NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2013-2014

MA3701 – AERODYNAMICS

AE2003 – AERODYNAMICS I

November/December 2013 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions. Support your answers with formulae and figures whenever appropriate.
3. All questions carry equal marks.
4. This is an **Open-book Examination.**

1(a) An ideal flow field is found to have velocity components described as:

$$u = x + 4y \qquad v = 4x - y$$

(i) List three assumptions for an idealized flow.

(3 marks)

(ii) Show that the above flow field is irrotational.

(2 marks)

(iii) Determine the stream function and velocity potential function of the above ideal flow field.

(5 marks)

(b) It is known that a doublet can be formed by a combination of a source and a sink with equal but opposite strengths located at some distance apart from each other. Using Figure 1 provided below and starting with the basic potential flow equations, show in detail that the resulting combined stream function can be approximated as

$$\psi = -\frac{Qar\sin\theta}{\pi(r^2 - a^2)},$$

where Q is the magnitude of the source's and sink's strengths and P is an arbitrary point within the flow field.

(9 marks)

Note: Question 1 continues on page 2, and
Figure 1 appears on Page 2.

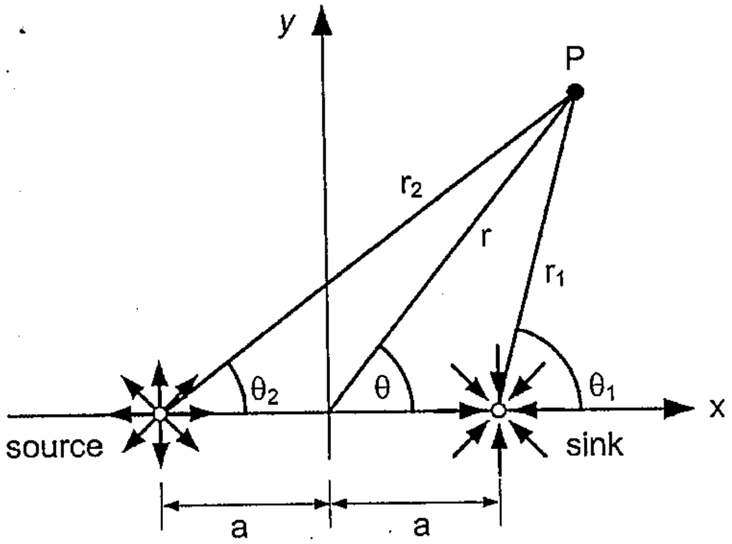

Figure 1

(c) Thin aerofoil theory is a classical theory that enables us to predict certain aerodynamic characteristics of aerofoils.

(i) Under what conditions will the thin aerofoil theory be invalid?

(3 marks)

(ii) Explain why thin aerofoil theory predicts the same maximum lift slope for both symmetric and asymmetric aerofoils.

(3 marks)

2(a) The camber lines for a 2D aerofoil are given as

$$\frac{z}{c} = 0.111\left[0.6\left(\frac{x}{c}\right) - 0.5\left(\frac{x}{c}\right)^2\right] \qquad 0 \le \frac{x}{c} \le 0.6$$

$$\frac{z}{c} = 0.0625\left[-0.4 + 2.4\left(\frac{x}{c}\right) - 2\left(\frac{x}{c}\right)^2\right] \qquad 0.6 < \frac{x}{c} \le 1.0$$

Assuming standard definitions for the parameters and applying Thin Aerofoil Theory, determine the lift coefficient, c_L, at α=4°.

(17 marks)

(b) A finite-span rectangular wing with elliptic load distribution with zero-lift angle-of-attack of -1.5° is observed to produce a maximum circulation of 32m^2/s along the plane of symmetry. This wing is designed to allow an aircraft to fly straight-and-through at 80m/s at an altitude of 4km above sea level. If the wing aspect-ratio and chord are 10 and 1.2m respectively, and by making use of information provided in Figure 2, determine

(i) the maximum aircraft weight that the wing can support under the above flight conditions, and

(6 marks)

(ii) the resulting downwash velocity.

(2 marks)

Altitude (m)	**Temperature (°C)**	**Acceleration of Gravity, g (m/s^2)**	**Pressure, p [N/m^2(abs)]**	**Density, ρ (kg/m^3)**	**Dynamic Viscosity, μ (N·s/m^2)**
−1,000	21.50	9.810	1.139 E + 5	1.347 E + 0	1.821 E − 5
0	15.00	9.807	1.013 E + 5	1.225 E + 0	1.789 E − 5
1,000	8.50	9.804	8.988 E + 4	1.112 E + 0	1.758 E − 5
2,000	2.00	9.801	7.950 E + 4	1.007 E + 0	1.726 E − 5
3,000	−4.49	9.797	7.012 E + 4	9.093 E − 1	1.694 E − 5
4,000	−10.98	9.794	6.166 E + 4	8.194 E − 1	1.661 E − 5
5,000	−17.47	9.791	5.405 E + 4	7.364 E − 1	1.628 E − 5

<u>Figure 2</u>

2

3(a) You are tasked with the job of conducting a flow visualization measurement experiment on the wake structures produced by a round cylinder immersed in a free-stream within a water tunnel.

(i) Which flow visualization technique would you use to obtain clear cross-section images of the wake structures and why?

(3 marks)

(ii) Which measurement technique would you use to obtain the wake vortex-shedding frequency and why?

(3 marks)

(iii) Which measurement technique would you use to measure the global velocity field of the wake structures? Describe the experimental equipment and procedures associated with your selected technique.

(5 marks)

(b) A very thin flat plate is immersed within a Mach number of M=3.2 free-stream and if linearized supersonic flows can be assumed, determine the lift and wave drag coefficients for angles-of-attack at 3° and 6° respectively.

(4 marks)

(c) With the aid of appropriate figures or plots, explain why a thicker aerofoil will incur a low critical Mach number.

(3 marks)

(d) Describe the key characteristics that a supercritical aerofoil should possess.

(4 marks)

(e) Vortex systems associated with sharp aircraft nose cones can become asymmetrical under some circumstances. Briefly describe these circumstances.

(3 marks)

4. A particular surface geometry, as shown in Figure 3 below, is to be tested within a supersonic wind tunnel at a Mach number of $M_\infty=2.2$ and static pressure of $p_\infty=0.5$bar.

(a) Sketch out the oblique shocks and expansion fans formed along the surface geometry.
(3 marks)

(b) Determine the Mach number and static pressures at regions 1, 2, 3 and 4 respectively.
(22 marks)

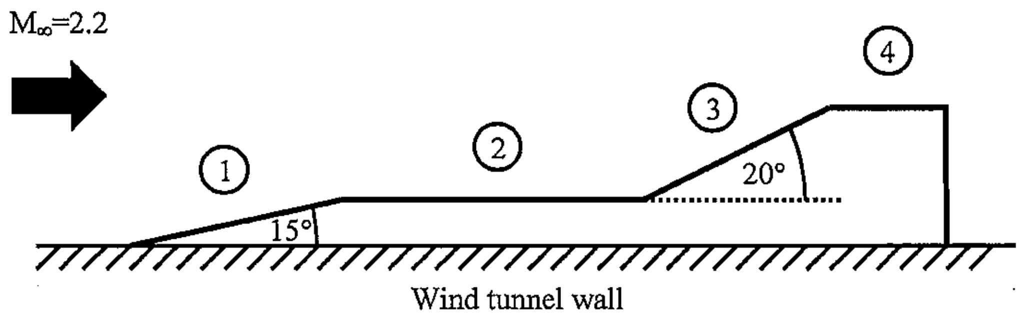

Figure 3

End of Paper

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1. a. i) - Invisid flow
- steady
- two-dimensional

ii) $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}(4x-y) - \frac{\partial}{\partial y}(x+4y)$

$= 4 - 4 = 0$

$\therefore$ flow field is irrotational

iii) $\frac{\partial \psi}{\partial y} = u = x + 4y$; integrate this eqn

$\psi = xy + 2y^2 + f(x)$; differentiate w.r.t. x

$\frac{\partial \psi}{\partial x} = y + f'(x)$

we also know that $\frac{\partial \psi}{\partial x} = -v = -(4x-y)$

comparing both equations, we have $f'(x) = -4x$

thus $f(x) = -2x^2$

$\therefore \psi = xy + 2y^2 - 2x^2$ //

$\frac{\partial \phi}{\partial x} = x + 4y$

$\phi = \frac{1}{2}x^2 + 4xy + f(y)$; diff. w.r.t y

$\frac{\partial \phi}{\partial y} = 4x + f'(y)$

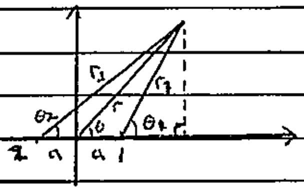
we know that $\frac{\partial \phi}{\partial y} = v = 4x - y$

by comparison, $f'(y) = -y$

$f(y) = -\frac{1}{2}y^2$

$\therefore \phi = \frac{1}{2}x^2 + 4xy - \frac{1}{2}y^2$ //

b)



stream equation by a source and a sink is given by:

