NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2013-2014

MA3702/AE3006 – AIRCRAFT PROPULSION

November/December 2013 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED BOOK** Examination.

1(a) A front-mounted turbofan operates at the sea-level take-off condition of 300 K and 1.01 bar. The engine has the following specifications:

Core engine nozzle entrance conditions:	170120 Pa, 808 K
Core engine fuel-air ratio:	0.02
Bypass ratio:	5
Fan pressure ratio:	1.5
Fan adiabatic efficiency:	0.85

The inlets operate isentropically. The core and fan streams exhaust separately and the nozzles are to be assumed isentropic also. The gas properties for the uncombusted air are c_{pa} = 1005 J/kg-K, γ_a = 1.4. For the combusted gas, c_{pg} = 1148 J/kg-K, γ_g = 1.333. The gas constant for all gases is R = 287 J/kg-K.

(i) Calculate the specific thrust delivered by the core flow mass.

(5 marks)

(ii) Calculate the specific thrust for the fan and for the entire engine (both based on core flow).

(7 marks)

(iii) Calculate the thrust specific fuel consumption (in kg/kN-hr, based on core air flow) of the engine.

(5 marks)

Note: Question No. 1 continues on page 2.

(b) A subsonic aircraft is to be designed to cover a range of 20,000 km. The aircraft is to cruise at an altitude of 40,000 ft at which the lift/drag ratio (L/D) is 21.5. The overall efficiency of the engines is 25%. The fuel used has a heating value of 45 MJ/kg. What is the ratio of fuel mass to the total vehicle mass in order to accomplish the flight mission? The gravitation constant is 9.81 m/s^2.

(8 marks)

2(a) An axial turbine stage has a hub diameter of 1.0 m and the tip diameter is 1.25 m. The mass flow rate is 50 kg/s and the rotor speed is 10,000 RPM. The stage is based on the free-vortex design and has zero swirl at the rotor exit. Assume the stage to be normal so that the flow velocity at the stage entrance is equal to that at the exit. The axial velocity is constant across the stage and along the blade span.

At the rotor tip, the axial velocity is 300 m/s and the degree of reaction is 0.5. The total temperature at stage entrance is 1500 K. Assume the gas properties to be: R = 287 J/kg-K, c_p = 1148 J/kg-K, γ = 1.3333.

(i) Calculate the rate of work output by the stage in MJ/kg.
(ii) Calculate the total and static temperatures at the exit of the stage.
(iii) Calculate the absolute flow velocity at the hub at rotor entrance.
(iv) Calculate the reaction at the hub section.

(20 marks)

(b) In axial compressors, the diffusion factor, D, is defined as:

$$D \approx \frac{V_{\max} - V_2}{V_1}$$

where V_1 and V_2 are the velocities at the inlet and exit of the blade, respectively, and V_{max} is the maximum velocity along the blade surface. Discuss the physical meaning of this parameter. Use diagrams to support your answer.

(5 marks)

3(a) Consider an aviation fuel that is given by the chemical formula $C_{11}H_{21}$. If the fuel-air mixture is to burn in a stoichiometric proportion so that all the fuel and the oxygen in the mixture are consumed, calculate the fuel-air mass ratio of the mixture. Assume the air contains, in terms of moles, 21% of O_2 and 79% of N_2. Let the atomic mass of C, H, O, and N to be 12, 1, 16, and 14 kg/kmol, respectively.

(7 marks)

(b) For each ton of the above aviation fuel burned, what is the mass of CO_2 generated? If a biofuel vegetable oil ($C_{54}H_{100}O_7$) is used, how much CO_2 is generated per ton of fuel burn?

(6 marks)

(c) List and discuss the different factors when considering what fuel-air ratios are used in aircraft combustors.

In aircraft combustor design, how is the airflow apportioned to satisfy the conflicting requirements of fuel-air ratio?

(12 marks)

4 (a) Consider the rocket engine in Figure 1. Show that the thrust coefficient C_F is given by the expression:

$$C_F = \frac{F}{A^* p_0} = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{p_e}{p_0}-\frac{p_a}{p_0}\right)\frac{A_e}{A^*}$$

where the flow in the nozzle is isentropic and γ is assumed constant throughout the engine. The symbols in the above equation have the usual meanings. You may use the following equation for the mass flow rate in the nozzle:

$$\dot{m} = \frac{A^* p_0}{\sqrt{RT_0}}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

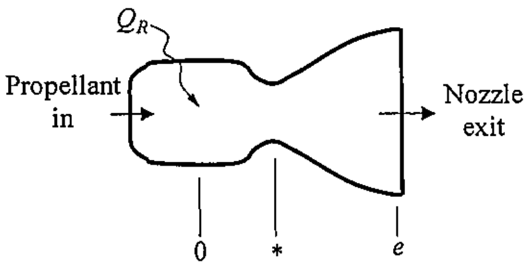


Figure 1: Rocket chamber and nozzle

(12 marks)

Note: Question No. 4 continues on page 4.

(b) A spaceship carries a load of balls as shown in Figure 2. The vehicle is ejecting balls at a rate of 1 ball a second, each ball having a mass of 2 kg and the ejection velocity of the balls (relative to the spaceship) is 300 m/s.

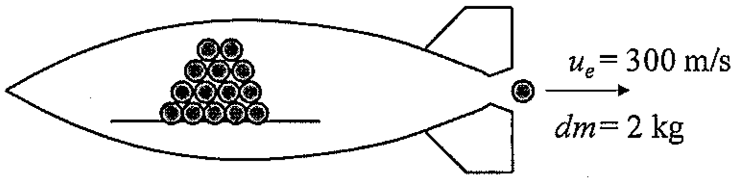


Figure 2: Spaceship ejecting balls

(i) Calculate the thrust generated on the spaceship.

(6 marks)

(ii) The spaceship is in a location where there is negligible gravity. Initially, the total mass of the spaceship with the balls is 20,000 kg and the ship carries 8,000 balls. If the spaceship is initially at velocity V_I, what is its final velocity when all the balls are ejected? Neglect all friction.

