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# NANYANG TECHNOLOGICAL UNIVERSITY

## SEMESTER 2 EXAMINATION 2016-2017

## MA1001 – DYNAMICS

April/May 2017 Time Allowed: $2^1/_2$ hours

**MATRICULATION NUMBER:** | | | | | | | | | |

**SEAT NUMBER:** | | | | | |

**INSTRUCTIONS**

1. This question and answer booklet contains **FOUR (4)** questions. It consists of **NINETEEN (19)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided A4 reference sheet is allowed.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

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For examiners:

| Questions | 1<br>(25) | 2<br>(25) | 3<br>(25) | 4<br>(25) | Total<br>(100) |
|---|---|---|---|---|---|
| Marks | | | | | |



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**Q1 (25 marks)** /25

(a) A projectile is launched from a point $A$ and moves in a vertical plane with a constant acceleration due to gravity, $g = 9.8$ m/s$^2$. Its motion is tracked by a radar at point $O$, located in the same vertical plane. At the instant shown in Figure Q1(a), the projectile is at position $B$, where $r = 500$ m, $\theta = \pi/6$ rad, $\dot{r} = -20$ m/s, and $\dot{\theta} = \sqrt{3}/25$ rad/s. Assuming that the effect of air resistance is negligible, determine

(i) the speed, $v$, of the projectile at position $B$, (4 marks)

(ii) the distance of the projectile from the radar at $O$ when it has fallen to the same height as $O$ (when $\theta = \pi$ rad), (8 marks)

***Solution:***

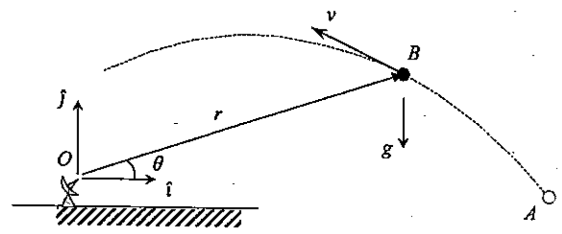


Figure Q1(a)



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**Q1**

(b) Block $A$ is translating vertically at a velocity of 20 mm/s downward and at an acceleration of 4 mm/s$^2$ upward. Block $B$ is connected to block $A$ by a cable passing over two pulleys, $C$ and $D$, as shown in Figure Q1(b). The centre of pulley $C$ is fixed in position. The centre of pulley $D$ is fixed to block $A$, and moves with the same velocity as $A$. Determine, at the instant shown,

(i) the velocity of block $B$, (6 marks)

(ii) the acceleration of block $B$. (7 marks)

Express your answers as vectors in the form ( )$\hat{\imath}$ + ( )$\hat{\jmath}$.

***Solution:***

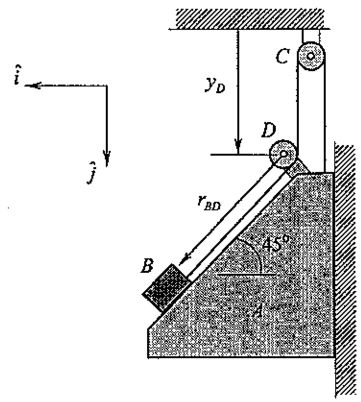


Figure Q1(b)



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**Q2 (25 marks)** /25

In Figure Q2, rod $AB$ of length $L$ = 1.0 m is pin-joined to sliders $A$ and $B$. Slider $B$ is sliding along the rod $OC$, which rotates about a fixed pivot $O$. Slider $A$ moves at a velocity $\vec{v} = 3.0\hat{\imath}$ m/s and an acceleration $\vec{a} = -0.5\hat{\imath}$ m/s$^2$. Rod $AB$ rotates at a constant angular velocity $\vec{\omega} = 2.0\hat{k}$ rad/s. Determine, at the instant shown,

(i) the velocity of end $B$ of rod $AB$, (4 marks)

(ii) the angular velocity of rod $OC$, (6 marks)

(iii) the velocity of slider $B$ relative to rod $OC$, (6 marks)

(iv) the angular acceleration of rod $OC$. (9 marks)

Express your answers as vectors in the form ( )$\hat{\imath}$ + ( )$\hat{\jmath}$.

***Solution:***

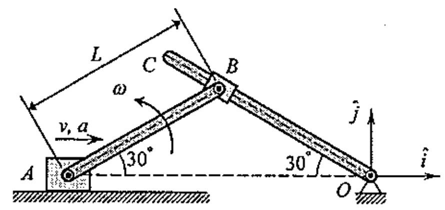


Figure Q2

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**Q3 (25 marks)** /25

(a) The block of 5 kg has an initial speed of 2 m/s at the position $A$ and slides along the path from $A$ to $C$ via $B$ in the vertical plane, as shown in Figure Q3(a). The path is smooth from $A$ to $B$ and rough from $B$ to $C$. The speed of the block becomes 5 m/s at the position C.

(i) Determine the speed of the block at $B$ (4 marks)

(ii) Determine the work $U_f$ done on the block by friction during the motion from $B$ to $C$. (8 marks)

***Solution:***

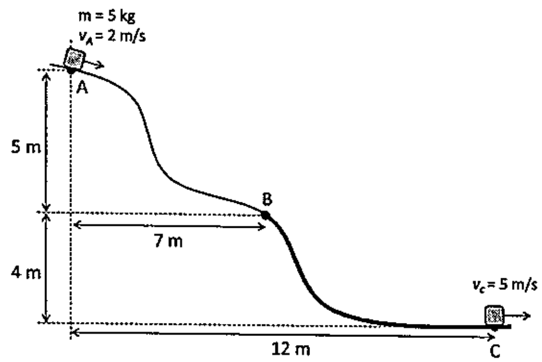


Figure Q3(a)



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Q3
(b) As shown in Figure Q3(b), the collar of mass $m$ [kg] slides up the vertical shaft under the action of a force $F$ [N] of constant magnitude but variable direction. The force direction is $\theta = \pi/2 - \omega t$ [rad] and $\omega$ [rad/s] is constant. The collar starts from rest when $\theta = \pi/2$ [rad]. The coefficient of kinetic friction between the collar and shaft is $\mu_k$.

(i) Draw free body diagram of the collar. (3 marks)

(ii) Find the acceleration of the collar in terms of $m$, $F$, $\omega$, $t$ and $\mu_k$. (5 marks)

(iii) Determine the magnitude of $F$ of the force which will result in the velocity of the collar becoming zero when $\theta$ is 0 [rad]. (5 marks)

***Solution:***

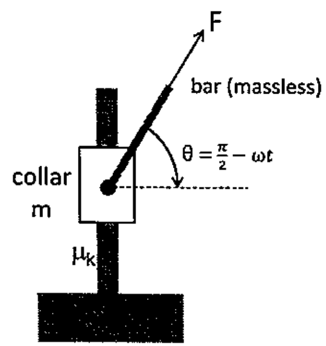


Figure Q3(b)



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**Q4 (25 marks)** | /25

(a) The spool in Figure Q4(a) has a mass of $m = 5$ kg, inner radius of $r = 0.3$ m, outer radius of $R = 0.6$ m, and a radius of gyration of $k_G = 0.4$ m. Assuming that the cords have negligible mass and that the spool rolls without slipping along the cords, determine the spool's angular acceleration, $\alpha$, when a constant force, $F = 150$ N, is applied to the cord.

(i) Derive the spool's angular acceleration using all given variables. (10 marks)

(ii) Substitute the values into the expression above, and determine the spool's angular acceleration numerically. (2 marks)

***Solution:***

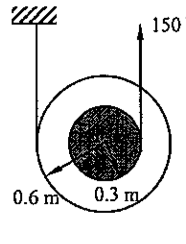


Figure Q4(a)



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**Q4**

(b) Two cubes, $A$ and $B$ with mass of 2 kg and 3 kg, respectively, and size of 0.1 m and 0.2 m, respectively, are attached to the ends of a rod which has a mass of 1 kg and length of 1 m. This rigid body has an initial translational velocity of $2\hat{i}$ m/s (no rotation). At the instant shown in Figure Q4(b), another cube, $C$, with a mass of 2 kg and size of 0.1 m is moving at a velocity of $5\hat{i}$ m/s. If $C$ is attached to $A$ after the collision, what are the velocity of the combined mass centre, $G$, and the angular velocity of the rod?