$\psi = -\frac{Q}{2\pi}(\theta_2 - \theta_1)$ $\because$ 2 = source & 1 = sink

from trigonometry, we know that $r_2 \sin\theta_2 = r_1 \sin\theta_1 = r\sin\theta$

also, $r\cos\theta - a = r_1\cos\theta_1$; divide this eqn by $r_1\sin\theta_1 = r\sin\theta$

$\frac{r\cos\theta - a}{r\sin\theta} = \frac{r_1\cos\theta_1}{r_1\sin\theta_1}$

$\tan\theta_1 = \frac{r\sin\theta}{r\cos\theta - a}$ ... (a)

also, $r_2\cos\theta_2 = a + r\cos\theta$ divide this eqn by $r_2\sin\theta_2 = r\sin\theta$ and some manipulations,

$\tan\theta_2 = \frac{r\sin\theta}{r\cos\theta + a}$ ... (b) back to stream equation, rearrange, $2\pi \frac{\psi}{Q} = \theta_2 - \theta_1$

$$\tan\left(2\pi \frac{\psi}{Q}\right) = \tan(\theta_2 - \theta_1)$$

$$= \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$$

substitute $\tan\theta_1$ and $\tan\theta_2$, $= \frac{2ar\sin\theta}{a^2 - r^2}$

thus, $\psi = \frac{Q}{2\pi}\tan^{-1}\left(\frac{2ar\sin\theta}{a^2 - r^2}\right)$

approximation,

$$\psi \approx \frac{-Qar\sin\theta}{\pi(r^2 - a^2)}$$ // QED

c.i) - large angle of attack
- thick airfoil
- when boundary layer exists

ii) because both cases are based on assumption that the airfoil is thin such that free vortices can be relocated to the chordline. This similar assumption results in same maximum lift slope for both cases.

2.a) $\frac{dz}{dx} = 0.111\left(0.6 - \frac{x}{c}\right) \qquad 0 \le \frac{x}{c} \le 0.6$

$\frac{dz}{dx} = 0.0625\left(2.4 - 4\frac{x}{c}\right) \quad 0.6 \le \frac{x}{c} \le 1.0$

$\frac{x}{c} = \frac{1}{2}(1 - \cos\theta)$; for $\frac{x}{c} = 0.6$, $\theta = 1.77215$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\, d\theta$$

$$= \alpha - \frac{1}{\pi}\left(\int_0^{1.77215} 0.111\left(0.6 - \frac{1}{2}(1-\cos\theta)\right) d\theta + \int_{1.77215}^{\pi} 0.0625\left(2.4 - 2(1-\cos\theta)\right) d\theta\right)$$

$$= \alpha - \frac{1}{\pi}\left(0.111\left[0.1\theta + 0.5\sin\theta\right]_0^{1.77215} + 0.0625\left[0.4\theta + 2\sin\theta\right]_{1.77215}^{\pi}\right)$$

$$= \alpha + 0.00451648$$

$$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos\theta\, d\theta$$

$$= \frac{2}{\pi}\left(0.111\int_0^{1.77215} 0.1\cos\theta + 0.5\cos^2\theta\, d\theta + 0.0625\int_{1.77215}^{\pi} 0.4\cos\theta + 2\cos^2\theta\, d\theta\right)$$

we know that $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$. thus

$$A_1 = \frac{2}{\pi}\left(0.111\left[0.1\sin\theta + 0.25\theta + 0.125\sin 2\theta\right]_0^{1.77215} + 0.0625\left[0.4\sin\theta + \theta + 0.5\sin 2\theta\right]_{1.77215}^{\pi}\right)$$

$$= 0.08146034$$

$$C_L = \pi(2A_0 + A_1)$$

$$C_L\big|_{\alpha = 4^\circ} = \pi \times \left(2 \times \left(\frac{4}{180}\pi + 0.00451648\right) + 0.08146034\right)$$

$$= 0.722942$$ //

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. The MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

2.b. i) $\Gamma_0 = 32$

$\Gamma_0 = \frac{L}{\rho U_\infty \frac{b}{4}\pi}$ where $b = AR \times c = 12\,m$ and from table, at 4km, $\rho = 8.194 \times 10^{-1}\ kg\,m^{-3}$

$L = \Gamma_0 \rho U_\infty \frac{b}{4}\pi = 32 \times 0.8194 \times 80 \times \frac{12}{4}\pi$
$= 19770.02\ N$

at level flight, $L = W$,
thus weight that the wing can support $= 19770.02\ N$ //

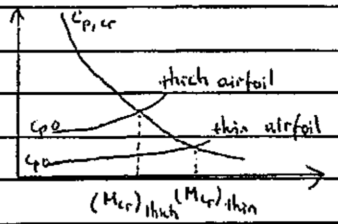
ii) $\omega = \frac{\Gamma_0}{2b} = \frac{32}{2\times 12} = 1.333\ ms^{-1}$ //

3.a. i) Laser-induced Flourescence
↳ laser is very precise in exciting flourescence dye of a plane of interest and thus getting cross-sectional view of that plane.
ii) Hot-wire anemometry (HWA)
↳ high response frequency
iii) Particle Image Velocimetry
↳ equipment needed - trackable small particles (smoke for air / hollow polymer sphere for water)
- laser and CCD camera
↳ procedures : - fluid with small particles is to be directed to flow on through a test section
- laser fired double laser pulses at small Δt along the same plane
- CCD camera is then triggered by the laser to capture laser pulse

b.) for AOA = 3°,
$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4}{\sqrt{3.2^2-1}} \times \frac{3}{180} \times \pi = 0.0689$
$C_D = \frac{4\alpha^2}{\sqrt{M_\infty^2-1}} = \frac{4}{\sqrt{3.2^2-1}} \times \left(\frac{3}{180}\times\pi\right)^2 = 0.0036076$

for AOA = 6°
$C_L = \frac{4}{\sqrt{3.2^2-1}} \times \frac{6}{180} \times \pi = 0.1378$
$C_D = \frac{4}{\sqrt{3.2^2-1}} \times \left(\frac{6}{180}\times\pi\right)^2 = 0.01443$

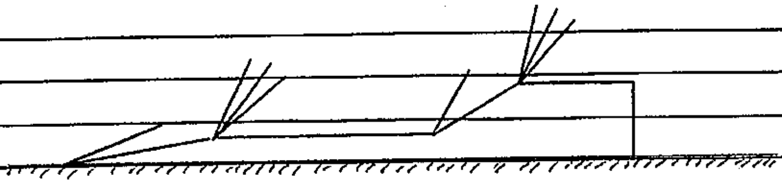
c.)



→ the thicker the aerofoil, local surface pressures are lower. This produces higher C_p magnitude and from the graph, this indicates that it will incur lower critical mach number

d) - relatively flat upper surface to achieve supersonic flow at lower local M values
- Negative camber up to 60% from L.E → reduce lift → cusped T.E. to impose +ve chamber
- More stable shock formation on upper surface and terminating shock is weaker and further downstream → ↓drag.

4.a.



b. at region ①, $M_\infty = 2.2$, from oblique shock chart, $\beta = 41.2°$

$M_{n\infty} = M_\infty \sin\beta = 1.44917$

from normal shock tables, for $M_{n\infty} = 1.44917$, and after extrapolation, $\frac{p_1}{p_\infty} = 2.283$, $M_{n1} = 0.71988$

ie $p_1 = 2.283 \times 0.5 = 1.1415$ bar #

$M_1 = \frac{M_{n1}}{\sin(\beta-\theta)} = 1.63051$ #

at region ②, expansion fan, from Prandtl-Meyer function, for $M_1 = 1.63051$, $\nu(M_1) = 15.744°$

$\nu(M_2) = \nu(M_1) + \theta = 30.744°$

referring back to the table, $M_2 = 2.163$ #

now reffering to isentropic flow table, for $M_1 = 1.63051$, $\frac{p_{01}}{p_1} = 4.4476$

for $M_2 = 2.163$, $\frac{p_{02}}{p_2} = 10.0918357$

since $p_{01} = p_{02}$, $p_2 = \frac{4.4476}{10.0918357} \times p_1 = 0.503074$ bar #

at region ③, from oblique shock table, $\beta = 47.95°$

$M_{n2} = M_2 \sin\beta = 1.6061586$

from normal shock table, for $M_{n2} = 1.6061586$, $\frac{p_3}{p_2} = 2.843$, $M_{n3} = 0.66659$

ie $p_3 = 2.843 \times 0.503074 = 1.43024$ bar #

$M_3 = \frac{M_{n3}}{\sin(\beta-\theta)} = 1.4222$ ~~bar~~ #

at region ④, Prandtl-Meyer function, for $M_3 = 1.4222$, $\nu(M_3) = 9.617°$

$\nu(M_4) = \nu(M_3) + \theta = 29.617°$

referring back to the table, $M_4 = 2.12$ #

from isentropic flow table, for $M_3 = 1.4222$, $\frac{p_{03}}{p_3} = 3.28278$

for $M_4 = 2.12$, $\frac{p_{04}}{p_4} = 9.435743$

since $p_{03} = p_{04}$, $p_4 = \frac{3.28278}{9.435743} \times 1.43024$ bar $= 0.49759$ bar ##

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2014-2015

MA3701 – AERODYNAMICS I

AE 2003 - AERODYNAMICS

November/December 2014 Time allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises of **SIX (6)** pages.
2. Answer **ALL** questions. Support your answers with formulae and figures whenever appropriate.
3. All questions carry equal marks.
4. This is an **OPEN-BOOK** examination.