(7 marks)

END OF PAPER

MA3702 - PYP Nov/Dec 2013

1 a.i) Given that $P_5 = 170120\ Pa$, $\gamma_g = 1.333$

$T_5 = 808\ K$, $c_{pg} = 1148\ J\,kg^{-1}K^{-1}$

$p_{cr} = p_5\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 91,836.547\ Pa < p_a = 1.01\times10^5\ Pa$

$\therefore$ non-choked condition $p_7 = p_a$

$\frac{T_7}{T_5} = \left(\frac{p_7}{p_5}\right)^{\frac{\gamma-1}{\gamma}}$ ie $T_7 = T_5\left(\frac{p_7}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = 709.325\ K$

$T_5 = T_7 + \frac{C_7^2}{2c_{pg}}$ ie $C_7 = \sqrt{2c_{pg}(T_5 - T_7)} = 475.9806\ ms^{-1}$

specific thrust $= \frac{T}{\dot m_a} = \frac{\dot m_a\left((1+f)u_e - u\right) + (p_e - p_a)A_e}{\dot m_a}$

$= (1+f)\,u_e$

$= 485.5\ N/kgs^{-1}$

ii) Fan, ② → ⑧

$\frac{T_8}{T_2} = 1 + \frac{1}{\eta_f}\left(\left(\frac{p_8}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)$ where $T_2 = T_a = 300K$, $\frac{p_8}{p_2} = 1.5$, $\eta_f = 0.85$, $\gamma = 1.4$, $c_{pa} = 1005$

$p_2 = p_a = 1.01\ bar$

$T_8 = 343.3497\ K$

$p_8 = 1.5\times p_2 = 1.515\ bar$

Fan exit nozzle, ⑧ → ef

$\frac{T_{ef}}{T_8} = \left(\frac{p_{ef}}{p_8}\right)^{\frac{\gamma-1}{\gamma}}$ where $p_{ef} = p_a = 1.01\ bar$

$T_{ef} = 343.3497\left(\frac{1.01}{1.515}\right)^{\frac{0.4}{1.4}} = 305.791165\ K$

$C_{ef} = \sqrt{2c_{pa}(T_8 - T_{ef})} = 274.75941\ ms^{-1}$

specific thrust of fan $= \frac{B\dot m_a u_{ef}}{\dot m_a} = 5\times 274.75941$

$= 1373.797\ N/kgs^{-1}$ //

specific thrust of the whole engine = specific thrust of fan + specific thrust of core

$= 1.64856\ N/kgs^{-1}$ //

iii) $TSFC = \frac{\dot m_f}{T} = \frac{f\dot m_a}{\dot m_a(1+f)u_e + B\dot m_a u_{ef}} = \frac{f}{(1+f)u_e + Bu_{ef}} = 1.078675\times10^{-3}\ kg/Ns^{-1}$

$= 30.724\ kg/kN\,h^{-1}$ //

b. Breguet Range Equation, $S = \eta_0\frac{Q_R}{g}\frac{L}{D}\ln\frac{m_1}{m_2}$ ie $\frac{m_1}{m_2} = \exp\left(\frac{S}{\eta_0\frac{Q_R}{g}\frac{L}{D}}\right) = 2.25052$

Since $m_{fuel} = m_1 - m_2$, $\frac{m_{fuel}}{m_{initial}} = \frac{-\frac{m_1}{2.25052} + m_1}{m_1} = 0.555659$ //

2.a)i) zero swirl ⇒ $C_{\theta 3} = 0$, $\alpha_3 = 0$

at tip, $R = 0.5$; thus $\beta_{2t} = \alpha_{3t}$ ie $W_{\theta 2t} = C_{\theta 3} = 0$

$U_t = 0.625 \times 10\,000 \times \frac{2\pi}{60} = 654.4985 \ ms^{-1}$

$\beta_{2t} = \alpha_{2t} = \tan^{-1}\left(\frac{U_t}{C_x}\right) = 65.37488°$

free vortex design, $r_m C_{\theta 2m} = r_t C_{\theta 2t}$ ∴ $\tan \alpha_{2m} = \frac{r_t}{r_m} \tan \alpha_{2t}$ where $r_m = \frac{0.625 + 0.5}{2} = 0.5625$

$\alpha_{2m} = 67.5824°$

$\dot{W} = \dot{m} U_m C_x \tan \alpha_{2m}$ ∵ $\tan \alpha_{3m} = 0$ (zero swirl)

$= 50 \times 0.5625 \times 10\,000 \times \frac{2\pi}{60} \times 300 \times \tan 67.5824° = 21.418 \ MJ/kg$ //

ii) $w_{st} = h_{01} - h_{03} = c_p (T_{01} - T_{03})$

$T_{03} = T_{01} - \frac{w_{st}}{c_p} = T_{01} - \frac{\dot{W}}{\dot{m} c_p} = 1500 - \frac{21.418 \times 10^6}{50 \times 1148} = 1126.86 \ K$ //

$T_{03} = T_3 + \frac{C_3^2}{2c_p}$ ie $T_3 = T_{03} - \frac{C_3^2}{2c_p} = 1126.86 - \frac{300^2}{2 \times 1148} = 1087.66 K$ //

iii) free vortex design, $r_t C_{\theta 2t} = r_h C_{\theta 2h}$ ie $\tan \alpha_{2h} = \frac{r_t}{r_h} \tan \alpha_{2t}$

$\alpha_{2h} = 69.86237°$

absolute flow velocity at hub ⇒ $C_{2h} = \frac{C_x}{\cos \alpha_{2h}} = 871.393 \ ms^{-1}$ //

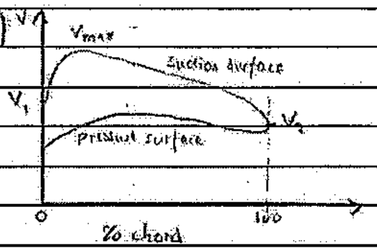
iv) $U_h = 0.5 \times 10\,000 \times \frac{2\pi}{60} = 523.59878 \ ms^{-1}$

$\tan \beta_{3h} = \frac{U_h}{C_x}$ ie $\beta_{3h} = 60.189112°$

$R_h = \frac{1}{2} + \frac{C_x}{2U_h} (\tan \beta_{3h} - \tan \alpha_{2h})$

$= \frac{1}{2} + \frac{300}{2 \times 523.59878} \times (\tan 60.189112° - \tan 69.86237°)$

$= 0.21875$ //

b) 

diffusion factor is a representation of a correlation between total pressure loss and the deceleration (diffusion) on the suction surface. The greater the deceleration, the greater the danger of separation.