(13 marks)

***Solution:***

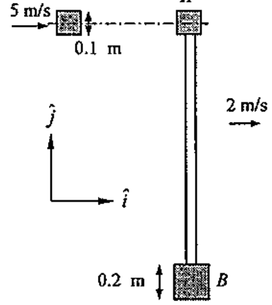


Figure Q4(b)

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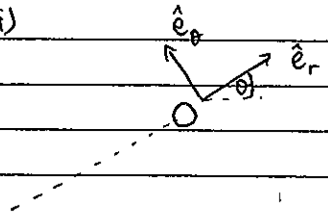
① (a) i)




$\hat{e}_r = \cos\theta \, \hat{i} + \sin\theta \, \hat{j}$

$\hat{e}_\theta = -\sin\theta \, \hat{i} + \cos\theta \, \hat{j}$

$\vec{V}_B = \dot{r} \, \hat{e}_r + r\dot{\theta} \, \hat{e}_\theta$

$= -20 \, \hat{e}_r + 20\sqrt{3} \, \hat{e}_\theta$

$V = |\vec{V}_B| = \sqrt{(-20)^2 + (20\sqrt{3})^2}$

$= 40 \text{ m/s}$

ii) Projectile motion → switch to rectangular coordinate for ease

$\vec{V}_B = -20(\cos\theta \, \hat{i} + \sin\theta \, \hat{j}) + 20\sqrt{3}(-\sin\theta \, \hat{i} + \cos\theta \, \hat{j})$

$= -20(\frac{1}{2}\sqrt{3} \, \hat{i} + \frac{1}{2} \, \hat{j}) + 20\sqrt{3}(-\frac{1}{2} \, \hat{i} + \frac{1}{2}\sqrt{3} \, \hat{j})$

$= -20\sqrt{3} \, \hat{i} + 20 \, \hat{j}$

$\vec{V}_x$ is constant $= -20\sqrt{3} \, \hat{i}$ m/s

$\vec{r}_B = r\cos\theta \, \hat{i} + r\sin\theta \, \hat{j}$

$= (250\sqrt{3} \, \hat{i} + 250 \, \hat{j})$ m

In y-direction

$y_f = y_i + V_{By} t - \frac{1}{2} g t^2$

$0 = 250 + 20t - \frac{1}{2} \times 9.8 \times t^2$

$t = 9.4695$ s

In x-direction

$x_f = x_o + V_{Bx} t$

$x_f = 250\sqrt{3} - 20\sqrt{3} \times 9.4695$

$x_f = 104.98$ m

(b) (i) Length of string is constant

$2y_D + r_{BD} = C$

→ differentiating

$2\dot{y}_D + \dot{r}_{BD} = 0$ ---- (1)

$2\ddot{y}_D + \ddot{r}_{BD} = 0$ ---(2)

$\vec{V}_A = 20\hat{j}$ mm/s $= \dot{y}_D$

(1) → $2\dot{y}_D + \dot{r}_{BD} = 0$

$\dot{r}_{BD} = -40$ mm/s



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$\vec{V}_{B/A} = \dot{r}_{BD} \angle 45^\circ$

$= (-20\sqrt{2}\,\hat{i} - 20\sqrt{2}\,\hat{j})$ mm/s

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$= 20\hat{j} - 20\sqrt{2}\,\hat{i} - 20\sqrt{2}\,\hat{j}$

$= -20\sqrt{2}\,\hat{i} + (20 - 20\sqrt{2})\hat{j}$

$= (-28.28\,\hat{i} - 8.284\,\hat{j})$ mm/s

ii) $\vec{a}_A = -4\hat{j}$ mm/s² $= \ddot{\vec{y}}_D$

(2) → $2\ddot{y}_D + \ddot{r}_{BD} = 0$

$\ddot{r}_{BD} = 8$ mm/s²

$\vec{a}_{B/A} = \ddot{r}_{BD} \angle 45^\circ$

$= (4\sqrt{2}\,\hat{i} + 4\sqrt{2}\,\hat{j})$ mm/s²

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$= -4\hat{j} + 4\sqrt{2}\,\hat{i} + 4\sqrt{2}\,\hat{j}$

$= 4\sqrt{2}\,\hat{i} + (4\sqrt{2} - 4)\hat{j}$

$= (5.657\,\hat{i} + 1.657\,\hat{j})$ mm/s²

② i) $\vec{V}_B = \vec{V}_A + \vec{\omega} \times \vec{r}_{B/A}$

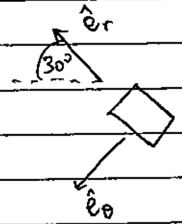
$= 3\hat{i} + 2\hat{k} \times 1\angle 30^\circ$

$= 3\hat{i} + 2\angle 120^\circ$

$= (2\hat{i} + \sqrt{3}\,\hat{j})$ m/s

$= 2.646 \angle 40.89^\circ$ m/s

ii)




$\hat{e}_r = -\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j}$

$= -\frac{1}{2}\sqrt{3}\,\hat{i} + \frac{1}{2}\hat{j} = 1\angle 150^\circ$

$\hat{e}_\theta = -\sin 30^\circ\,\hat{i} - \cos 30^\circ\,\hat{j}$

$= -\frac{1}{2}\hat{i} - \frac{1}{2}\sqrt{3}\,\hat{j} = 1\angle 240^\circ$

$\vec{V}_B = V_{B/f}\,\hat{e}_r + \dot{\theta}\, r_{B/O}\,\hat{e}_\theta$ ; $r_{B/O} = 1$ m (isosceles triangle)

$2\hat{i} + \sqrt{3}\,\hat{j} = V_{B/f}\left(-\frac{1}{2}\sqrt{3}\,\hat{i} + \frac{1}{2}\hat{j}\right) + \dot{\theta}\left(-\frac{1}{2}\hat{i} - \frac{1}{2}\sqrt{3}\,\hat{j}\right)$

$2\hat{i} + \sqrt{3}\,\hat{j} = \left(-\frac{1}{2}\sqrt{3}\,V_{B/f} - \frac{1}{2}\dot{\theta}\right)\hat{i} + \left(\frac{1}{2}V_{B/f} - \frac{1}{2}\sqrt{3}\,\dot{\theta}\right)\hat{j}$

$\hat{i}$ : $2 = -\frac{1}{2}\sqrt{3}\,V_{B/f} - \frac{1}{2}\dot{\theta}$ $\xrightarrow{\times 1}$ $2 = -\frac{1}{2}\sqrt{3}\,V_{B/f} - \frac{1}{2}\dot{\theta}$

$\hat{j}$ : $\sqrt{3} = \frac{1}{2}V_{B/f} - \frac{1}{2}\sqrt{3}\,\dot{\theta}$ $\xrightarrow{\times\sqrt{3}}$ $3 = \frac{1}{2}\sqrt{3}\,V_{B/f} - \frac{3}{2}\dot{\theta}$ +

$5 = -2\dot{\theta}$

$\dot{\theta} = -2.5$ rad/s

$\vec{\omega}_{OC} = -2.5\hat{k}$ rad/s



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iii) $\hat{\imath}$ : $2 = -\frac{1}{2}\sqrt{3}\ V_{B/f} - \frac{1}{2}\dot{\theta}$

$2 = -\frac{1}{2}\sqrt{3}\ V_{B/f} + 1.25$

$V_{B/f} = -0.866$ m/s

$\vec{V}_{B/f} = -0.866\ \hat{e}_r$

$= -0.866\ \angle 150^\circ$

$= 0.866\ \angle -30^\circ$ m/s $= (0.75\ \hat{\imath} - 0.433\ \hat{\jmath})$ m/s

iv) $\vec{a}_B = \vec{a}_A - \omega_A^2\ \vec{r}_{B/A}$

$= -0.5\ \hat{\imath} - 4 \times 1\ \angle 30^\circ$

$= -0.5\ \hat{\imath} - 4\ (\cos 30^\circ \hat{\imath} + \sin 30^\circ \hat{\jmath})$

$= (-2\sqrt{3} - 0.5)\ \hat{\imath} - 2\ \hat{\jmath}$

$= (-3.964\ \hat{\imath} - 2\ \hat{\jmath})$ m/s² $= 4.44\ \angle 206.77^\circ$ m/s²

$\vec{a}_B = (\ddot{r} - \dot{\theta}^2 r_{B/O})\ \hat{e}_r + (\ddot{\theta} r_{B/O} + 2\dot{\theta}\dot{r})\ \hat{e}_\theta$ ; where $\dot{r} = V_{B/f}$