1(a) In a two-dimensional idealized flow, a source and a sink are located at (-4 m, 0 m) and (4 m, 0 m) respectively in a Cartesian coordinate system, where their corresponding strengths are 6 m^2/s and -6 m^2/s. They are immersed in a uniform free stream which flows along the x-axis direction with a velocity of 15 m/s. Assuming that a Rankine oval is formed by these flow conditions, determine

(i) the full length of the Rankine oval, (2 marks)
(ii) the axial location of the stagnation point along the negative x-axis. (4 marks)

(b) For a two-dimensional idealized flow, a doublet located within a uniform free-stream can be used to approximate the inviscid flow behaviour of a round cylinder immersed in a free-stream. Starting with the basic stream function and velocity potential expressions for a doublet and a uniform flow, show <u>in detail</u> that the following stream function ψ and velocity potential ϕ expressions can be obtained for the above combined flow scenario:

$$\psi = Ur\left(1 - \frac{a^2}{r^2}\right)\sin\theta$$

$$\phi = Ur\left(1 + \frac{a^2}{r^2}\right)\cos\theta$$

where U is the uniform flow velocity, r is radial distance away from the coordinate origin, a is the doublet radius and θ is the angular displacement from the positive x-axis.

(4 marks)

Note: Question 1 continues on page 2.

6

(c) For an experimental research project, an air jet is discharged from a 5 cm diameter round nozzle with an exit velocity of 30 m/s into a quiescent environment within a laboratory. To investigate and understand the air jet flow characteristics, flow visualizations and measurements will have to be conducted.

(i) Recommend and describe briefly a flow visualization technique if the instantaneous cross-sections of the jet flow structures are to be visualized. Justify your recommended technique.

Fl

(4 marks)

(ii) If the exit velocity is increased to 300 m/s, recommend and describe briefly a flow visualization technique by which the air jet should be visualized. Justify your recommended technique.

(4 marks)

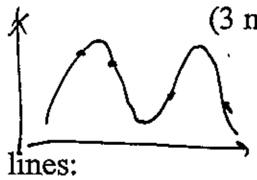
(iii) For the flow conditions described in Part (ii), recommend and describe briefly a flow measurement technique by which the global instantaneous velocity field of air jet should be measured. Justify your recommended technique.

Part

(4 marks)

(iv) Using simple illustrations, describe aliasing effects and their relationship with Nyquist frequency within the context of flow measurements based on data-acquisition devices.

$f_{ny} = \frac{1}{2} f_{sample}$



(3 marks)

2(a) A two-dimensional aerofoil has the following mean camber lines:

$$\frac{z}{c} = 1.1072\left(\frac{x}{c}\right)^3 - 0.9632\left(\frac{x}{c}\right)^2 + 0.2523\left(\frac{x}{c}\right) \qquad 0 \le \frac{x}{c} \le 0.29$$

$$\frac{z}{c} = 0.027\left(1 - \frac{x}{c}\right) \qquad 0.29 \le \frac{x}{c} \le 1$$

Assuming standard definitions for the above equations and that Thin Aerofoil Theory is valid, determine

(i) the coefficients A_0, A_1 and A_2,

(15 marks)

(ii) the zero-lift angle-of-attack, $\alpha_{L=0}$, and

(3 marks)

(iii) the lift-coefficient at $\alpha = 5°$

(2 marks)

Note: Question 2 continues on page 3.

(b) A small unmanned aerial vehicle is to have a tapered wing based on a SD7032 aerofoil, where the wing half-span geometry has the dimensions shown in Figure 1. Provided that the lift curve for the SD7032 aerofoil is provided in Figure 2 and that Figure 3 shows the relationship between δ and the taper-ratio, determine the lift and induced drag coefficient of the entire wing at a geometric angle-of-attack of $\alpha = 4°$. Assume standard definitions for the variables used in this question and that $\tau = \delta$.

(5 marks)

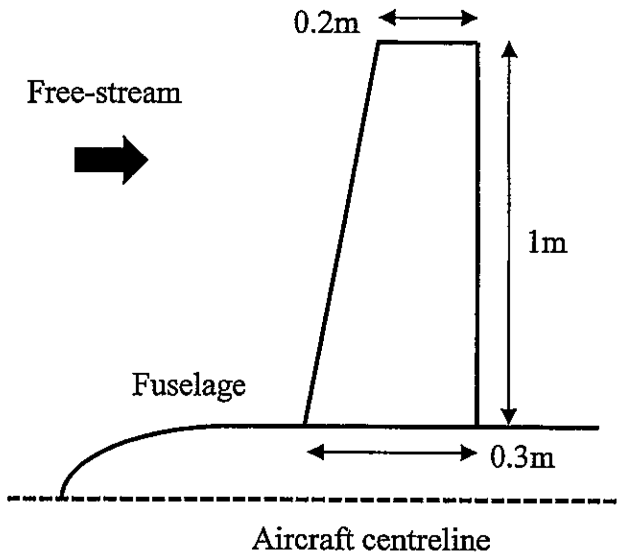


Figure 1: Geometry of the wing half-span

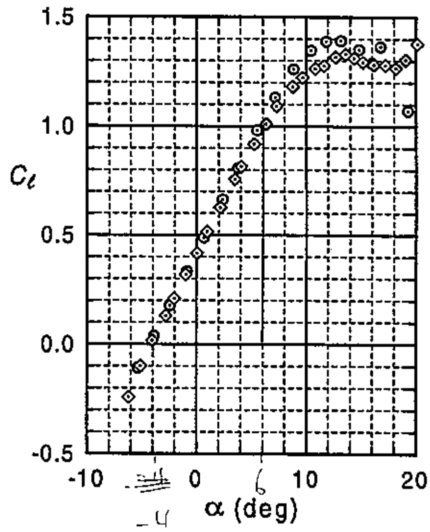


Figure 2: Lift curve for SD7032 aerfoil

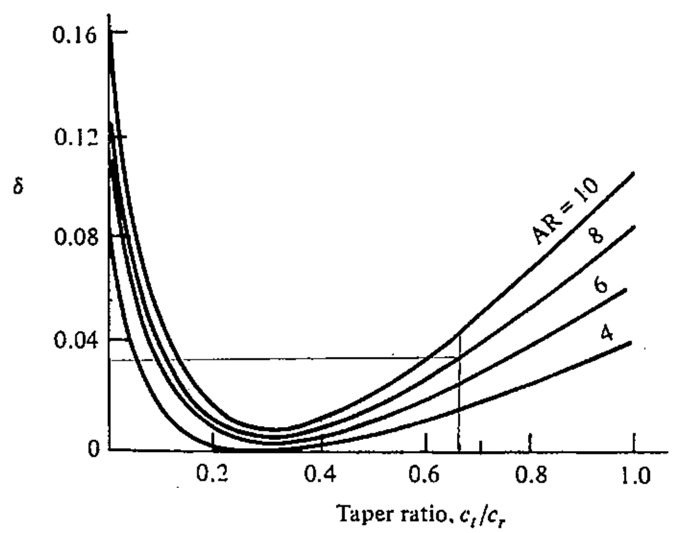


Figure 3: Graphical relationship between δ and taper-ratio

3. Consider the following geometry of a supersonic wind tunnel (Figure 4):

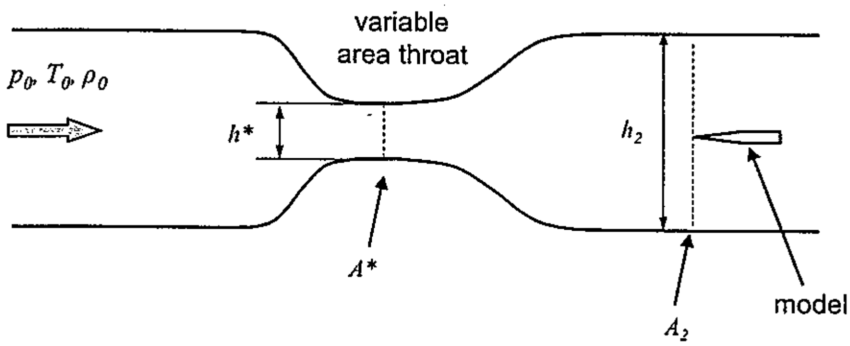


Figure 4: Geometry of a supersonic wind tunnel with a variable area throat.