3.a) $C_{11}H_{21} + 154.762 (0.21 O_2 + 0.79 N_2) \rightarrow 11 CO_2 + 10.5 H_2O + 122.262 N_2$

Fuel-air mass ratio $= \frac{11 \times 12 + 21}{154.762 ((0.21 \times 2 \times 16) + (0.79 \times 2 \times 14))} = 0.03428$ //

b) $\frac{m_{CO_2}}{m_{C_{11}H_{21}}} = \frac{11 \times (12 + 2 \times 16)}{11 \times 12 + 21} = 3.163399$

ie for 1 ton of $C_{11}H_{21}$, $m_{CO_2} = 3.1634$ tons //

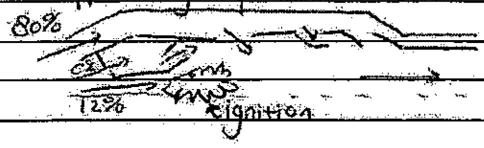
for $C_{54}H_{100}O_7$, $\frac{m_{CO_2}}{m_{C_{54}H_{100}O_7}} = \frac{54(12+2\times16)}{54\times12+100+7\times16} = 2.76279$

$\therefore$ for 1 ton of $C_{54}H_{100}O_7$, $m_{CO_2} = 2.7628$ tons

c.) - flame stability $\Rightarrow$ ~~needed~~ right fuel-air ratio is needed to ensure that flame is at burning region and does not blow off because of moving air

- material selection of combustion chamber $\Rightarrow$ maximum temperature inside combustion chamber is limited by the ability of the material to withstand heat due to combustion

Apportioning airflow in combustion chamber



- fuel is injected to small portion of air (primary air) to ensure sufficient fuel-air ratio for ignition to take place.
- After combustion, products are diluted and cooled by remaining air
- remaining air at liner helps protecting the metal chamber liner from direct contact with hot gas and cool down ~~the~~ some of combustion chamber metal parts.

4.a) isentropic relationship between combustion chamber and nozzle exit,

$$\left(\frac{p_0}{p_e}\right) = \left(\frac{T_0}{T_e}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow T_e = T_0\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \quad \ldots (a)$$

$$T_0 = T_e + \frac{u_e^2}{2c_{p,g}}$$

$\Rightarrow u_e = \sqrt{2c_{p,g}(T_0 - T_e)}$; substitute eqn (a),

$u_e = \sqrt{2c_{p,g}T_0\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$ and we know that $c_p = \frac{\gamma R}{\gamma-1}$, thus

$$u_e = \sqrt{\frac{2\gamma R T_0}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

We know that $F = \dot{m}u_e + (p_e - p_a)A_e$, thus

$$C_F = \frac{F}{A^* p_0} = \frac{1}{A^* p_0}\left(\left(\frac{A^* p_0}{\sqrt{RT_0}}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\right)\sqrt{\frac{2\gamma R T_0}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + (p_e - p_a)A_e\right)$$

$$= \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \left(\frac{p_e}{p_0} - \frac{p_a}{p_0}\right)\frac{A_e}{A^*}$$

QED

(2)

4b. i) 2^{nd} Newton's law of motion, $F = \frac{dp}{dt}$

$$= \frac{dm}{dt} V \qquad \text{where } \frac{dm}{dt} = 2\,kg\,s^{-1}$$

$$= 600N \qquad V = 300\,ms^{-1}$$

Due to 3^{rd} Newton's law of motion, thrust generated on the spaceship = rate of change of momentum $= 600\,N$

ii) by POCOM (Principle of Conservation of Momentum), $p_{initial} = p_{final}$

After 1^{st} ball is ejected, let m_b = mass of ball

$$m_{ss} V_1 = (m_{ss} - m_b) V_2 + m_b (V_1 - V_b) \qquad m_{ss} = \text{mass of spaceship initially}$$

$$V_2 = \frac{(m_{ss} - m_b) V_1 + m_b V_b}{m_{ss} - m_b} = \frac{m_b V_b}{m_{ss} - m_b} + V_1$$

after 2^{nd} ball is ejected,

$$(m_{ss} - m_b) V_2 = (m_{ss} - 2m_b) V_3 + m_b (V_2 - V_b)$$

$$V_3 = \frac{(m_{ss} - 2m_b) V_2 + m_b V_b}{m_{ss} - 2m_b} = \frac{m_b V_b}{m_{ss} - 2m_b} + V_2$$

after 3^{rd} ball is ejected,

$$(m_{ss} - 2m_b) V_3 = (m_{ss} - 3m_b) V_4 + m_b (V_3 - V_b)$$

$$V_4 = \frac{(m_{ss} - 3m_b) V_3 + m_b V_b}{m_{ss} - 3m_b} = \frac{m_b V_b}{m_{ss} - 3m_b} + V_3$$

from the equations above, we can obtain emphirical formula

$$V_K = \left(\sum_{n=1}^{K-1} \frac{m_b V_b}{m_{ss} - n m_b} \right) + V_1$$

where V_K is the velocity of the spaceship after $(K-1)$ ball(s) are ejected

Therefore, final velocity, V_f:

$$V_f = V_{8001} = \sum_{n=1}^{8000} \frac{m_b V_b}{m_{ss} - n m_b} + V_1$$

$$= (482.89 + V_1)\ ms^{-1}$$

Note: rocket equation in lecture note can't be used (though its result is more or less similar) because in this problem, the propulsion is discrete (1 ball at a time), where as the rocket equation assumes a continous function of propellant ejected (and hence the integration & the derivation).

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2014-2015

MA3702 – AIRCRAFT PROPULSION

November/December 2014 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED BOOK** Examination.

1(a) Explain the main difference between turbofan and turbojet engine in the way they produce thrust.

(2 marks)

(b) Explain the meaning of specific thrust and thrust specific fuel consumption. Provide the equations and for the thrust specific fuel consumption also show how this relates to overall engine performance.

(4 marks)

(c) Provide the expression for specific thrust and thrust specific fuel consumption for an after-mounted turbofan engine. How does this differ from those of a turbojet engine?

(4 marks)

(d) Derive the propulsion efficiency for an airbreathing propulsive device assuming negligible fuel to air ratio ($f <<< 1$) and negligible thrust due to pressure difference between the inlet and exit.

(5 marks)

(e) Explain how the propulsion efficiency differs from the thermal efficiency (provide the final expression for the thermal efficiency and explain the qualitative difference between thermal and propulsion efficiency)

(3 marks)

(f) Explain the advantage and disadvantage of mixed exhaust turbofan.

