance;

$W_7' = (W_0 - W_7) \times 110\%$ ($W_0 - W_7$ = total fuel used).

$= 643968.55\,N$

d)

$$R' = \frac{V_g}{C_t}\left(\frac{L}{D}\right)\ln\frac{W_2}{W_3}$$

$$12000\times10^3 = \frac{250.75+50}{6.8125\times10^{-5}}(16)\ln\frac{W_2}{W_3}$$

$$\frac{W_3}{W_2} = 0.844$$

$$0.995\times0.98\times0.844\times0.99\times0.982\times0.995\times0.995 = \frac{W_7^*}{2300\times10^3}$$

$$W_7^* = 1980270.216\,N$$

$$-W_7^* + W_0 = (2500\times10^3 - 1980270.216)N$$

$$= 519729.78\,N \quad \text{(total fuel used)}$$

Saved amount $= 585425.95 - 519729.78$

$= 65696.17\,N$ //