The stagnation point values upstream of the throat are $T_0 = 450\text{K}$, $p_0 = 7.82$ bar, and $\rho_0 = 6.06$ kg/m^3. The height of the wind tunnel test section is $h_2 = 0.4$ m. The extend of the wind tunnel in the third dimension is $z = 0.4$ m throughout the entire wind tunnel, including the throat. The flow is assumed to be adiabatic and isentropic.

(a) Determine the area of the throat section A^* and its height h^* , if the Mach number in the test section is to be $M_2 = 2$.

(2 marks)

(b) Determine the flow velocity in the throat.

(3 marks)

(c) Determine the mass flow rate in the throat.

(3 marks)

(d) If the wind tunnel is fed upstream by a pressure vessel, and the pressure vessel contains 1.4 tons of air, determine how long it takes to empty the vessel.

(2 marks)

(e) Determine the flow velocity in the test section.

(3 marks)

(f) Determine the pressure in the test section.

(2 marks)

(g) Determine the stagnation point temperature on the model in the test section.

(2 marks)

Note: Question 3 continues on page 5.

(h) Which parameter needs to be changed to increase the flow velocity in the test section to $M_2 = 2.5$? Determine the mass flow rate in the throat under these conditions and the time it takes to empty the pressure vessel.

(6 marks)

(i) Comment on your results in (d) and (h).

(2 marks)

4(a) Consider a supersonic flow on a ramp consisting of two segments (Figure 5) under flow conditions found at 13,000 m altitude ($p_\infty = 16{,}500$ Pa, $T_\infty = 216$K, $p_\infty = 0.265$ kg/m^3) and a freestream Mach number of $M_\infty = 2$.

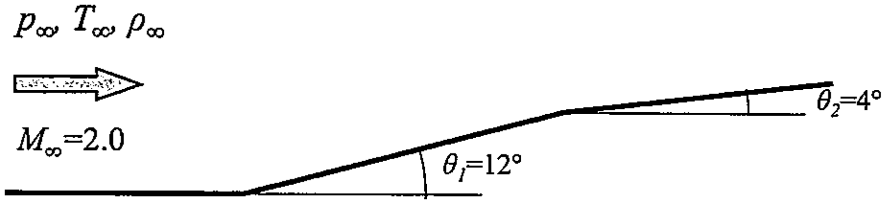


Figure 5: Supersonic flow on a ramp

(i) Sketch the shock/expansion pattern.

(2 marks)

(ii) Determine the pressure coefficient on the two ramp surfaces using Prandtl-Meyer theory.

(6 marks)

(iii) Determine the pressure coefficient on the two ramp surfaces using linear theory.

(2 marks)

(iv) Comment on the difference between the two results from (ii) and (iii) and explain, which result you consider more accurate.

(2 marks)

(b) Determine the pressure coefficient c_p in terms of temperature and freestream velocity.

(5 marks)

Note: Question 4 continues on page 6.

(c) Consider the Boeing B-52 Stratofortress, which is a further development of the Boeing B-29 Superfortress (Figure 6). Since the B-52 is commonly operating in transonic conditions, explain the aerodynamic design choices that lead to the final design of the B-52 (Figure 7).

(8 marks)

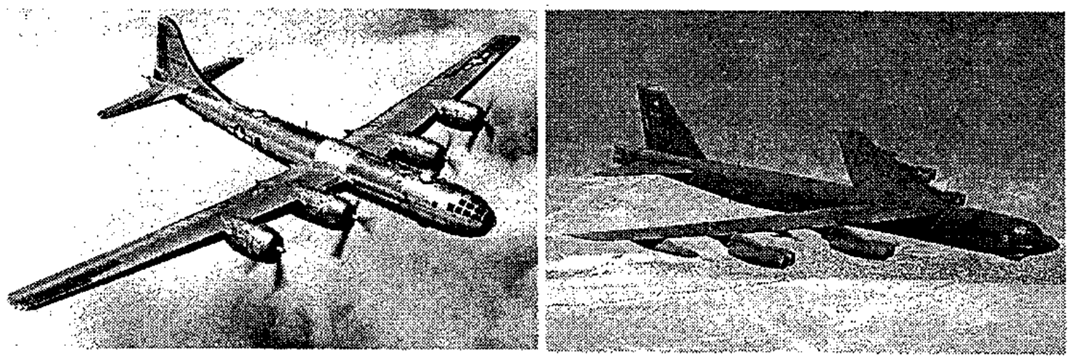


Figure 6: Comparison of the Boeing B-29 Superfortress (left) and the B-52 Stratofortress (right)

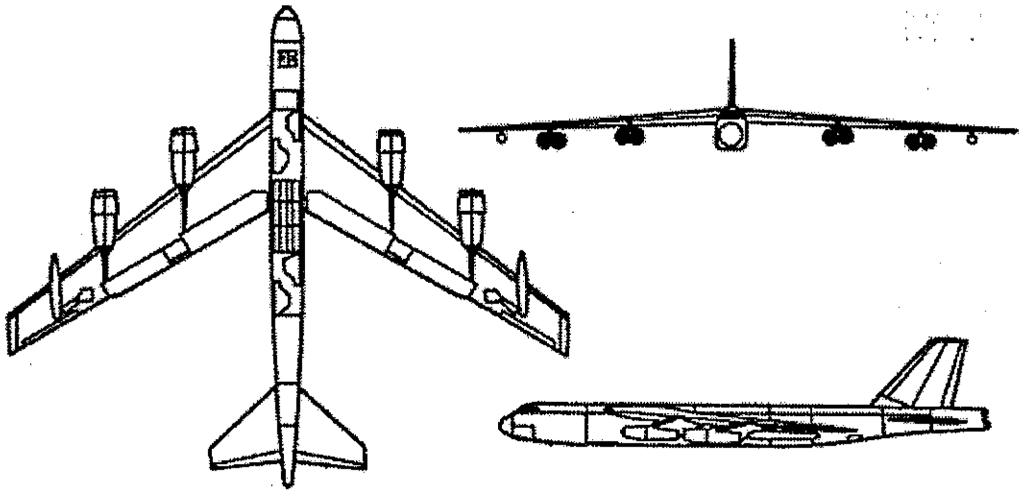


Figure 7: Boeing B-52 Stratofortress 3-side view.

END OF PAPER

November December 2014

MA3701 Date No.

1) (a)(i) Rankine oval half length $= \left(\frac{Qa}{\pi U} + a^2\right)^{1/2}$

$$= \left[\frac{(6\,m^3/s)(4\,m)}{\pi(15\,m/s)} + (4\,m)^2\right]^{1/2}$$

$$= 4.06\,m$$

$\therefore$ Full length $= 2 \times 4.06\,m = 8.13\,m$

(ii) Let $u = U + \frac{Q}{2\pi}\left[\frac{x-a}{(x-a)^2+y^2} - \frac{x+a}{(x+a)^2+y^2}\right] = 0$

$$15 + \frac{6}{2\pi}\left[\frac{x-4}{(x-4)^2+0} - \frac{x+4}{(x+4)^2+0}\right] = 0$$

$$\frac{1}{x-4} + \frac{1}{x+4} = 5\pi$$

$$\frac{2x}{x^2-16} = 5\pi$$

$$5\pi x^2 - 2x - 80\pi = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5\pi)(-80\pi)}}{2(5\pi)}$$

$$= \frac{2 \pm \sqrt{4 + 1600\pi^2}}{10\pi}$$

$$= 4.06 \text{ (rej) or } -3.94$$

$\therefore x = -3.94$

(b) For uniform flow / For doublet,

$\psi = Uy = Ur\sin\theta$ / $\psi = -\frac{K\sin\theta}{r}$

$\phi = Ux = Ur\cos\theta$ / $\phi = \frac{K\cos\theta}{r}$

For doublet in uniform flow, $\psi = Ur\sin\theta - \frac{K\sin\theta}{r}$

Let $\psi = 0$ be the boundary $\Rightarrow 0 = Ua\sin\theta - \frac{K\sin\theta}{a}$

$$K = Ua^2$$

$\therefore \psi = Ur\sin\theta - \frac{Ua^2\sin\theta}{r} = Ur\left(1 - \frac{a^2}{r^2}\right)\sin\theta$

$\therefore \phi = Ur\cos\theta + \frac{Ua^2\cos\theta}{r} = Ur\left(1 + \frac{a^2}{r^2}\right)\cos\theta$

(c) (i) - Laser-induced fluorescence

- Dyes absorb energy at excitation frequency and re-emit fluorescence
- Needs laser of certain frequency (~532nm)
- Useful for taking flow cross-sections

(ii) - Schlieren photography

- Typically high speed flows with shocks
- Change in density causes change in refractive index
- Formation of patterns for flow visualisation

A'ZONE

__DISCLAIMER:__ The solutions are done by students who scored A or above in this subject. The MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.