(2 marks)

Note: Question No. 1 continues on Page No. 2

(g) In a T-S diagram, show the static and total pressure and temperature for the expansion process within a turbine.

Assume:

- Inlet turbine: station 1
- Exit turbine: station 2
- Flow velocity C_1 & C_2 are different than zero

(5 marks)

2. A turbojet aircraft is flying at an altitude of 6 km in which ambient pressure and temperature are 45800 Pascal and -20°C respectively. The following parameters for the turbojet engine are given:

Flight speed:	850 km/h
Mass flow rate of air entering the engine:	45 kg/s
Mass flow rate of fuel:	2 kg/s
Isentropic compressor exit temperature:	468.397 K
Power input to the compressor:	10MW
Pressure loss in the combustor:	0.2 bar
Turbine inlet temperature:	960 K
Diffuser efficiency:	0.93
Transmission efficiency:	0.97
Turbine efficiency:	0.9
Nozzle efficiency:	0.9

Gas properties for air: c_{pa} = 1005 J/kg-K, γ_a = 1.4, R = 287 J/kg-K
Gas properties for combusted gases: c_{pg} = 1147 J/kg-K, γ_g = 1.333, R = 287 J/kg-K
Assume 0°C = 273°K
Assume mechanical efficiency for both compressor and turbine equal to 100%.

(a) Draw a sketch of the system and a *T-s* diagram of the thermodynamic cycle.

(3 marks)

(b) Calculate the compressor efficiency.

(7 marks)

(c) Determine the turbine exit pressure.

(6 marks)

Note: Question 2 continues on page 3.

(d) Determine the critical pressure for the nozzle and explain whether or not the nozzle is choked.

(4 marks)

(e) Explain what happens in a converging nozzle when it chokes (Mach number =1).

(2 marks)

(f) Assuming the nozzle exit pressure to be the same as the atmospheric pressure, determine the exit area of the nozzle (nozzle has to be treated as not choked).

(3 marks)

Please consider the following stations:

1 - 2: Diffuser
2 - 3: Compressor
3 - 4: Combustor
4 - 5: Turbine
5 - 6: Nozzle

Assume zero flow velocity in the compressor, combustor and turbine: $C_2=C_3=C_4=C_5=0$

3. The inlet of an axial compressor has the following stagnation pressure and temperature, 101.5 kPa and 287.0 K respectively. The compressor mean radius is 0.29 m and it turns at 900 rad/s. The axial velocity at the exit of the rotor is 1.1 times the inlet axial velocity. The stage inlet absolute angle is 40° and corresponding Mach number is 0.75. The compressor is designed to have the same absolute flow angle and Mach number at the entrance and exit of the stage (i.e. exit stator has the same Mach number and absolute angle as inlet to the rotor). The change in total temperature across the stage is 13.0 K.

For all compressor calculations the ratio of specific heats may be taken to be 1.4, the specific heat capacity to be $c_p = 1005$ J/kg K and the gas constant $R = 287$ J/kg K.

(a) Calculate the compressor inlet static pressure and temperature.

(3 marks)

(b) Draw the velocity triangle at the rotor inlet and find all magnitudes of velocities in the absolute and relative frames of reference, and the corresponding angles.

(7 marks)

(c) Draw the velocity triangle at the exit of the rotor and find all the magnitudes of velocities in the absolute and relative frames of reference, and the corresponding angles.

(7 marks)

(d) Draw the velocity triangle at the exit of the stator finding the magnitudes of velocities as well as the corresponding angles.

(4 marks)

(e) Calculate the loading coefficient for the stage and the flow coefficient referred to the rotor inlet conditions.

(4 marks)

6

4(a) A ramjet burner is being designed for an overall stagnation temperature ratio of 3 and outlet Mach number of 0.9. A schematic of the burner is shown in Figure 1. If the burner cross-sectional area is to be varied in the stream direction in such a way as to maintain the Mach number uniform within the burner. Assuming that the static pressure varies linearly with burner cross-sectional area and the frictional losses are negligible and taking γ=1.4, determine the ratio of the outlet and inlet areas and the ratio of the outlet and inlet stagnation pressures.

(10 marks)

(b) If the outlet Mach number is reduced to 0.8 and other conditions remain the same as given in a), then determine the ratio of the outlet and inlet areas and briefly discuss the relationship between the outlet Mach number and the area ratio.

(5 marks)

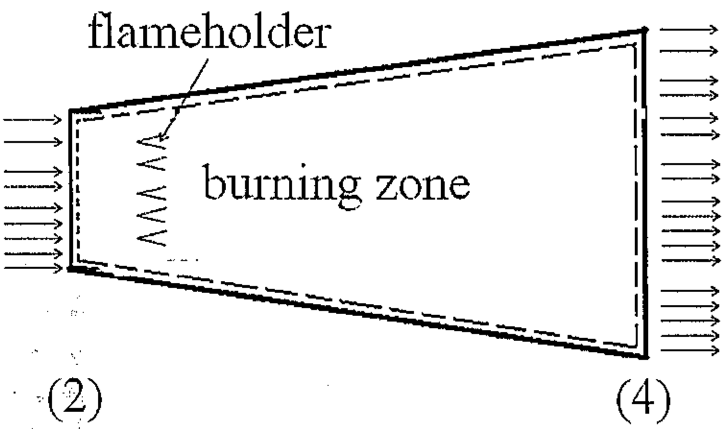


Figure1: Simplified 1D ramjet burner with inlet denoted by stage (2) and outlet by stage (4).

(c) Given A mole of CO and B mole of O_2 and the equilibrium constants K_p in terms of partial pressure for all dissociation reaction, consider a complete set of four products for a carbon monoxide-oxygen reaction of combustion

$$ACO + BO_2 \rightarrow n_{CO_2}CO_2 + n_{O_2}O_2 + n_{CO}CO + n_O O$$

List the governing equations for atom conservation and the equilibrium equations in terms of equilibrium constant K_n based on mole fraction (Hint: The reaction $2CO_2 \leftrightarrow 2CO + O_2$ and $O_2 \leftrightarrow 2O$ are involved in the combustion process). Then briefly describe what is chemical equilibrium and its characteristics.