$-3.964\ \hat{\imath} - 2\ \hat{\jmath} = (\ddot{r} - 6.25)\ \angle 150^\circ + (\ddot{\theta} + 4.33)\ \angle 240^\circ$

dot product with $1\angle 240^\circ$

$-3.964 \cos 240^\circ - 2\cos 150^\circ = \ddot{\theta} + 4.33$

$\ddot{\theta} = -0.616$ rad/s²

$\vec{\alpha}_{OC} = -0.616\ \hat{k}$ rad/s²

③ (a) i) Take point C as reference point $y = 0$

$E_A = E_B$ (conservation of energy)

$mg y_A + \frac{1}{2} m V_A^2 = mg y_B + \frac{1}{2} m V_B^2$

$9.8 \times 9 + \frac{1}{2} \times 2^2 - 9.8 \times 4 = \frac{1}{2} V_B^2$

$V_B = 10.1$ m/s

ii) $U_{B \to C} = T_C - T_B$

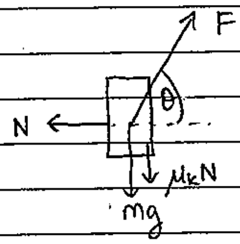
$U_f + (-mg)(y_C - y_B) = \frac{1}{2} m V_C^2 - \frac{1}{2} m V_B^2$

$U_f + 5 \times 9.8 \times (4 - 0) = \frac{1}{2} \times 5 \times (5^2 - 10.1^2)$

$U_f = -388.5$ J

Work done by friction $= |U_f| = 388.5$ J

(b) i)



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ii) $\sum F_x = 0$

$F\cos\theta - N = 0$

$N = F\cos\theta$

$\sum F_y = ma$

$F\sin\theta - mg - \mu_k N = ma$

$F\sin\theta - mg - \mu_k F\cos\theta = ma$

$$a = \frac{F}{m}(\sin\theta - \mu_k\cos\theta) - g$$

$$= \frac{F}{m}\left(\sin\left(\frac{\pi}{2} - \omega t\right) - \mu_k\cos\left(\frac{\pi}{2} - \omega t\right)\right) - g$$

iii) When $\theta = 0$

$0 = \frac{\pi}{2} - \omega T \rightarrow T = \frac{\pi}{2\omega}$

$$\int_0^T a\,dt = \int_0^0 dv$$

$$\int_0^T \left(\frac{F}{m}\left(\sin\left(\frac{\pi}{2} - \omega t\right) - \mu_k\cos\left(\frac{\pi}{2} - \omega t\right)\right) - g\right)dt = 0$$

$$\left[\frac{F}{m\omega}\left(\cos\left(\frac{\pi}{2} - \omega t\right) + \mu_k\sin\left(\frac{\pi}{2} - \omega t\right)\right) - gt\right]_0^{\frac{\pi}{2\omega}} = 0$$

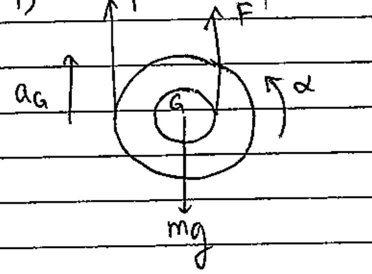
$$\frac{F}{m\omega}(\cos 0 + \mu_k\sin 0) - \frac{g\pi}{2\omega} - \frac{F}{m\omega}\left(\cos\frac{\pi}{2} + \mu_k\sin\frac{\pi}{2}\right) + 0 = 0$$

$$\frac{F}{m\omega} - \frac{g\pi}{2\omega} - \frac{\mu_k F}{m\omega} = 0$$

$$\frac{F}{m\omega}(1 - \mu_k) = \frac{g\pi}{2\omega}$$

$$F = \frac{mg\pi}{2(1 - \mu_k)}$$

(4) (a) i)




$\sum F_y = m a_G$

$F + T - mg = m a_G$ ---- (1)

$\sum M_G = I\alpha$

$Fr - TR = m k_G^2 \alpha$ --- (2)

Rolls without slipping

$a_G = \alpha R$ ---- (3)



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(1) → $T = m a_G + mg - F$

$= m\alpha R + mg - F$ ----(4)

(4) → (2) : $Fr - (m\alpha R + mg - F)R = m k_G^2 \alpha$

$Fr - m\alpha R^2 - mgR + FR = m k_G^2 \alpha$

$F(R+r) - mgR = (mR^2 + mk_G^2)\alpha$

$$\alpha = \frac{F(R+r) - mgR}{m(k_G^2 + R^2)}$$

ii) $$\alpha = \frac{150(0.6+0.3) - 5\times9.8\times0.6}{5(0.4^2 + 0.6^2)} = 40.62 \text{ rad/s}^2$$

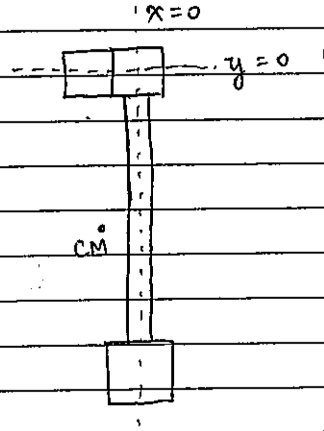
$\vec{\alpha} = 40.62\,\hat{k}$ rad/s²

(b) Linear momentum is conserved

$m_c \vec{V}_c + (m_A + m_B + M)\vec{V} = (m_A + m_B + m_c + M)\vec{V}_f$

$2\times5\hat{i} + (2+3+1)\times2\hat{i} = (2+3+2+1)\vec{V}_f$

$\vec{V}_f = 2.75\,\hat{i}$ m/s




$$y_{cm} = \frac{m_A y_A + m_B y_B + m_c y_c + My}{m_A + m_B + m_c + M}$$

$$= \frac{2\times0 + 3\times1.15 + 2\times0 + 1\times0.55}{2+3+2+1}$$

$= 0.5$ m

$$x_{cm} = \frac{m_A x_A + m_B x_B + m_c x_c + Mx}{m_A + m_B + m_c + M}$$

$$= \frac{2\times0 + 3\times0 + 2\times(-0.1) + 1\times0}{2+3+2+1}$$

$= -0.025$ m

$I_A = \frac{1}{12}\times2\times(0.1^2 + 0.1^2) = 0.003$ kg m²

$I_C = \frac{1}{12}\times2\times(0.1^2 + 0.1^2) = 0.003$ kg m²

$I_B = \frac{1}{12}\times3\times(0.2^2 + 0.2^2) = 0.02$ kg m²

$I_{rod} = \frac{1}{12}\times1\times1^2 = 0.083$ kg m²

$I_{cm} = (I_A + m_A(y_{cm} - y_A)^2 + m_A(x_{cm} - x_A)^2) + (I_B + m_B(y_{cm} - y_B)^2$
$+ m_B(x_{cm} - x_B)^2) + (I_C + m_c(y_{cm} - y_c)^2 + m_c(x_{cm} - x)^2)$
$+ (I_{rod} + M(y_{cm} - y)^2 + M(x_{cm} - x)^2)$



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$$I_{CM} = (0.003 + 2(0.5-0)^2 + 2(-0.025-0)^2)$$
$$+ (0.02 + 3(0.5-1.15)^2 + 3(-0.025-0)^2)$$
$$+ (0.003 + 2(0.5-0)^2 + 2(-0.025+0.1)^2)$$
$$+ (0.083 + 1(0.5-0.55)^2 + 1(-0.025-0)^2)$$
$$= 2.394 \text{ kg m}^2$$

Angular momentum about CM is conserved

$$\vec{L}_A + \vec{L}_B + \vec{L}_C + \vec{L}_{rod} = \vec{L}_f$$
$$m_A(\vec{r}_{A/CM} \times \vec{V}_A) + m_B(\vec{r}_{B/CM} \times \vec{V}_B) + m_c(\vec{r}_{C/CM} \times \vec{V}_C) + M(\vec{r} \times \vec{V})$$
$$= I_{CM}\vec{\omega}_f$$
$$2(0.5\hat{j} \times 2\hat{i}) + 3(-0.65\hat{j} \times 2\hat{i}) + 2(0.5\hat{j} \times 5\hat{i})$$
$$+ 1(-0.05\hat{j} \times 2\hat{i}) = 2.394\vec{\omega}_f$$
$$-2\hat{k} + 3.9\hat{k} - 5\hat{k} + 0.1\hat{k} = 2.394\vec{\omega}_f$$
$$\vec{\omega}_f = -1.253\hat{k} \text{ rad/s}$$



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# NANYANG TECHNOLOGICAL UNIVERSITY

## SEMESTER 2 EXAMINATION 2017-2018

## MA1001 – DYNAMICS

April/May 2018 Time Allowed: $2^1/_2$ hours

**MATRICULATION NUMBER:** | | | | | | | | | |

**SEAT NUMBER:** | | | | | |

**INSTRUCTIONS**

1. This question and answer booklet contains **FOUR (4)** questions. It consists of **SEVENTEEN (17)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided A4 size reference sheet is allowed.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

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For examiners:

| Questions | **1** (25) | **2** (25) | **3** (25) | **4** (25) | **Total** (100) |
|---|---|---|---|---|---|
| Marks | | | | | |



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**Q1 (25 marks)**

/25

(a) A bullet train starts from rest and moves along a straight track. The acceleration-time graph of the train is shown in Figure Q1(a).