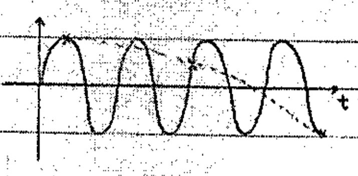
P(1)

(iii) - Particle image velocimetry based on stroboscopic photography of moving particles in fluid flows

- Can track flows reasonably well for sufficiently small particles
- Determine velocity components with cross-correlation technique for 2 images taken in quick succession
- Global measurement technique which measures entire flow field for that instance

(iv) - Frequencies higher than sampling frequency may appear as a low frequency

- Only frequencies lower than Nyquist frequency can be measured correctly, $f_{Nyquist} = \frac{1}{2} f_{sample}$
- Use a low pass filter to block frequencies higher than $f_{Nyquist}$



2) (a) (i) $\frac{z}{c} = 1.1072\left(\frac{x}{c}\right)^3 - 0.9632\left(\frac{x}{c}\right)^2 + 0.2523\left(\frac{x}{c}\right) \qquad 0 \le \frac{x}{c} \le 0.29$

$\frac{z}{c} = 0.027\left(1 - \frac{x}{c}\right) \qquad 0.29 \le \frac{x}{c} \le 1$

Let $\frac{x}{c} = 0.5(1-\cos\theta) = 0.29$

$\theta = 1.137 \text{ rad}$

$\Rightarrow \frac{dz}{dx} = 3.3216\left(\frac{x}{c}\right)^2 - 1.9264\left(\frac{x}{c}\right) + 0.2523 \qquad \Rightarrow \frac{dz}{dx} = -0.027 \quad 1.137 \le \theta \le \pi$

$= 3.3216(0.5)^2(1-\cos\theta)^2 - 1.9264(0.5)(1-\cos\theta) + 0.2523$

$= 0.8304(1 - 2\cos\theta + \cos^2\theta) - 0.9632(1-\cos\theta) + 0.2523$

$= -0.8304\cos^2\theta - 0.6976\cos\theta + 0.1195 \qquad 0 \le \theta \le 1.137$

$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\, d\theta$

$= \alpha - \frac{1}{\pi}\int_0^{1.137} -0.8304\cos^2\theta - 0.6976\cos\theta + 0.1195\, d\theta - \frac{1}{\pi}\int_{1.137}^{\pi} -0.027\, d\theta$

$= \alpha - \frac{1}{\pi}\int_0^{1.137} -0.4152(\cos 2\theta + 1) - 0.6976\cos\theta + 0.1195\, d\theta - \frac{1}{\pi}\int_{1.137}^{\pi} -0.027\, d\theta$

$= \alpha + \frac{1}{\pi}\int_0^{1.137} 0.4152\cos 2\theta + 0.6976\cos\theta + 0.2957\, d\theta + \frac{1}{\pi}\int_{1.137}^{\pi} 0.027\, d\theta$

$= \alpha + \frac{1}{\pi}\left[0.2076\sin 2\theta + 0.6976\sin\theta + 0.2957\theta\right]_0^{1.137} + \frac{1}{\pi}\left[0.027\theta\right]_{1.137}^{\pi}$

$= \alpha + 0.3589 + 0.01723$

$= \alpha + 0.3762$

Note:

$\cos^2\theta = 0.5(\cos 2\theta + 1)$

$\cos^3\theta = 0.25(\cos 3\theta + 3\cos\theta)$

$\cos^4\theta = 0.125(3 + 4\cos 2\theta + \cos 4\theta)$

$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos\theta\, d\theta$

$= \frac{2}{\pi}\int_0^{1.137} -0.8304\cos^3\theta - 0.6976\cos^2\theta + 0.1195\cos\theta\, d\theta + \frac{2}{\pi}\int_{1.137}^{\pi} -0.027\cos\theta\, d\theta$

$= \frac{2}{\pi}\int_0^{1.137} -0.2076(\cos 3\theta + 3\cos\theta) - 0.3488(\cos 2\theta + 1) + 0.1195\cos\theta\, d\theta + \frac{2}{\pi}\int_{1.137}^{\pi} -0.027\cos\theta\, d\theta$

$= \frac{2}{\pi}\int_0^{1.137} -0.2076\cos 3\theta - 0.3488\cos 2\theta - 0.5033\cos\theta - 0.3488\, d\theta + \frac{2}{\pi}\int_{1.137}^{\pi} -0.027\cos\theta\, d\theta$

$= \frac{2}{\pi}\left[-0.0692\sin 3\theta - 0.1744\sin 2\theta - 0.5033\sin\theta - 0.3488\theta\right]_0^{1.137} + \frac{2}{\pi}\left[-0.027\sin\theta\right]_{1.137}^{\pi}$

$= -0.6162 + 0.01560$

$= -0.6006$

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$A_2 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx} \cos 2\theta \, d\theta$

$= \frac{2}{\pi}\int_0^{1.137} (-0.8304\cos^2\theta - 0.6976\cos\theta + 0.1195)(2\cos^2\theta - 1)\, d\theta + \frac{2}{\pi}\int_{1.137}^{\pi} -0.027\cos 2\theta \, d\theta$

$= \frac{2}{\pi}\int_0^{1.137} -1.6608\cos^4\theta - 1.3952\cos^3\theta + 1.0694\cos^2\theta + 0.6976\cos\theta - 0.1195 \, d\theta + \frac{2}{\pi}\int_{1.137}^{\pi} -0.027\cos 2\theta \, d\theta$

$= \frac{2}{\pi}\int_0^{1.137} -0.4152(3 + 4\cos 2\theta + \cos 4\theta) - 0.3488(\cos 3\theta + 3\cos\theta) + 0.5347(\cos 2\theta + 1) + 0.6976\cos\theta - 0.1195 \, d\theta + \frac{2}{\pi}\int_{1.137}^{\pi} -0.027\cos 2\theta \, d\theta$

$= \frac{2}{\pi}\int_0^{1.137} -0.4152\cos 4\theta - 0.3488\cos 3\theta - 1.1261\cos 2\theta - 0.3488\cos\theta - 0.8304 \, d\theta + \frac{2}{\pi}\int_{1.137}^{\pi} -0.027\cos 2\theta \, d\theta$

$= \frac{2}{\pi}\left[-0.1038\sin 4\theta - \frac{0.3488}{3}\sin 3\theta - 0.56305\sin 2\theta - 0.3488\sin\theta - 0.8304\theta\right]_0^{1.137} + \frac{2}{\pi}\left[-0.0135\sin 2\theta\right]_{1.137}^{\pi}$

$= -0.9910 + 0.006552$

$= -0.9845$

(ii) $\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta - 1)\, d\theta$

$= -\frac{1}{2}(-0.6006) - 0.3762$

$= -0.07587$ rad

$= -4.35°$

(iii) At $\alpha = 5° = \frac{5\pi}{180}$ rad,

$A_0 = \alpha + 0.3762$

$= \frac{5\pi}{180} + 0.3762$

$= 0.4634$

$\therefore C_L = \pi(2A_0 + A_1)$

$= \pi\left[2(0.4634) - 0.6006\right]$

$= 1.025$

(b) From figure 2,

$\alpha = 4° \Rightarrow C_L = 0.8$

$S = \frac{1}{2}(1m)(0.2m + 0.3m)$

$= 0.25m^2$

$AR = \frac{b^2}{S}$

$= \frac{(1m)^2}{0.25m^2}$

$= 4$

Taper ratio $\frac{C_t}{C_r} = \frac{0.2m}{0.3m} = 0.67$

From figure 3,

$AR = 4$, $\frac{C_t}{C_r} = 0.67 \Rightarrow \delta = 0.015$

$\therefore C_{D,i} = \frac{C_L^2}{\pi AR}(1+\delta)$

$= \frac{(0.8)^2}{4\pi}(1.015)$

$= 0.0517$

3) (a) $A_2 = h_2 z$

$= (0.4m)(0.4m)$

$= 0.16m^2$

For sonic conditions at throat,

$$\frac{A^*}{A_2} = M_2\left\{\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_2^2\right)\right\}^{-(\gamma+1)/2(\gamma-1)}$$

$$\frac{A^*}{0.16m^2} = 2\left[\frac{2}{2.4}\left(1+0.2\times 2^2\right)\right]^{-\frac{2.4}{0.8}}$$

$\therefore A^* = 0.0948m^2 \approx 0.095m^2$

$h^* = \frac{A^*}{z}$

$= \frac{0.0948m^2}{0.4m}$

$= 0.237m \approx 0.28m$

(b) For isentropic flow,

$T_0^* = T_{0_1} = 450K$

$\Rightarrow \frac{T_0^*}{T^*} = 1+\frac{\gamma-1}{2}M^{*2}$

$\frac{450K}{T^*} = 1+0.2(1)^2$

$T^* = 375K$

$\therefore U^* = M^*\sqrt{\gamma R T^*}$

$= 1\times\sqrt{(1.4)(280J/kgK)(375K)}$

$= 388.2m/s$

(c) For isentropic flow,

$\rho_0^* = \rho_{0_1} = 6.06kg/m^3$

$\Rightarrow \frac{\rho_0^*}{\rho^*} = \left(1+\frac{\gamma-1}{2}M^{*2}\right)^{1/\gamma-1}$