(10 marks)

END OF PAPER

Nov/Dec. 2014

MA3702

1. a). — Turbofan engine consists of an additional fan placed in front of the compressor.
— Turbofan engine accelerates a larger mass of air to a velocity than turbojet engine to produce thrust.

b). — Specific thrust is the thrust per unit mass flow rate of air; it is a measure of how effective an engine is to produce thrust for a given amount of air flow rate. $T_s = \frac{T}{\dot{m}_a}$
— Thrust specific fuel consumption is the amount of fuel consumed per unit thrust produced. $TSFC = \frac{\dot{m}_f}{T}$
— Overall engine performance is measured by overall efficiency, which is defined as the ratio of the thrust power generated by the engine to the total energy consumption rate due to the fuel. $\eta_o = \frac{TC_a}{\dot{m}_f Q_R}$

$\therefore TSFC = \frac{\dot{m}_F}{T} = \frac{C_a}{\eta_o Q_R} \Rightarrow \eta_o = \frac{C_a}{TSFC \times Q_R}$

c). Assumption: $P_e = P_a$.

For turbojet engine, $T = \dot{m}_a[(1+f)C_e - C_a]$

$T_s = \frac{T}{\dot{m}_a} = (1+f)C_e - C_a$

$TSFC = \frac{\dot{m}_f}{T} = \frac{\dot{m}_f}{\dot{m}_a(1+f)C_e - C_a} = \frac{f}{(1+f)C_e - C_a}$

For aft-mounted turbofan engine, $T = (1+f)\dot{m}_a C_e + B\dot{m}_a C_{ef} - (1+B)\dot{m}_a C_a$

$T_s = \frac{T}{\dot{m}_a} = (1+f)C_e + BC_{ef} - (1+B)C_a$

$TSFC = \frac{\dot{m}_f}{T} = \frac{f}{(1+f)C_e + BC_{ef} - (1+B)C_a}$

d). Assumptions: $f \ll 1$, $p_e = p_a$

$T = \dot{m}_a[(1+\cancel{f})C_e - C_a] + \cancel{(P_e - P_a)A_e} = \dot{m}_a(C_e - C_a)$

$\eta_p = \frac{\text{Thrust + Power}}{\Delta KE_{propellant}} = \frac{TC_a}{\frac{1}{2}\dot{m}_a[(1+\cancel{f})C_e^2 - C_a^2]} = \frac{\dot{m}_a(C_e - C_a)C_a}{\frac{1}{2}\dot{m}_a(C_e - C_a)(C_e + C_a)} = \frac{2C_a}{C_e + C_a}$

$= \frac{2}{C_e/C_a + 1}$

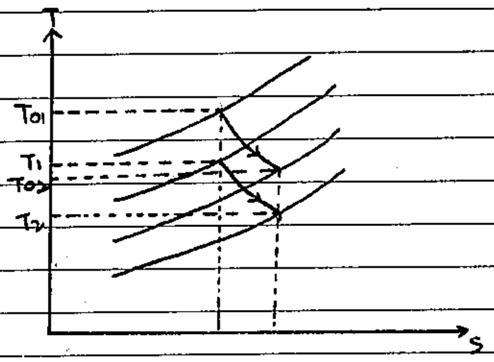
(e). — Propulsive efficiency is the ratio of thrust power to the rate of production of propellant kinetic energy.
⇒ measure of effectiveness of propulsion device to convert potentially useful kinetic energy to thrust power to propel aircraft.
— Thermal efficiency is the ratio of rate of addition of propellant kinetic energy to total energy consumption rate due to the fuel ⇒ measure of effectiveness of energy conversion from fuel to potentially useful KE for propulsion.

$$\eta_{th} = \frac{\Delta KE_{propellant}}{\dot{Q}_{fuel}} = \frac{\frac{1}{2}\dot{m}_a[(1+f)C_e^2 - C_a^2]}{\dot{m}_f Q_R} = \frac{\frac{1}{2}[(1+f)C_e^2 - C_a^2]}{f Q_R} = \frac{C_e^2 - C_a^2}{2 f Q_R} \text{ assuming } f \ll 1$$

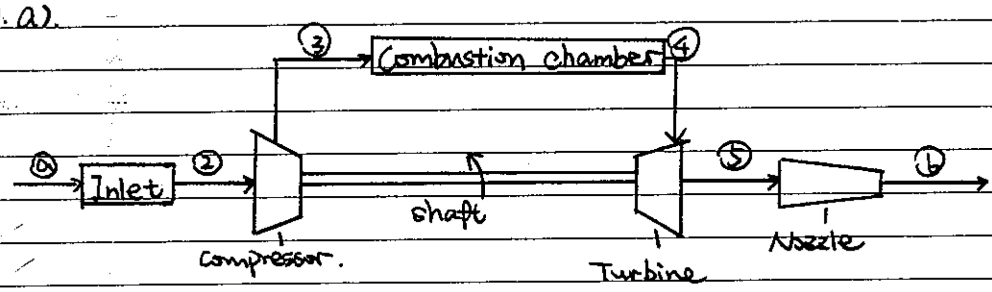
f) Advantages (choose 1)
- small increase in thrust => significant improvement in efficiency
- lower exhaust velocity => reduce jet noise
- lower exhaust gas temperature

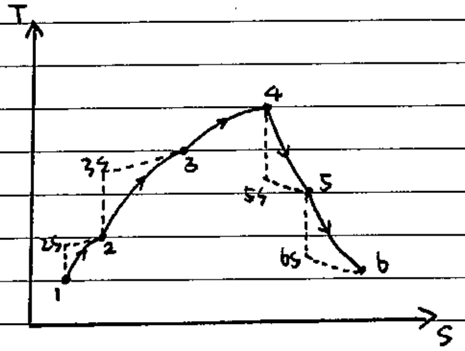
Disadvantages (choose 1)
- longer engine casing to channel fan flow to exhaust nozzle => heavier engine
- fully mixing is unlikely => difficult to analyse performance

g).



2). a).





b). Ambient: state 1.

$P_1 = 45.8\,kPa$ $T_1 = -20°C = 253K$

$C_1 = \frac{850 \times 10^3 m}{3600s} = 236.11 m/s$

Diffuser: 1-2

$T_{2s} = T_{01} = T_1 + \frac{C_1^2}{2C_{pa}} = 253K + \frac{(236.11 m/s)^2}{2(1005 J/kg\cdot k)} = 280.74\,k$

$\eta_d = \frac{T_{2s} - T_1}{T_2 - T_1}$

$0.93 = \frac{280.74k - 253k}{T_2 - 253k} \Rightarrow T_2 = 282.82k$

Using isentropic relations.