(i) Determine the speed of the train at $t = 10$ s. (3 marks)

(ii) Determine the speed of the train at $t = 110$ s. (5 marks)

(iii) Determine the elapsed time $t'$ before the train again comes to rest. (4 marks)

***Solution:***

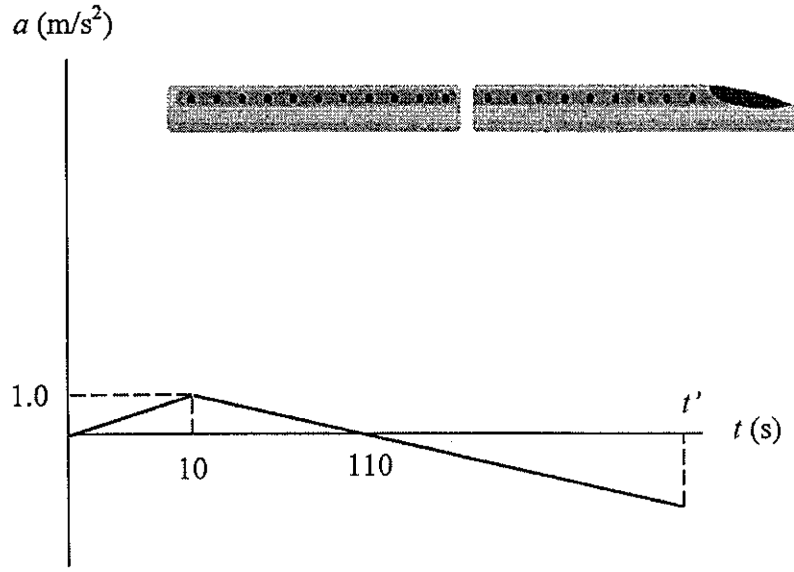


Figure Q1(a)

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**Q1**

(b) Two cars, $A$ and $B$, travel at the same constant speed of 25 m/s. $A$ travels along the circular track of radius 400 m, while $B$ travels along the diameter of the circle, as shown in Figure Q1(b).

(i) Determine the velocity of $A$ relative to $B$. (6 marks)

(ii) Determine the angular velocity of car $A$. (2 marks)

(iii) If a reference frame $f$ is attached to car $A$, what is the velocity of the frame at position $B$, $\vec{v}_{B'}$? (2 marks)

(iv) Determine the velocity of $B$ relative to the reference frame $f$, $\vec{v}_{B/f}$. (Hint: $\vec{v}_B = \vec{v}_{B'} + \vec{v}_{B/f}$) (3 marks)

Express your answers as vectors in the form $(\ )\hat{\imath} + (\ )\hat{\jmath} + (\ )\hat{k}$.

***Solution:***

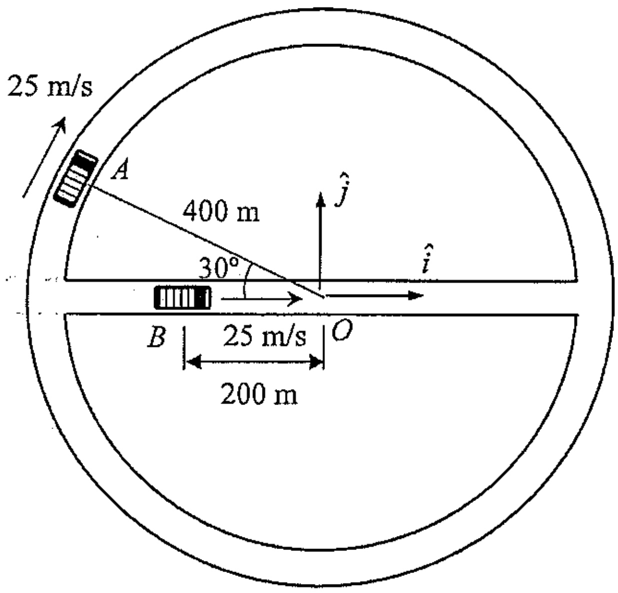


Figure Q1(b)



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**Q2 (25 marks)** /25

Figure Q2(a) shows a block $A$ moving with a velocity $v_A = 2.0$ m/s and an acceleration $a_A = 1.0$ m/s$^2$. Link $AB$ slides freely along its slot on the fixed peg $C$. Using $\vec{v}_C = \vec{v}_{C'} + \vec{v}_{C/f}$ and $\vec{a}_C = \vec{a}_{C'} + \vec{a}_{C/f} + \vec{a}_C^{\text{Coriolis}}$, determine

(i) the angular velocity of the slotted link $AB$ at the instant shown, (6 marks)

(ii) the sliding velocity of the slotted link $AB$ relative to the fixed peg $C$, (6 marks)

(iii) the angular acceleration of the slotted link $AB$ at the instant shown, (7 marks)

(iv) the sliding acceleration of the slotted link $AB$ relative to the fixed peg $C$. (6 marks)

***Solution:***

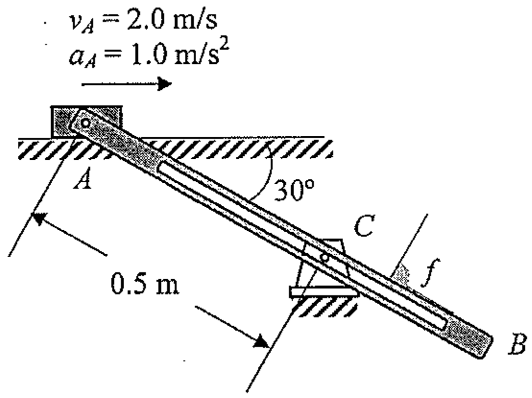


Figure Q2(a)

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**Q3 (25 marks)** /25

(a) As shown in Figure Q3(a), the disk is rotating at a constant angular velocity of $\omega = 20$ [rad/s] about its centre. The block $A$ is connected to the taut cable whose length $L = 1$ [m]. Another end of the cable is connected to the centre of the disk. At time $t = 0$ [s], the block $A$ (mass = 0.5 [kg]) is placed on the disk with zero initial velocity and an external force ($F = 2t$ [N]) is exerted on the top surface of the block $A$ in the direction normal to the disk until the block stops slipping. Knowing that the coefficient of kinetic friction, $\mu_K$, is 0.5, determine the time, $t_f$ [s], required for the block to stop slipping.

(13 marks)

***Solution:***

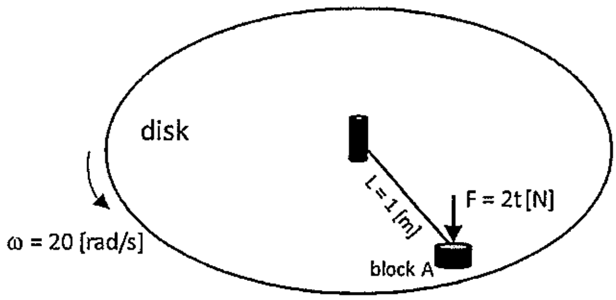


Figure Q3(a)



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**Q3**

(b) As shown in Figure Q3(b), the ball is thrown from the point $A$ toward the wall at a velocity of 50 [m/s] in the direction $\theta$ [degree] with respect to the +X direction. After striking the wall, the ball comes back exactly to the point $A$. Knowing that the coefficient of restitution of the wall impact is $e = 0.1$, determine $\theta$ [degree]. Assume that the motion of the ball is in a vertical plane and that air resistance is negligible.