$\frac{6.06kg/m^3}{\rho^*} = \left[1+0.2(1)^2\right]^{1/0.4}$

$\rho^* = 3.84kg/m^3$

$\therefore \dot{m}^* = \rho^* A^* U^*$

$= (3.84kg/m^3)(0.0948m^2)(388.2m/s)$

$= 141.4kg/s$

(d) Time taken $= \frac{1.4\ tons}{\dot{m}^*}$

$= 1400kg \div 141.4kg/s$

$= 9.90s$

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(e) For isentropic flow,

$T_{o_2} = T_{o_1} = 450K$

$\Rightarrow \frac{T_{o_2}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2$

$\frac{450K}{T_2} = 1 + 0.2(2)^2$

$T_2 = 250K$

$\therefore U_2 = M_2 \sqrt{\gamma R T_2}$

$= 2 \times \sqrt{(1.4)(287 J/kgK)(250K)}$

$= 633.9 m/s$

(f) For isentropic flow,

$P_{o_2} = P_{o_1} = 7.82 \text{ bar}$

$\frac{P_{o_2}}{P_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\gamma/\gamma-1}$

$\frac{7.82 bar}{P_2} = \left[1 + 0.2(2)^2\right]^{1.4/0.4}$

$\therefore P_2 = 1.00 \text{ bar}$

(g) For isentropic flow,

$T_{o_2} = T_{o_1} = 450K$

(h) For $M_2 = 2.5$ at test section,

$\frac{A^*}{A_2} = M_2 \left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_2^2\right)\right]^{-\gamma+1/2(\gamma-1)}$

$\frac{A^*}{0.16m^2} = 2.5 \left[\frac{2}{2.4}\left(1 + 0.2 \times 2.5^2\right)\right]^{-\frac{2.4}{0.8}}$

$A^* = 0.06068 m^2$

$h^* = \frac{A^*}{2}$

$= \frac{0.06068 m^2}{0.4m}$

$= 0.152 m$

$\therefore$ Reduce height of throat area to $h^* = 0.15m$

For sonic flow at throat,

$U^* = 388.2 m/s$, $\rho^* = 3.84 kg/m^3$ from parts (b), (c)

$\therefore \dot{m}^* = \rho^* A^* U^*$

$= (3.84 kg/m^3)(0.06068 m^2)(388.2 m/s)$

$= 90.49 kg/s = 90.5 kg/s$

$\therefore$ Time taken $= \frac{1.4 \text{ tons}}{\dot{m}^*}$

$= 1400 kg \div 90.49 kg/s$

$= 15.5s$

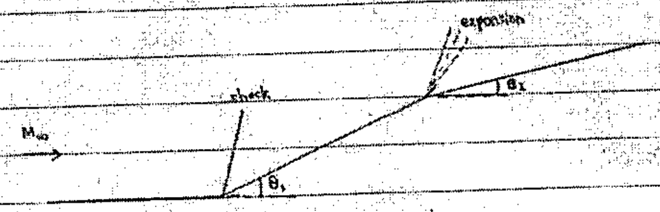
A'ZONE

P3

(i) - Takes longer time in (h) despite higher flow velocity in test section of Mach 25

- Mass flow rate constrained by variable throat area which is smaller in (h)

4) (a)(i)



(ii) From oblique shock graph (pg 613),

$M_\infty = 2.0,\ \theta_1 = 12° \Rightarrow \beta_1 = 45°$

$M_{n_\infty} = M_\infty \sin\beta_1$

$= 2.0 \sin 45°$

$= 1.414$

Using shock relations,

$$\frac{p_1}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_\infty}^2 - 1)$$

$$\frac{p_1}{16500\text{Pa}} = 1 + \frac{2.8}{2.4}(1.414^2 - 1)$$

$p_1 = 35750\text{Pa}$

$$\therefore C_{p_1} = \frac{2}{\gamma M_\infty^2}\left(\frac{p_1}{p_\infty} - 1\right)$$

$$= \frac{2}{(1.4)(2)^2}\left(\frac{35750}{16500} - 1\right)$$

$= 0.417$

Using shock relations,

$$M_{n_1}^2 = \frac{1 + \frac{\gamma-1}{2}M_{n_\infty}^2}{\gamma M_{n_\infty}^2 - \frac{\gamma-1}{2}}$$

$$= \frac{1 + 0.2(1.414)^2}{1.4(1.414)^2 - 0.2}$$

$M_{n_1} = 0.7338$

$$M_1 = \frac{M_{n_1}}{\sin(\beta_1 - \theta_1)}$$

$= 0.7338 \div \sin(45° - 12°)$

$= 1.347$

Using Appendix C,

$\nu(M_2) = \theta_2 + \nu(M_1)$

$= 8° + \nu(1.347)$

$= 8° + 7.279°$

$= 15.28° \Rightarrow M_2 = 1.620$

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For isentropic flow,

$$\frac{P_2}{P_1} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\gamma/\gamma-1}$$

$$\frac{P_2}{35750Pa} = \left[\frac{1+0.2(1.343)^2}{1+0.2(1.629)^2}\right]^{1.4/0.4}$$

$P_2 = 24140Pa$

$$\therefore C_{p_2} = \frac{2}{\gamma M_\infty^2}\left(\frac{P_2}{P_\infty}-1\right)$$

$$= \frac{2}{(1.4)(2)^2}\left(\frac{24140}{16500}-1\right)$$

$= 0.165$

(ii) Using linear theory,

$$C_{p_1} = \frac{2\theta_1}{\sqrt{M_\infty^2-1}}$$

$$= \frac{2\left(12\times\frac{\pi}{180}\text{rad}\right)}{\sqrt{2^2-1}}$$

$= 0.242$

$$C_{p_2} = \frac{2\theta_2}{\sqrt{M_\infty^2-1}}$$

$$= \frac{2\left(4\times\frac{\pi}{180}\text{rad}\right)}{\sqrt{2^2-1}}$$

$= 0.0806$

(iii) - Prandtl-Meyer theory yields higher C_p values than linear theory

- Linear theory assumes small perturbations which may be invalid in particular at section 1 due to large AOA

(b) For isentropic flow,

$$\frac{T}{T_\infty} = \left(\frac{P}{P_\infty}\right)^{\gamma-1/\gamma}$$

$$\frac{P}{P_\infty} = \left(\frac{T}{T_\infty}\right)^{\gamma/\gamma-1}$$

$$P = P_\infty\left(\frac{T}{T_\infty}\right)^{\gamma/\gamma-1}$$

$$C_p = \frac{P-P_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$$

$$= \frac{P_\infty\left[\left(\frac{T}{T_\infty}\right)^{\gamma/\gamma-1}-1\right]}{\frac{1}{2}\rho_\infty U_\infty^2}$$

$$= \frac{2\rho_\infty RT_\infty\left[\left(\frac{T}{T_\infty}\right)^{\gamma/\gamma-1}-1\right]}{\rho_\infty U_\infty^2}$$ ~~for ideal gas~~

$$= \frac{2RT_\infty\left[\left(\frac{T}{T_\infty}\right)^{\gamma/\gamma-1}-1\right]}{U_\infty^2}$$

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(c) - Thinner aerofoil ⇒ increase critical Mach number

- Increased wing sweep ⇒ increase critical Mach number
- Supercritical aerofoil with relatively flat top ⇒ creates weak shock nearer to trailing edge ⇒ more lift generated
- Area rule applied to fuselage ⇒ thinner at certain sections to avoid sudden changes in aircraft cross-section area ⇒ less drag

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

MA3701 – AERODYNAMICS

November/December 2015 | Time allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises of **FIVE (5)** pages.
2. Answer **ALL** questions. Support your answers with formulae and figures whenever appropriate.
3. All questions carry equal marks.
4. This is an **OPEN-BOOK examination.**

---

1 (a) In a two-dimensional idealized flow, a source of strength $5m^2/s$ and a sink of strength $3m^2/s$ are located at (-3m, 1m) and (2m, 4m) respectively. Assuming Potential Flow Theory is valid, determine

(i) the resultant velocity magnitude at (0m, 0m), and

(7 marks)

(ii) the direction of the resultant velocity vector, with respect to the positive x-axis.

(2 marks)

(b) A student is tasked to conduct a flow visualization and measurement investigation on a sharp-tipped cylinder immersed in a water tunnel as shown in Figure 1 below.

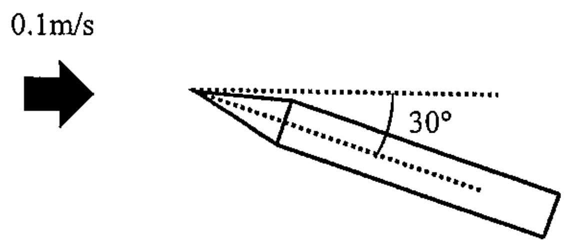

Figure 1

Note: Question 1 continues on page 2.

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(i) If the vortex systems associated with the flow past the sharp-tipped cylinder are to be visualized three-dimensionally, recommend and justify a suitable flow visualization technique.