$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma_a - 1/\gamma_a}$

$\frac{280.74k}{253k} = \left(\frac{P_2}{45.8kPa}\right)^{0.4/1.4} \Rightarrow P_2 = 65.92kPa$

compressor: 2-3

$\dot{W}_c = \dot{m}_a C_{pa}(T_3 - T_2)$

$10MW = (45kg/s)(1005J/kg\cdot k)(T_3 - 282.82k)$

$T_3 = 503.94k$

$\therefore \eta_c = \frac{T_{3s} - T_2}{T_3 - T_2} = \frac{468.397k - 282.82k}{503.94k - 282.82k} \times 100\% = 83.93\%$

c). Compressor: 2-3.

using isentropic relations. $\frac{T_{3s}}{T_2} = \left(\frac{P_3}{P_2}\right)^{\gamma_a - 1/\gamma_a}$

$\frac{468.397k}{282.82k} = \left(\frac{P_3}{65.92kPa}\right)^{0.4/1.4} \Rightarrow P_3 = 385.33kPa$

Combustor: 3-4

$T_4 = 960k$ $P_4 = P_3 - 0.2\,bar = 385.33kPa - 20kPa = 365.33kPa$

Turbine: 4-5

$\dot{W}_a = \eta_{transmission} \times \dot{W}_t = 0.97(\dot{m}_a + \dot{m}_f)C_{pg}(T_4 - T_5)$

$10MW = 0.97(47kg/s)(1147J/kg\cdot k)(960k - T_5)$

$T_5 = 768.77k$

$\eta_t = \frac{T_4 - T_5}{T_4 - T_{5s}} \Rightarrow 0.9 = \frac{960k - 768.77k}{960k - T_{5s}} \Rightarrow T_{5s} = 747.52k$

Using isentropic relations. $\frac{T_{5s}}{T_4} = \left(\frac{P_5}{P_4}\right)^{\gamma_g - 1/\gamma_g}$

$\frac{747.52k}{960k} = \left(\frac{P_5}{365.33kPa}\right)^{0.333/1.333}$

$\therefore P_5 = 134.20kPa$

d). Nozzle: 5-6

$\frac{P_c}{P_5} = \left(1 - \frac{1}{\eta_n}\frac{\gamma_g - 1}{\gamma_g + 1}\right)^{\gamma_g/\gamma_g - 1}$

$\frac{P_c}{134.20kPa} = \left[1 - \frac{0.333}{(0.9)(2.333)}\right]^{1.333/0.333}$

$P_c = 67.23kPa > P_a \Rightarrow$ Nozzle is choked.

Since the ambient pressure $P_a = 45.8\,kPa$ is lower than the critical pressure, the flow expands to Mach 1 and becomes choked.

e). — choked flow occurs in the throat (smallest area) of a converging nozzle where mass flow rate is maximum
— This occurs at sonic conditions (Mach 1) thus the flow cannot expand to atmospheric pressure

f). Nozzle: 5-6
$P_c > P_a \Rightarrow P_6 = P_c = 67.23\,kPa$
Using isentropic relations. $\frac{T_{6s}}{T_5} = \left(\frac{P_6}{P_5}\right)^{\gamma_g - 1/\gamma_g}$

$$\frac{T_{6s}}{768.77k} = \left(\frac{67.23kPa}{134.20kPa}\right)^{0.333/1.333}$$

$$T_{6s} = 646.84k$$

$$\eta_n = \frac{T_5 - T_6}{T_5 - T_{6s}}$$

$$0.9 = \frac{768.77k - T_6}{768.77k - 646.84k}$$

$$T_6 = 659.04k$$

Using ideal gas equation.

$$P_6 = \rho_6 R T_6$$

$$67.23kPa = \rho_6 (287J/kg\cdot k)(659.04k)$$

$$\rho_6 = 0.3554\,kg/m^3$$

$$T_5 = T_{06} = T_6 + \frac{C_6^2}{2C_{pg}}$$

$$768.77k = 659.04k + \frac{C_6^2}{2(1147J/kg\cdot k)}$$

$$C_6 = 501.72m/s.$$

$$\dot{m}_6 = \rho_6 A_6 C_6$$

$$47kg/s = (0.3554\,kg/m^3)\,A_6\,(501.72m/s)$$

$$\therefore A_6 = 0.264\,m^2$$

3) a). $C_1 = M_1\sqrt{\gamma R T_1}$

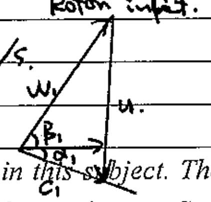
$$T_{01} = T_1 + \frac{C_1^2}{2C_p} = T_1 + \frac{M_1^2 \gamma R T_1}{2C_p} = \left(1 + \frac{M_1^2 \gamma R}{2C_p}\right)T_1$$

$$\cancel{287.0k}\ 287.0k = \left[1 + \frac{(0.75)^2(1.4)(287J/kg\cdot k)}{2(1005J/kg\cdot k)}\right]T_1$$

$$\therefore T_1 = 257.99k.$$

Using isentropic relations.

$$\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1}\right)^{\gamma-1/\gamma} \quad \frac{287.0k}{257.99k} = \left(\frac{101.5kPa}{P_1}\right)^{0.4/1.4} \quad \therefore P_1 = 69.90kPa$$



b) $\therefore C_1 = M_1\sqrt{\gamma R T_1} = 0.75\sqrt{(1.4)(287J/kg\cdot k)(257.99k)} = 241.47m/s.$

$\alpha_1 = 40°$ (given)

$\therefore C_{z1} = C_1 \cos\alpha_1 = (241.47m/s)\cos 40° = 184.98m/s$

$\therefore u = \omega r = (900\,rad/s)(0.29\,m) = 261\,m/s.$

$u = C_{z1}(\tan\beta_1 + \tan\alpha_1)$

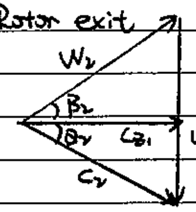
$261\,m/s = (184.98\,m/s)(\tan\beta_1 + \tan 40°).$