(12 marks)

***Solution:***

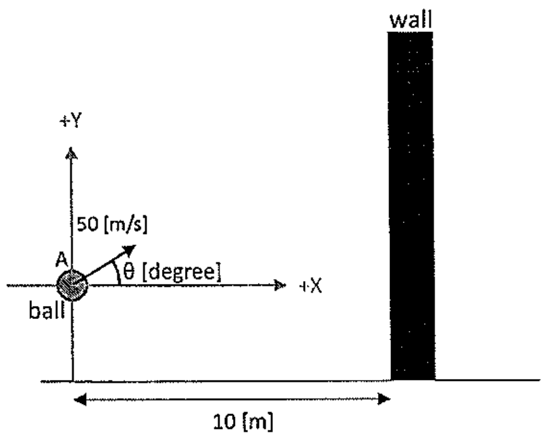


Figure Q3(b)

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**Q4 (25 marks)** /25

(a) The cord is wrapped around the inner core of the spool as shown in Figure Q4(a). If a 1-kg block $B$ is suspended from the cord and released from rest, determine the spool's angular velocity when $t$ = 3 s. The spool has a weight of 10 kg. The radius of gyration about the axle $A$ is $k_A$ = 0.5 m. Neglect the mass of the cord.

(10 marks)

***Solution:***

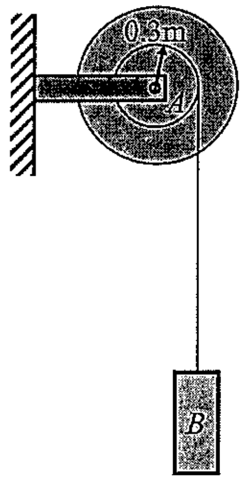


Figure Q4(a)



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**Q4**

(b) A 2-kg mass of putty $D$ strikes the uniform 10-kg plank $ABC$ with a velocity of 10 m/s, as shown in Figure Q4(b). Initially plank $ABC$ is stationary. If the putty remains attached to the plank, determine the maximum angle $\theta$ of swing before the plank momentarily stops. Neglect the size of the putty and friction torque on the axis. The motion is in a vertical plane.

(15 marks)

***Solution:***

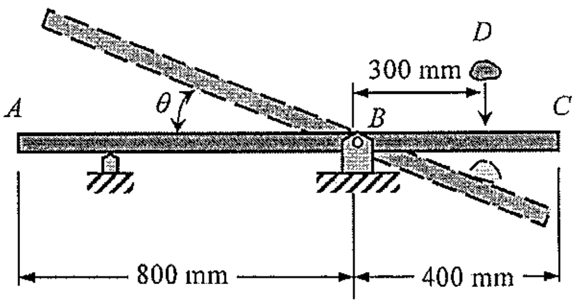



Figure Q4(b)

Semester 2 2017 – 2018 Dynamics MA1001

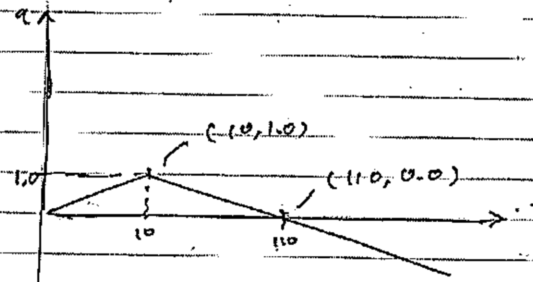
1ai: acceleration – time graph given. Since $a = \frac{dv}{dt}$; integrating across time $t_0$ to $t_1$: $\int_{t_0}^{t_1} a \, dt = \int_{t_0}^{t_1} dv$

∴ change in speed $= \int_{t_0}^{t_1} a \, dt$ = area under graph

Since train at rest at $t=0$,

Speed at $t=10$: $\frac{1}{2}(1.0)(10) + 0 = 5 \, m s^{-1}$

1aii:




note: a – t graphs Before and After $t=10$ not to scale.

$a \begin{cases} 0.1t & \text{for } t < 10 \\ \text{let } t' \text{ be dt after } t=10 & \\ 1.0 - 0.01t' & \text{for } t > 10 \end{cases}$

Speed of train at $t = 110s$:

$\int_0^{10} 0.1t \, dt + \int_0^{100} 1.0 - 0.01t' \, dt' + 0$

$= \left[\frac{0.1}{2}t^2\right]_0^{10} + \left[t' - \frac{0.01}{2}t'^2\right]_0^{100} = 5 + 50 = 55 \, m s^{-1}$

1aiii: Once again, let $t'$ be time elapsed after $t=10$ (or $v=5$)

$a = 1.0 - 0.01t'$. To reach zero velocity reduce $5 \, m s^{-1}$

$\int_0^{t_f} 1.0 - 0.01t' \, dt = \int_0^{-5} dv$

$\left[t' - \frac{0.01}{2}t'^2\right]_0^{t_f} = -5 \rightarrow t_f = 205$

total time elapsed: $205 + 10 = 215s$

note: we have to introduce new variable $t'$, instead of $\int_{10}^{110} 1.0 - 0.01t \, dt$ because this eqn assumes $t=10$, $a = 0.9$, which does not correspond to actual point $(10, 1.0)$

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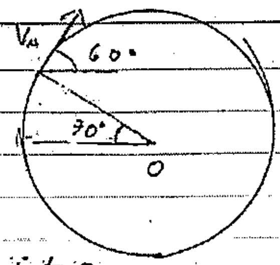
1(b i) $\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$

$= 25 \angle 60° - 25\,\hat{i}$

$= -12.5\,\hat{i} + 21.65\,\hat{j}$



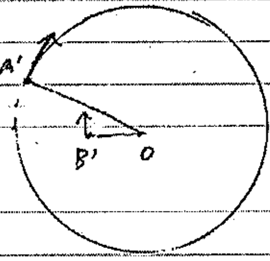

ii) Consider A a rotating about fixed point O

$\vec{V}_A = \omega_A \times \vec{r}_{A/O}$ $\qquad \vec{V}_A = 25 \angle 60°$

$\vec{\omega}_A = \frac{25}{400} = 0.0625(-\hat{k})$ $\qquad \vec{V}_B = 25\,\hat{i}$

clockwise

iii) Since our reference frame is attached to A, A must be seen as stationary. We can do this by imagining a circular piece of cardboard spinning about O. Point A' follows motion of A exactly, while B' happens to be at position of B at that instant, while it rotates about O.




$\vec{V}_{B'} = \vec{\omega} \times \vec{r}_{B'/O}$

$= -0.0625\,\hat{k} \times -200\,\hat{i}$

$= 12.5\,\hat{j}$

iv) $\vec{V}_B = \vec{V}_{B'} + \vec{V}_{B/f}$

$\therefore \vec{V}_{B/f} = \vec{V}_B - \vec{V}_{B'} = 25\,\hat{i} - 12.5\,\hat{j}$

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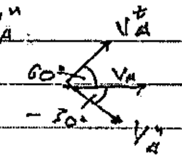
2 i) We can break down $V_A$ into $V_A^t$ and $V_A^n$.




$V_A = V_A^t \measuredangle 60° + V_A^n \measuredangle -30°$

dotting along $\measuredangle 60°$ : $V_A^t = V_A \cos 60°$

$= 2 \cos 60°$

$= 1 \, m s^{-1} \measuredangle 60°$

$\vec{V_A^t} = \vec{\omega_A} \times \vec{r_{A/C}}$ } Apply right hand grip rule to find direction.