(4 marks)

(ii) If the cross-sections of the vortex systems associated with the flow past the sharp-tipped cylinder are to be visualized, recommend and justify a suitable flow visualization technique.

(4 marks)

(iii) If the global instantaneous velocity field associated with the vortex systems associated with the flow past the sharp-tipped cylinder are to be measured, recommend and justify a suitable flow measurement technique.

(4 marks)

(iv) If the above flow scenario is to be tested using a Mach number = 2 supersonic free-stream, recommend and justify a suitable flow visualization technique to visualize the shock systems.

(4 marks)

2 (a) A two-dimensional aerofoil has the following mean camber lines:

$$\frac{z}{c} = 0.4\left[\left(\frac{x}{c}\right) - 2.5\left(\frac{x}{c}\right)^2\right] \qquad 0 \le \frac{x}{c} \le 0.2$$

$$\frac{z}{c} = 0.0625\left[0.6 + 0.4\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right] \qquad 0.2 \le \frac{x}{c} \le 1$$

Assuming standard definitions for the above equations and that Thin Aerofoil Theory applies, determine

(i) the coefficients A_0, A_1 and A_2,

(15 marks)

(ii) the zero-lift angle-of-attack, $\alpha_{L=0}$, and

(2 marks)

(iii) the lift-coefficient at α=6°.

(2 marks)

(b) An aerofoil profile with a lift-slope of 0.2/° and zero-lift angle-of-attack of -5° is selected to be the cross-section of a rectangular wing with a wing-span of 5.2m and wing-chord of 50cm. For an operating angle-of-attack of 3°, determine

(i) the lift coefficient of the wing, and

(4 marks)

(ii) the induced-drag coefficient of the wing.

(2 marks)

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3 (a) Figure 2(a) shows a tapered, untwisted wing with the physical dimensions shown. Assuming that the wing is based on an SD7032 aerofoil cross-section and that it is immersed in a free-stream at an angle-of-attack of 3°, by making use of the information provided in Figure 2(b), determine

(i) the wing aspect-ratio and taper-ratio,

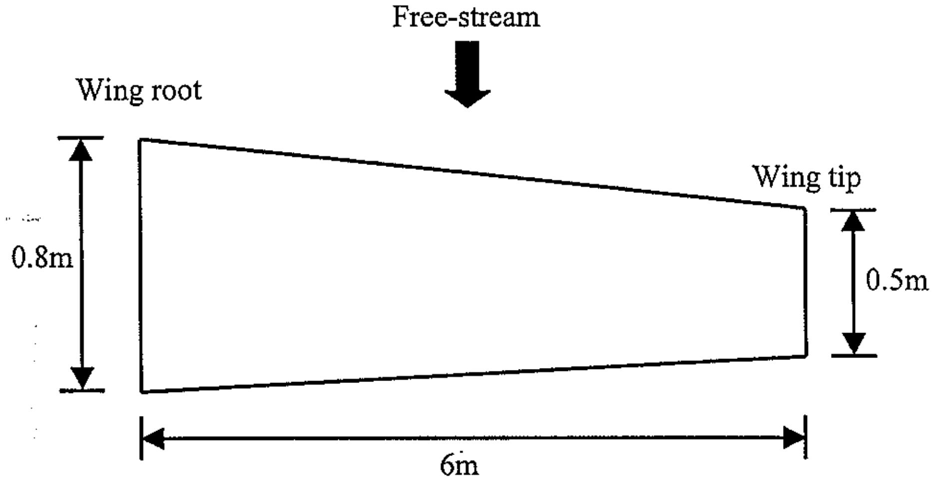
(4 marks)

(ii) wing surface area and parameter μ, and

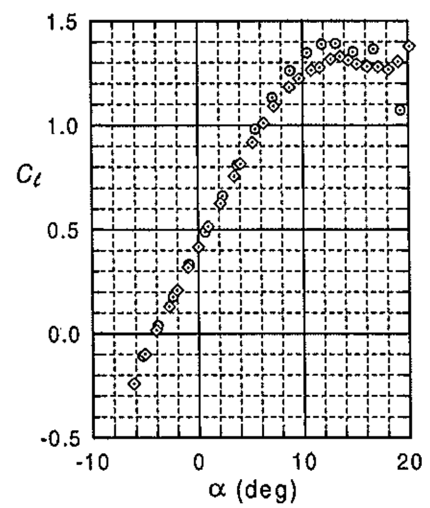
(4 marks)

(iii) the simplified monoplane equations associated with ϕ=22.5° and 45°.

(10 marks)



(a)



(b)

Figure 2

Note: Question 3 continues on page 5.

(b) Delta-wing planforms have proved to be very popular in the last generation of military aircrafts. Explain

(i) the engineering rationale behind such popularity, and

(4 marks)

(ii) how the use of canards offer delta-wing aircrafts achieve better flight stability, as compared to elevators or tail-planes.

(3 marks)

4 (a) Figure 3 shows the plan-view of an experimental aerodynamic body immersed in a M_∞=2.8, p_∞=0.8bar supersonic free-stream. Based on the information provided in Figure 3,

(i) sketch out all the oblique shocks and expansions fans formed at the various physical junctions, and

(2 marks)

(ii) by assuming only weak shocks are formed, determine the static pressures and Mach numbers at Regions 1 and 2.

(10 marks)

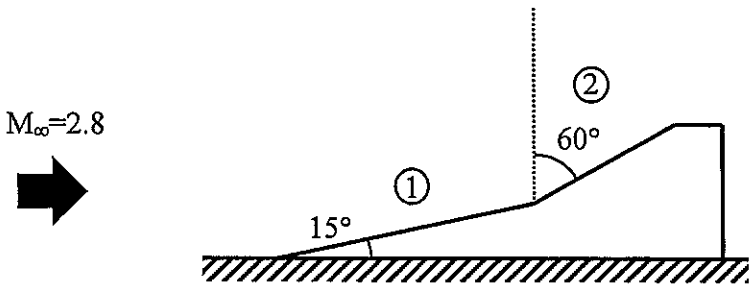


Figure 3

(b) With the aid of drawings, briefly explain how the Whitcomb area-rule and supercritical aerofoil work.

(10 marks)

(c) Why a starting vortex is always produced when a wing accelerates from rest?

(3 marks)

End of Paper

MA3701

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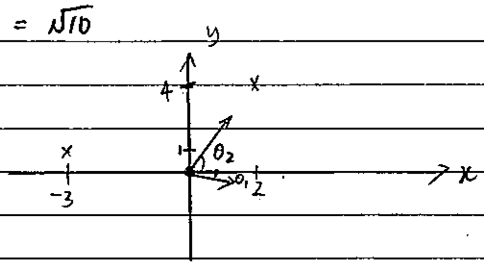
November / December 2015

1a) V_{r_1} at (0,0) due to source, $r = \sqrt{1^2+3^2} = \sqrt{10}$

$V_{r_1} = \frac{5}{2\pi\sqrt{10}} = 0.25165$ m/s

~~$\theta_1 = \tan^{-1}(1/3) = 18.43°$~~



V_{r_2} due to sink, $r = \sqrt{2^2+4^2} = \sqrt{20}$

$V_{r_2} = \frac{3}{2\pi\sqrt{20}} = 0.10676$ m/s

$\theta_2 = \tan^{-1}(4/2) = 63.43°$

Resultant velocity magnitude $= \sqrt{(0.25165\cos 18.43° + 0.10676\cos 63.43)^2 + (0.10676\sin 63.43 - 0.25165\sin 18.43)^2}$

$= 0.28694$ m/s *

ii) $V_x = 0.25165\cos 18.43° + 0.10676\cos 63.43° = 0.286496$ m/s

$V_y = 0.10676\sin 63.43 - 0.25165\sin 18.43 = 0.015927$ m/s

$\theta = \tan^{-1}\left(\frac{0.015927}{0.286496}\right) = 3.182°$ with respect to positive x-axis.

b i) Dye visualization – Dye should be neutrally buoyant as not to affect flow of water.

– Dye has colours to make flow field visible.

ii) Laser induced fluorescent

- Useful for taking flow field cross sections
- Dye absorbs energy at excitation frequency and re-emit fluorescence.

iii) Particle image velocimetry.

- Global measurement technique which measures entire flow field for that instance.

iv) Schlieren photography

- High speed flows where shocks are present.
- Density variation produce different refractive index and hence patterns.