$\therefore \beta_1 = 29.76°$

$C_{z1} = W_1 \cos\beta_1$

$184.98\,m/s = W_1 \cos 29.76°$

$\therefore W_1 = 213.09\,m/s.$



c). $C_{z2} = 1.1\,C_{z1} = 1.1\,(184.98\,m/s) = 203.48\,m/s.$

$\frac{\dot{W}}{\dot{m}} = C_p \Delta T_0 = U(C_{\theta 2} - C_{\theta 1}) = U(C_{z2}\tan\alpha_2 - C_{z1}\tan\alpha_1)$

$(1005\,J/kg\cdot k)(13.0\,k) = (261\,m/s)[(203.48\,m/s)\tan\alpha_2 - (184.98\,m/s)\tan 40°].$

$\therefore \alpha_2 = 45.25°$

$C_{z2} = C_2 \cos\alpha_2$

$203.48\,m/s = C_2 \cos 45.25°$

$\therefore C_2 = 289.03\,m/s$

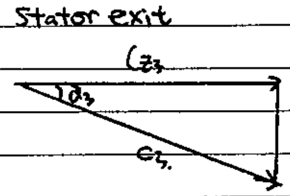
$u = C_{z2}(\tan\alpha_2 + \tan\beta_2)$

$261\,m/s = (203.48\,m/s)(\tan 45.25° + \tan\beta_2) \quad \therefore \beta_2 = 15.32°$

$C_{z2} = W_2 \cos\beta_2$

$203.48\,m/s = W_2 \cos 15.32°$

$\therefore W_2 = 210.97\,m/s.$



d). $C_3 = M_3\sqrt{\gamma R T_3}$

$T_{03} = T_{02} = T_{01} + 13.0\,k = 300\,k$

$T_{03} = T_3 + \frac{C_3^2}{2C_p} = T_3 + \frac{M_3^2 \gamma R T_3}{2C_p} = \left[1 + \frac{M_3^2 \gamma R}{2C_p}\right] T_3$

$300\,k = \left[1 + \frac{(0.75)^2(1.4)(287\,J/kg\,k)}{2(1005\,J/kg\cdot k)}\right] T_3.$

$T_3 = 269.68\,k$

$\therefore C_3 = M_3\sqrt{\gamma R T_3} = 0.75\sqrt{(1.4)(287\,J/kg\,k)(269.68\,k)} = 246.88\,m/s.$

$\therefore C_{z3} = C_3 \cos\alpha_3 = (246.88\,m/s)\cos 40° = 189.12\,m/s.$

e). $\psi = \frac{\Delta h_0}{u^2} = \frac{C_p \Delta T_0}{u^2} = \frac{(1005\,J/kg\cdot k)(13.0\,k)}{(261\,m/s)^2} = 0.192$

$\phi = \frac{C_{z1}}{u} = \frac{184.98\,m/s}{261\,m/s} = 0.709$

4 a). Momentum conservation.

$P_2A_2 - P_4A_4 + \frac{(P_2+P_4)}{2}(A_4 - A_2) - \frac{k}{2}\rho_2 u_2^2 A_2 = \dot{m}_4 u_4 - \dot{m}_2 u_2 = \rho_4 u_4^2 A_4 - \rho_2 u_2^2 A_2$

Divide throughout by P_4A_2.

$\frac{P_2}{P_4} - \frac{A_4}{A_2} + \frac{1}{2}\left(\frac{P_2}{P_4} + 1\right)\left(\frac{A_4}{A_2} - 1\right) - \frac{k}{2}\frac{\rho_2 u_2^2}{P_4} = \frac{\rho_4 u_4^2}{P_4}\frac{A_4}{A_2} - \frac{\rho_2 u_2^2}{P_4}$

Since $M^2 = \frac{u^2}{c^2} = \frac{u^2}{\gamma P/\rho} \Rightarrow \frac{\rho u^2}{P} = \gamma M^2$

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P.3

and assuming $\gamma_2 = \gamma_4$

$$\cancel{\frac{P_2}{P_4} - \frac{A_4}{A_2} + \frac{1}{2}\left(\frac{P_2}{P_4}+1\right)\left(\frac{A_4}{A_2}-1\right) - \frac{k}{2}\gamma M_2^2 \frac{P_2}{P_4} = \gamma M_4^2 \frac{A_4}{A_2} - \gamma M_2^2 \frac{P_2}{P_4}}$$

$$\cancel{\text{Since } M^2 = \frac{u^2}{c^2} = \frac{u^2}{\gamma P/\rho} \Rightarrow \frac{\rho u^2}{P} = \gamma M^2}$$

~~and~~ $$\frac{P_2}{P_4} - \frac{A_4}{A_2} + \frac{1}{2}\left(\frac{P_2}{P_4}+1\right)\left(\frac{A_4}{A_2}-1\right) - \frac{k}{2}\gamma M_2^2 \frac{P_2}{P_4} = \gamma M_4^2 \frac{A_4}{A_2} - \gamma M_2^2 \frac{P_2}{P_4}$$

$$\left[1 + \frac{1}{2}\left(\frac{A_4}{A_2}-1\right) + \left(1-\frac{k}{2}\right)\gamma M_2^2\right]\frac{P_2}{P_4} = \frac{A_4}{A_2} - \frac{1}{2}\left(\frac{A_4}{A_2}-1\right) + \gamma M^2 + \frac{A_4}{A_2}$$
$$= 1 + \frac{1}{2}\left(\frac{A_4}{A_2}-1\right) + \gamma M^2 + \frac{A_4}{A_2}$$

$$\frac{P_2}{P_4} = \frac{1 + \frac{1}{2}\left(\frac{A_4}{A_2}-1\right) + \gamma M_4^2 + \frac{A_4}{A_2}}{1 + \frac{1}{2}\left(\frac{A_4}{A_2}-1\right) + \left(1-\frac{k}{2}\right)\gamma M_2^2}$$