$\vec{\omega_A} = \frac{-1}{0.5} = -2\underline{k}$

ii) dotting along $\measuredangle -30°$ : sliding velocity $= 2 \cos 30° = 1.73$

$\vec{V_{A/S}} = 1.73 \measuredangle -30°$

iii) $\vec{a_A} = \vec{a_{A'}} + \vec{a_{A/S}} + \vec{a_A}^{coriolis}$ . recall $\vec{a_A}$

(1) $= \ddot{\theta} r \measuredangle 60° + \dot{\theta}^2 r \measuredangle -30 + \ddot{r} \measuredangle -30° + 2\dot{\theta}\dot{r} \measuredangle 60°$

angular acc. centripetal acc. relative acc. coriolis acc.

dotting to $\measuredangle 60°$:

$1 \cos 60° = [\ddot{\theta}(\frac{1}{2}) + -2(2 \times 1.73)]$

$\ddot{\theta} \; \alpha = 14.84$

$\vec{\alpha} = -14.84\underline{k}$

direction determine by right hand grip $2 \times \vec{\omega} \times \vec{v_{A/S}}$

since $\vec{\alpha} \times \vec{r_{A/C}} = \measuredangle 60°$

vectors $\vec{\alpha} \times \nwarrow = \uparrow$

By RHGR, $\vec{\alpha} = \alpha(-\underline{k})$

iv) Dotting (1) to $\measuredangle -30°$ :

$\frac{\sqrt{3}}{2} = \dot{\theta}^2 r + \ddot{r}$

$\frac{\sqrt{3}}{2} = 2^2 \times 0.5 + \ddot{r}$

$\ddot{r} = -1.13 \measuredangle -30°$

$= 1.13 \measuredangle 150°$

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3a) Recall $J^{ang}_{1-2} = \int_0^{t_f} M \, dt$ — angular impulse $= \int$ torque $dt$ = change in angular momentum.

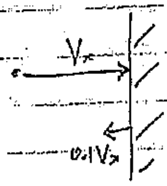
Recall $H_o = I_o w = mR^2 w$

change in angular momentum $= 0.5(1^2)20 - 0$
$= 10 \, kgm^2$

torque $= \frac{1}{2}(mg + 2t)$

Hence:
$$10 = \int_0^t \frac{1}{2}(0.5 \times 9.81 + 2t) \, dt$$
$$20 = t^2 + 4.905t$$
$$t = 2.65 s.$$

3b)




let $V_x$ be initial horizontal velocity. since $e = 0.1$, horizontal velocity after rebound $= 0.1V_x$

total travel time $t = \frac{10}{V_x} + \frac{10}{0.1V_x}$ (assuming collision is instantaneous)

$V_x = 50\cos\theta$

$t = \frac{1}{5\cos\theta} + \frac{10}{5\cos\theta} = \frac{11}{5\cos\theta}$

For entire trip, no vertical displacement (back at A)

$\therefore S_y = 50\sin\theta \, t - \frac{1}{2} g t^2 = 0$

$50\sin\theta\left(\frac{11}{5\cos\theta}\right) - \frac{1}{2} g \left(\frac{11}{5\cos\theta}\right)^2 = 0$

$110\tan\theta - 23.74\sec^2\theta = 0$

$110\tan\theta - 23.74(\tan^2\theta + 1) = 0$

solve quadratic eqn: $\tan\theta = 4.407 / 0.2264$

$\theta = 77.2° / 12.8°$

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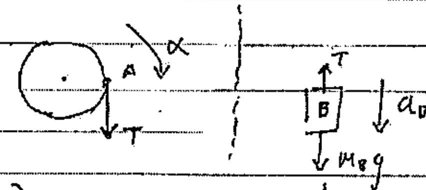
4a)



Since point A & B accelerate at same rate,

$a_B = 0.3\alpha$

$\Sigma M$ (+):

$T(0.3) = [10 \times 0.5^2]\,\alpha$ — (1)

$\Sigma F_y$ on B:

$9.81 - T = a_B(1) = 0.3\alpha$

(2) $-T: 9.81 - 0.3\alpha$

solve (1) & (2):

$\alpha = 1.136 \text{ rads}^{-2}$

$\omega = \alpha t = 1.136(3) = 3.41 \text{ rads}^{-1}$

b) Recall change in angular momentum = angular impulse

moment of inertia of rod & putty about B:

$\frac{10}{12} \times 1.2^2 + 10 \times 0.2^2 + 0.3^2 \times 2 = 1.78$

$I_{rod \text{ about c.g.}} + I_{rod \text{ about } B} + I_{putty \text{ about } B}$

$\omega\, 1.78 = (20 \times 0.3) \rightarrow \omega = 3.37$

If no gravity, rod & putty will spin at $\omega = 3.37$ about B.

∴ By C.O.E:

$\frac{1}{2}(1.78)(3.37^2) = -0.3\sin\theta(2 \times 9.81) + 0.2\sin\theta(10 \times 9.81)$

$\theta = 47.4°$

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**NANYANG TECHNOLOGICAL UNIVERSITY**

**SEMESTER 1 EXAMINATION 2018-2019**

**MA1001 – DYNAMICS**

November/December 2018 Time Allowed: $2^1/_2$ hours

**MATRICULATION NUMBER:**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

**SEAT NUMBER:**

| | | | | |
|---|---|---|---|---|
| | | | | |

**INSTRUCTIONS**

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For examiners:

| Questions | **1** (25) | **2** (25) | **3** (25) | **4** (25) | **Total** (100) |
|---|---|---|---|---|---|
| Marks | | | | | |



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**Q1 (25 marks)** /25

(a) The motion of peg $P$ is constrained by the curved slot in $OB$ and by the slotted arm $OA$. $OA$ rotates counterclockwise with an angular velocity of $\dot{\theta} = (3t^{3/2})$ rad/s, where $t$ is in seconds. When $t = 0$, $\theta = 0°$. Meanwhile, the distance of $OP$ is $r = \sqrt{4\cos(2\theta)}$. Determine the magnitudes of the velocity and acceleration of peg $P$ at $\theta = 30°$. Present the answer using the unit vectors of $\hat{\imath}$ and $\hat{\jmath}$.

(17 marks)

***Solution:***

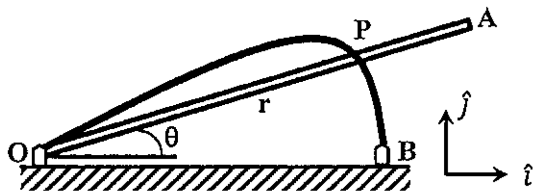


Figure Q1(a)

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**Q1**

(b) A ball is thrown from A at the angle of 30°. If it is required to clear another wall at B, determine the minimum magnitude of its initial velocity $v$.

(8 marks)

***Solution:***

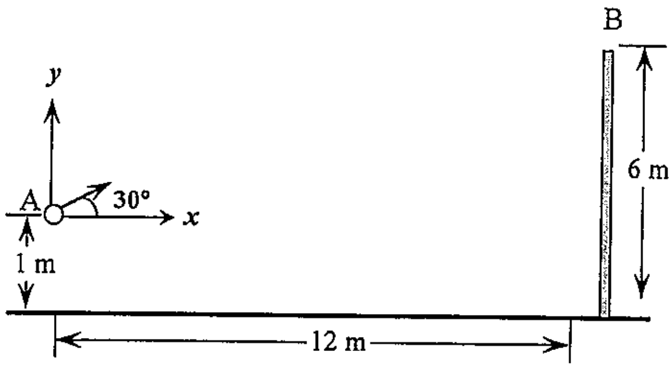


Figure Q1(b)



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**Q2 (25 marks)** /25

In the position shown in Figure Q2, the bar AO has a counterclockwise angular velocity $\omega_0$ = 10 [rad/s], and a counterclockwise angular acceleration $\alpha_0 = 2$ [rad/s$^2$].

(i) Determine the angular velocities of the rods AB and BC, $\omega_1$ and $\omega_2$. (10 marks)

(ii) Determine the angular accelerations of the rods AB and BC, $\alpha_1$ and $\alpha_2$. (15 marks)

***Solution:***

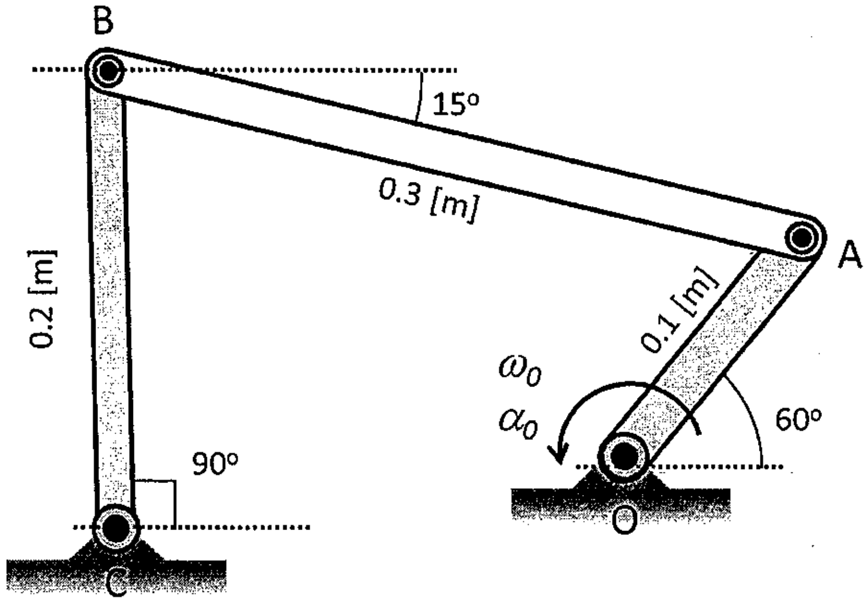


Figure Q2

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**Q3 (25 marks)** /25

(a) The 5 [kg] cart placed on a smooth surface is acted on by a horizontal force $F$, which varies with time $t$ as shown in Figure Q3(a). At $t = 0$ [s], the velocity of the cart is zero.