2a) $\left(\frac{z}{c}\right)_1 = 0.4\left[\left(\frac{x}{c}\right) - \frac{2}{5}\left(\frac{x}{c}\right)^2\right], \quad 0 \le \frac{x}{c} \le 0.2$

$\left(\frac{z}{c}\right)_2 = 0.0625\left[0.6 + 0.4\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right], \quad 0.2 \le \frac{x}{c} \le 1$

$\frac{x}{c} = \frac{1}{2}(1-\cos\theta)$

At $\frac{x}{c} = 0.2$

$0.2 = \frac{1}{2}(1-\cos\theta)$

$\theta = 0.927295$ rad

$\left(\frac{dz}{dx}\right)_1 = 0.4\left[1 - \frac{4}{5}\left(\frac{x}{c}\right)\right]$

$= 0.4\left[1 - 4/5\,(1-\cos\theta)1/2\right] = 0.24 + 0.16\cos\theta, \quad 0 \le \theta \le 0.927295$

$\left(\frac{dz}{dx}\right)_2 = 0.0625\left[0.4 - 2\left(\frac{x}{c}\right)\right] = 0.0625\left[0.4 - \frac{1}{2}(2)(1-\cos\theta)\right]$

$= -0.0375 + 0.0625\cos\theta, \quad 0.927295 \le \theta \le \pi$

i) $A_0 = \alpha - \frac{1}{\pi}\left[\int_0^{0.927295} 0.24 + 0.16\cos\theta\, d\theta + \int_{0.927295}^{\pi} -0.0375 + 0.0625\cos\theta\, d\theta\right]$

$= \alpha - \frac{1}{\pi}\left[0.35055 - 0.13304\right] = \alpha - 0.069236$

$A_1 = \frac{2}{\pi}\left[\int_0^{0.927295} (0.24 + 0.16\cos\theta)\cos\theta\, d\theta + \int_{0.927295}^{\pi} (-0.0375 + 0.0625\cos\theta)\cos\theta\, d\theta\right]$

$= \frac{2}{\pi}\left[0.30458 + 0.08419\right] = 0.247499$

$A_2 = \frac{2}{\pi}\left[\int_0^{0.927295}(0.24+0.16\cos\theta)\cos 2\theta\, d\theta + \int_{0.927295}^{\pi}(-0.0375+0.0625\cos\theta)\cos 2\theta\, d\theta\right]$

$= \frac{2}{\pi}\left[0.18859 - 0.01067\right] = 0.113267$

(The above integrations are done using mathematical calculations. However, they can be done using trigonometry identities as well).

ii) $\alpha_{L=0}$ when $C_l = 0$, $C_l = \pi(2A_0 + A_1)$

$0 = 2A_0 + A_1$

$2A_0 = -A_1$

$2(\alpha - 0.069236) = -0.247499$

$\alpha = -0.0545135 = -3.123°$ #

iii) $\alpha = 6° = \frac{6\pi}{180}$ $\quad C_l = \pi\left[2\left[\frac{6\pi}{180} - 0.069236\right] + 0.247499\right]$

$= 1.000492$ #

b) $a_0 = 0.2/° = 11.4592/rad$ $\quad \alpha_{L=0} = -5° = -\frac{5\pi}{180}$ rad

$b = 5.2m$ $\quad c = 0.5m$ $\quad \alpha = 3° = \frac{3\pi}{180}$

i) $a = \frac{a_0}{1+\frac{a_0}{\pi AR}} = \frac{11.4592}{1+\frac{11.4592}{\pi(10.4)}}$ $\quad AR = \frac{b}{c} = \frac{5.2}{0.5} = 10.4$

$= 8.4837/rad$

$C_L = a(\alpha - \alpha_{L=0}) = 8.4837\left[\frac{3\pi}{180} - \left(-\frac{5\pi}{180}\right)\right] = 1.1845$ #

ii) $C_{Di} = \frac{C_L^2}{\pi AR} = \frac{1.1845^2}{\pi(10.4)} = 0.042946$ #

3a) $\alpha = 3° = \frac{3\pi}{180}$, for two wings

i) $AR = \frac{b^2}{S} = \frac{12^2}{2(\frac{1}{2})(0.8+0.5)(6)} = 18.4615$ $\quad \lambda = \frac{C_t}{C_r} = \frac{0.5}{0.8} = 0.625$

ii) $A = 2(\frac{1}{2})(0.8+0.5)(6) = 7.8m^2$ #

$a_0 = \frac{0.2-0}{\left[-\frac{24\pi}{180} - \left(-\frac{4\pi}{180}\right)\right]} = 5.72958/rad$

$\mu = \frac{a_0}{2(1+\lambda)AR}\left[1+(\lambda-1)\cos\phi\right] = 0.095493(1-0.375\cos\phi)$

iii) $0.095493(1-0.375\cos\phi)\left(\frac{3\pi}{180}+\frac{4\pi}{180}\right)\sin\phi = A_1\sin\phi(\mu+\sin\phi) + A_3\sin 3\phi(3\mu+\sin\phi)$

At $\phi = 22.5°$, ($\mu = 0.062409$)

$0.0029178 = 0.170329A_1 + 0.52653A_3$

At $\phi = 45°$, ($\mu = 0.070172$)

$0.0060208 = 0.54962A_1 + 0.648857A_3$

$\therefore A_1 = 7.1386\times10^{-3}$ $\quad A_3 = 3.2323\times10^{-3}$

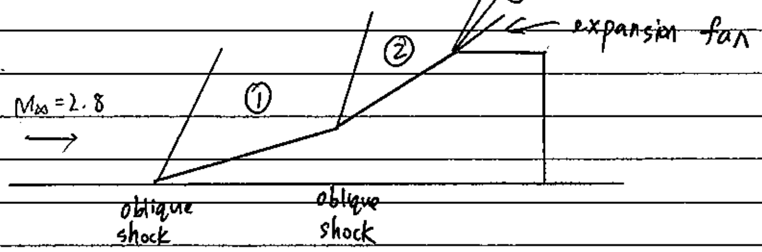
b) Delta wings can generate very high lift.
Delta wings can experience larger angle of attack before stalling.
Delta wings have large wing area with low wing loading, allowing high maneuverability.

Delta wings have larger internal volume for fuel and other storage.

ii) Delta wings tend to produce significant nose-down, negative pitching moment. Hence, canards are used to produce positive pitching moment.

4a) i) $M_\infty = 2.8$

$P_\infty = 0.8$



ii) $M_\infty = 2.8$, $\theta = 15°$, $\beta = 34°$

$$M_{n\infty} = 2.8 \sin 34° = 1.56574 \quad , \quad \gamma = 1.4$$

$$\frac{P_1}{P_\infty} = 1 + \frac{2(1.4)(1.56574^2 - 1)}{2.4} = 2.69347 = 1 + \frac{2\gamma(M_{n\infty}^2 - 1)}{\gamma + 1}$$

$$\therefore P_1 = 2.69347(0.8) = 2.1548 \text{ bar}_\#$$

$$M_{n1} = \sqrt{\frac{1 + \left(\frac{\gamma-1}{2}\right) M_{n\infty}^2}{\gamma M_{n\infty}^2 - \left(\frac{\gamma-1}{2}\right)}} = \sqrt{\frac{1 + 0.2(1.56574)^2}{1.4(1.56574^2) - 0.2}} = 0.67903$$

$$M_1 = \frac{0.67903}{\sin(34 - 15)} = 2.08569_\#$$

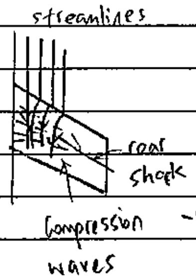
$\theta = 15°$, $M_1 = 2.08569$, $\beta \approx 43°$

$$M_{n1} = 2.08569 \sin 43° = 1.42244$$

$$\frac{P_2}{P_1} = 1 + \frac{2(1.4)(1.42244^2 - 1)}{2.4} = 2.19388$$

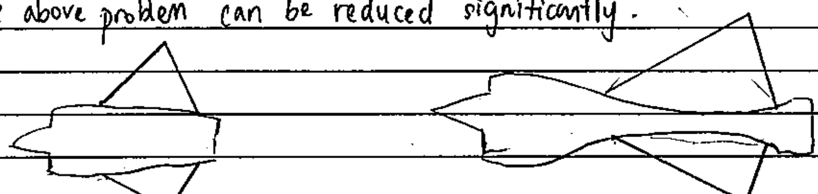
$$P_2 = 2.19388(2.1548) = 4.7274 \text{ bar}_\#$$

b)



Whitcomb area-rule states that transonic drag would be drastically reduced if variation of aircraft cross section is as smooth as possible.

- Streamlines over wing surface tend to curve inwards, due to variation in leading edge normal velocity component.
- Streamlines "bunched-up" near fuselage. Compression waves form from wing root and merged into rear shock. This causes wave drag.
- However, if fuselage is curved inwards to accomodate curved streamlines, the above problem can be reduced significantly.



(without area-rule) (with area rule)

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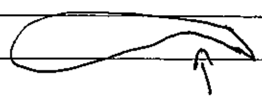

- The upper surface of supercritical airfoil is relatively flat whereas lower surface ~~has~~ is more curved.
- Supercritical airfoils increase the drag divergence Mach Number, delaying onset of significant drag growth.
- It also has more stable shock formation on upper surface.
- Because the upper surface is relatively flat, cusped trailing edge is used to impose positive camber.

c) This is because when a wing accelerate from rest, circulation is generated around the wing. However, since initially the wing is at rest and has no circulation, the total circulation when wing is accelerating must also be equal to zero. In order to balance out the circulation of wing, a starting vortex of same magnitude but opposite direction has to be produced.

(For more information, textbook pg 334)

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