Since $\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1}$

$$\cancel{\therefore \frac{P_{02}}{P_{04}} = \frac{P_2 T_4}{P_4 T_4} = \frac{u_4 A_4 T_2}{u_2 A_2 T_4} = \frac{M_4 A_4}{M_2 A_2}\sqrt{\frac{T_2}{T_4}} \Rightarrow}$$

$$\therefore \frac{P_{02}}{P_{04}} = \frac{1 + \frac{1}{2}\left(\frac{A_4}{A_2}-1\right) + \gamma M^2 + \frac{A_4}{A_2}}{1 + \frac{1}{2}\left(\frac{A_4}{A_2}-1\right) + \left(1-\frac{k}{2}\right)\gamma M_2^2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_4^2}\right]^{\gamma/\gamma-1}$$

$$\frac{\rho_2}{\rho_4} = \frac{P_2 T_2}{P_4 T_4} = \frac{u_4 A_4 T_2}{u_2 A_2 T_4} = \frac{M_4 A_4}{M_2 A_2}\sqrt{\frac{T_2}{T_4}} \Rightarrow \frac{T_2}{T_4} = \frac{M_2^2 A_2^2}{M_4^2 A_4^2}\frac{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+\gamma M_4^2 \frac{A_4}{A_2}\right]^2}{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+\left(1-\frac{k}{2}\right)\gamma M_2^2\right]^2}$$

Since $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2$

$$\frac{T_{02}}{T_{04}} = \frac{M_2^2 A_2^2}{M_4^2 A_4^2}\frac{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+\gamma M_4^2 \frac{A_4}{A_2}\right]^2}{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+\left(1-\frac{k}{2}\right)\gamma M_2^2\right]^2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_4^2}\right]$$

$$\therefore \frac{T_{04}}{T_{02}} = \frac{M_4^2 A_4^2}{M_2^2 A_2^2}\frac{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+\left(1-\frac{k}{2}\right)\gamma M_2^2\right]^2}{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+\gamma M_4^2 \frac{A_4}{A_2}\right]^2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_4^2}\right]$$

Using $\frac{T_{04}}{T_{02}} = 3$, $M_2 = M_4 = 0.9$, $\gamma = 1.4$, $k = 0$

$$3 = \left(\frac{A_4}{A_2}\right)^2 \frac{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+1.134\right]^2}{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+1.134\frac{A_4}{A_2}\right]^2} = \left(\frac{A_4}{A_2}\right)^2\left(\frac{0.5\frac{A_4}{A_2}+1.634}{1.634\frac{A_4}{A_2}+0.5}\right)^2$$

$\therefore \frac{A_4}{A_2} = 2.975$

Using $\frac{A_4}{A_2} = 2.975$, $M_2 = M_4 = 0.9$, $\gamma = 1.4$, $k = 0$

$$\frac{P_{02}}{P_{04}} = \frac{1+\frac{1}{2}(2.975-1)+(1.4)(0.9)^2(2.975)}{1+\frac{1}{2}(2.975-1)+(1.4)(0.9)^2} = 1.717$$

$\therefore \frac{P_{04}}{P_{02}} = \frac{1}{1.717} = 0.582$

b) Using $\frac{T_{04}}{T_{02}} = 3$, $M_2 = M_4 = 0.8$, $\gamma = 1.4$, $k = 0$

$$3 = \left(\frac{A_4}{A_2}\right)^2 \frac{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+0.896\right]^2}{\left[1+\frac{1}{2}\left(\frac{A_4}{A_2}-1\right)+0.896\frac{A_4}{A_2}\right]^2} = \left(\frac{A_4}{A_2}\right)^2\left(\frac{0.5\frac{A_4}{A_2}+1.396}{1.396\frac{A_4}{A_2}+0.5}\right)^2$$

$\therefore \frac{A_4}{A_2} = 2.69$

— Direct relation between outlet Mach number and area ratio

— Lower Mach number $\Rightarrow$ increased restriction on allowable flow expansion

$\Rightarrow$ outlet area decreases for certain inlet area $\Rightarrow$ lower area ratio

(c) $ACO + BO_2 \rightarrow n_{CO_2}CO_2 + n_{O_2}O_2 + n_{CO}CO + n_O O$

For carbon, $N_C = n_{CO_2} + n_{CO} = A$ —— ①

For oxygen, $N_O = 2n_{CO_2} + 2n_{O_2} + n_{CO} + n_O = 2B$ —— ②

$2CO_2 \leftrightarrow 2CO + O_2$

$$K_n = \frac{x^2_{CO}\, x_{O_2}}{x^2_{CO_2}} = \frac{\left(\frac{n_{CO}}{n_{CO_2}+n_{O_2}+n_{CO}+n_O}\right)^2 \left(\frac{n_{O_2}}{n_{CO_2}+n_{O_2}+n_{CO}+n_O}\right)}{\left(\frac{n_{CO_2}}{n_{CO_2}+n_{O_2}+n_{CO}+n_O}\right)^2}$$

$$= \frac{n^2_{CO}\, n_{O_2}}{n^2_{CO_2}(n_{CO_2}+n_{O_2}+n_{CO}+n_O)} \quad \text{—— ③}$$

$O_2 \leftrightarrow 2O$

$$K_n = \frac{x^2_O}{x_{O_2}} = \frac{\left(\frac{n_O}{n_{CO_2}+n_{O_2}+n_{CO}+n_O}\right)^2}{\frac{n_{O_2}}{n_{CO_2}+n_{O_2}+n_{CO}+n_O}} \quad \text{—— ④}$$

— Chemical equilibrium is the ~~store~~ state at which the forward and reverse reactions are occuring at the same rate.

— Molar concentration of each product is constant at equilibrium.

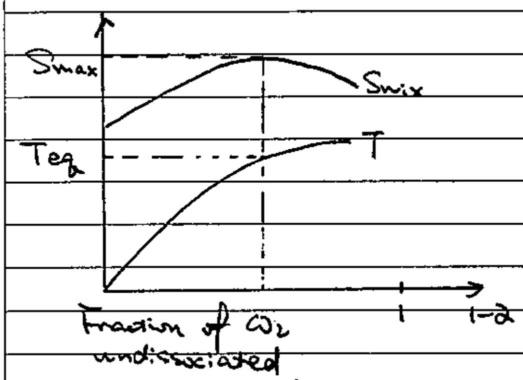
— Chemical equilibrium occurs when entropy of mixture ~~temperature~~ is maximised (2nd law of Thermodynamics).

— When $\alpha = 1$, no heat is released and mixture temperature, pressure, composition remain unchanged.

— When $\alpha = 0$, for complete combustion, the maximum amount of heat is ~~not~~ released, so temperature and pressure are highest possible allowed by 1st law of Thermodynamics.

— Entropy of mixture increases with $(1-\alpha)$ first then decreases.

— This gives a maximum entropy, where a chemical equilibrium state is reached.



P.4