(i) Determine the velocity of the cart at $t = 1$ [s]. (5 marks)

(ii) Determine the velocity of the cart at $t = 3$ [s]. (8 marks)

***Solution:***

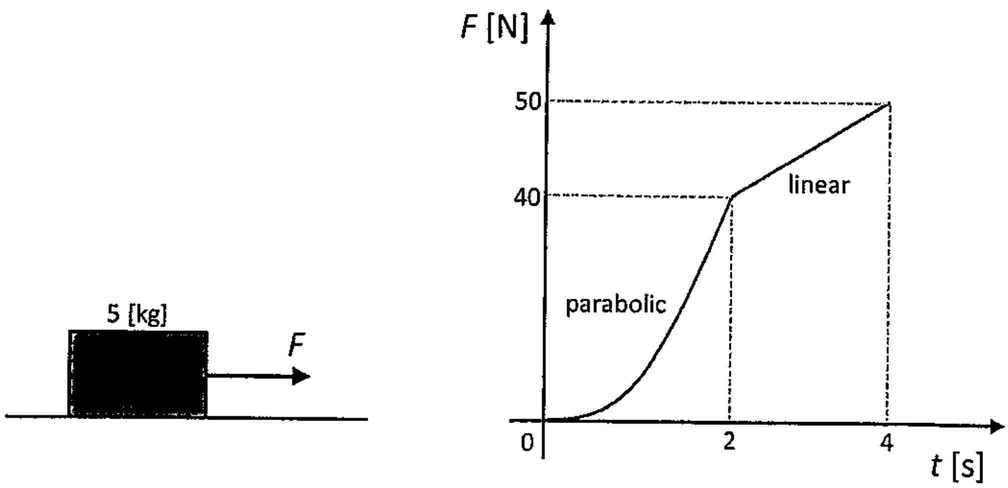


Figure Q3(a)



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**Q3**

(b) As shown in Figure Q3(b), the 5 [kg] block is released from rest in the position shown (the height $h$ of the block from the top surface of the spring is 200 [mm]) and falls on the spring, which has been initially pre-compressed by 100 [mm] with the massless wires. Knowing the stiffness of the spring is 2 [kN/m], determine the additional deflection of the spring produced by the falling block before it rebounds.

(12 marks)

***Solution:***

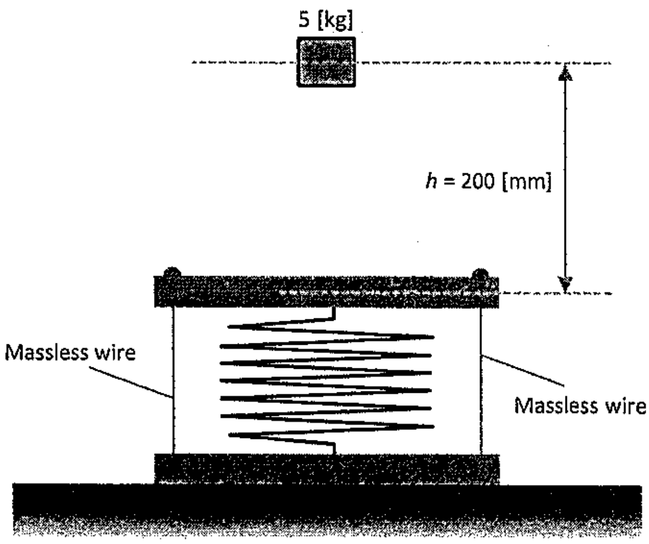


Figure Q3(b)

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**Q4 (25 marks)**

/25

(a) The spool has a weight of 10 kg and a radius of gyration $k_G$ = 0.5 m. A cord is wrapped around the spool's inner hub and its end subjected to a horizontal force $P$ = 10 N. Determine the spool's angular velocity in 4 s starting from rest. Assume the spool rolls without slipping.

(8 marks)

***Solution:***

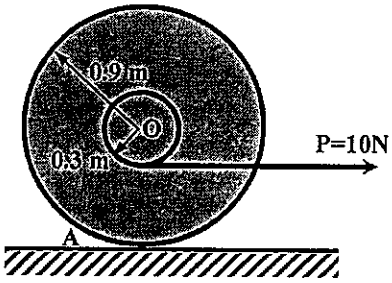


Figure Q4(a)



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Q4

(b) A wheel has a mass of 50 kg and a radius of gyration $k_G = 0.4$ m. If it rolls without slipping down the inclined plank, determine at the instant when the wheel is at the mid-point of the plank AB, the horizontal and vertical components of reaction at A, and the normal reaction at the smooth support B. The plank has negligible thickness and a mass of 20 kg.

(17 marks)

***Solution:***

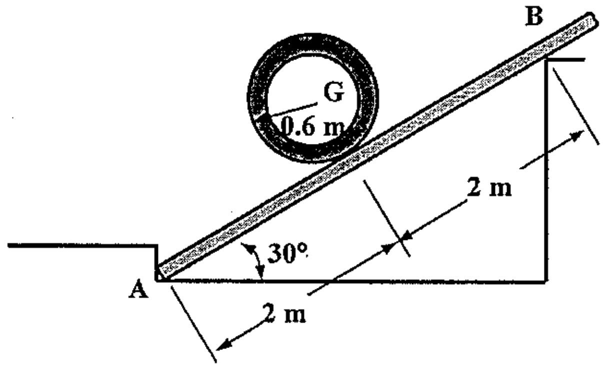


Figure Q4(b)

November / December 2018

1a) $\dot{\theta} = 3t^{1.5}$ rads$^{-1}$, $\theta = \int \dot{\theta}\, dt = \frac{3}{2.5}\theta^{2.5} + c$

when $t = 0$, $\theta = 0$, $c = 0$. $\theta = \frac{3}{2.5}\theta^{2.5}$

$\ddot{\theta} = 3(1.5)t^{0.5} = 4.5t^{0.5}$

$r = 2\cos^{0.5}(2\theta)$, $\dot{r} = \frac{dr}{dt} = 2\cos^{-0.5}(2\theta)(-\sin 2\theta)(\dot{\theta})$ ← given condition

$\ddot{r} = -2\sin 2\theta\,(\dot{\theta})[-\frac{1}{2}\cos^{-1.5}(2\theta)(-\sin 2\theta)\dot{\theta}]$

$+ 2\cos^{-0.5}(2\theta)\dot{\theta}[-2\cos(2\theta)\dot{\theta}]$

$+ 2\cos^{-0.5}(2\theta)(-\sin 2\theta)\ddot{\theta}$

$\vec{V_p} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$. At $\theta = 30° = \frac{\pi}{6}$, $t = \frac{3}{2}\sqrt{(\frac{1}{2}\frac{\pi}{6})} = 0.31231$

$\vec{V_p} = (-2(0.5)^{-0.5}(1)(-\frac{\sqrt{3}}{2})(0.3123)\hat{e}_r$

$+ 2(\frac{1}{\sqrt{2}})(\frac{3}{2.5})(0.3123)^{2.5}\hat{e}_\theta = 0.765\hat{e}_r + 0.0925\hat{e}_\theta$

since $\hat{e}_r = \cos(\frac{\pi}{6})\hat{i} + \sin(\frac{\pi}{6})\hat{j}$, and $\hat{e}_\theta = \cos\theta\hat{j} - \sin\theta\hat{i}$

$\vec{V_p} = (0.08 + 0.3825)\hat{j} + (0.6625 - 0.0463)\hat{i}$

$= 0.615\hat{i} + 0.463\hat{j}$

$\ddot{\theta} = 0.0659$, $\dot{\theta} = 2.515$

$\vec{a_p} = \ddot{r}\hat{e}_r - (\dot{\theta})^2 r\hat{e}_r + 2\dot{\theta}\dot{r}\hat{e}_\theta + \ddot{\theta}r\hat{e}_\theta$

$= [(-1.73)(0.0659)[2.83(\frac{\sqrt{3}}{2})0.0659] + 0.185(0.0659) - 6.16]\hat{e}_r$

$- [(0.0659)^2(\sqrt{2})]\hat{e}_r$

$+ 2(0.0659)(-0.16)\hat{e}_\theta + (2.515)(\sqrt{2})\hat{e}_\theta$

$= 0.674\hat{e}_r + 3.54\hat{e}_\theta$

$= -1.19\hat{i} + 3.403\hat{j}$

1b) let initial velocity be $V$

horizontal displacement = 12m, acceleration in x-direction = 0

$\therefore 12 = (V\cos 30°)t$

$\therefore t = \frac{12(2)}{V\sqrt{3}} = \frac{24}{\sqrt{3}V}$ — (1)

vertical displacement = 6 − 1 = 5m, acceleration is −g

$5 = (V\sin 30°)t - \frac{1}{2}(9.81)t^2$ — (2)

subst. (1) into (2):

$5 = \frac{V}{2}(\frac{24}{\sqrt{3}V}) - \frac{1}{2}(9.81)\frac{24^2}{3V^2}$

$\frac{24^2}{3V^2} = 0.393$

$V = 22.1$ ms$^{-1}$

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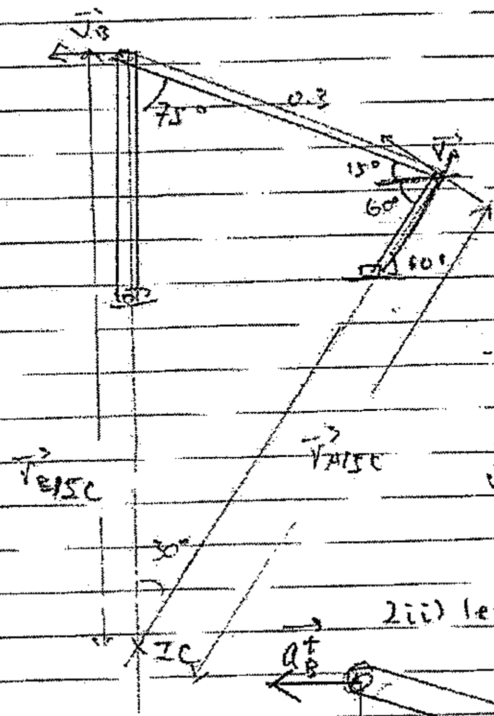
2i) $\vec{V_A} = (10 \times 0.1) \measuredangle 120° = 1 \measuredangle 120°\ ms^{-1}$



From IC, AB appears to be in rotation ∴

$\frac{V_A}{r_{A/IC}} = \frac{V_B}{r_{B/IC}} = \omega_{AB}$

sine rule:

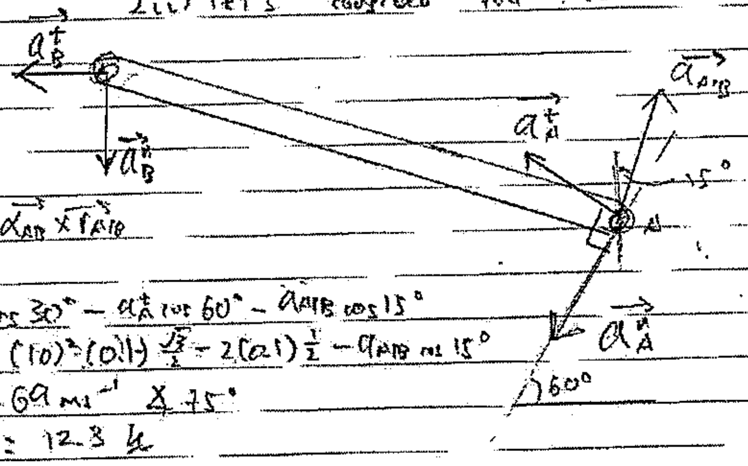
$\frac{\sin 30°}{0.3} = \frac{\sin 75}{r_{A/IC}}$ ∴ $r_{A/IC} = 0.58$

∴ $\omega_{AB} = \frac{1}{0.58} = 1.725\ \omega\ \hat{k}$

Since $r_{A/IC} = r_{B/IC}$, $V_B = V_A = 1 ms^{-1}$

$\vec{\omega_B} = \frac{V_B}{r_B} = 5\hat{k}$

2ii) let's consider rod AB



we know that

$\vec{a_{A/B}} = \vec{a_B} - \vec{a_A} = \alpha_{AB} \times \vec{r_{A/B}}$

along $-\hat{j}$:

$a_B^n = a_A^n \cos 30° - a_A^t \cos 60° - a_{A/B} \cos 15°$

∴ $5^2(0.2) = (10)^2(0.1)\frac{\sqrt{3}}{2} - 2(0.1)\frac{1}{2} - a_{A/B} \cos 15°$

$a_{A/B} = 3.69\ ms^{-1} \measuredangle 75°$

∴ $\vec{\alpha_{AB}} = 12.3\ \hat{k}$

to find $\vec{a_B^t}$, consider along $-\hat{i}$:

$a_B^t = 10 \cos 60° + 0.2 \cos 30° - 3.69 \cos 75°$

$\vec{a_B^t} = \vec{\alpha_B} \times \vec{r_B} = 4.22 ms^{-1} (-\hat{i})$

∴ $\vec{\alpha_B} = -21.1\ \hat{k}$

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3 i) impulse = area under F-t graph = change in velocity.
for $t \leq 2s$,
we see that $F = 10x^2$, since shape is parabolic
(we can use $F = Ax^2$, and subst. (2, 40) to find A)
Hence change in velocity: $\int_0^1 10x^2\,dx = \frac{10}{3}$
$\frac{1}{2}(5)(\Delta V)^2 = \frac{10}{3}$
$V_1 = \Delta V = 1.155\,ms^{-1}$

ii) at $t = 3$, impulse: $\int_0^2 10x^2\,dx + \frac{40+50}{2}(2) = 116.7$
$\frac{1}{2}(5)(\Delta V)^2 = 116.7$
$V_3 = \Delta V = 6.83\,ms^{-1}$

b) let mass of platform be m, deflection be z
initial energy: $5(9.81)(0.2+z) + \frac{1}{2}(2\times10^3)(0.1)^2 + m(9.81)z$
final energy: $\frac{1}{2}(2\times10^3)(0.1+z)^2$
By C.O.E: $(9.81 + 9.81m)z + 10 = 1000(0.1+z)^2$
$10 = 1000(0.1^2 + 0.2z + z^2) - (9.81 + 9.81m)z$
$0 = (190.2 - 9.81m)z + 1000z^2$
$z = \frac{9.81m - 190.2}{1000}\,m$

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4a) moment of inertia: $10(0.5)^2 = 2.5\ kgm^2$

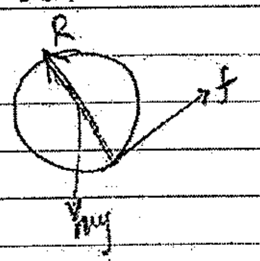
angular acceleration: $\frac{\tau}{I} = \frac{3}{2.5} = 1.2\ rads^{-2}$

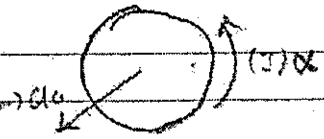
angular velocity after 4s: $4 \times \alpha = 4.8\ rads^{-1}$

4b) since wheel is rolling without slip: $a_G = \alpha r$

$a_G = 0.6\alpha$

FBD. KD




Here we can form 2 equations:

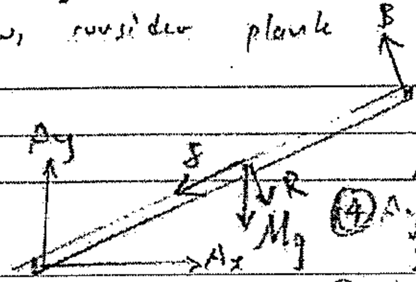
$mg \sin 30° - f = ma_G$ — ①

$f(0.6) = m(k_G)^2\alpha = m(k_G)^2\left(\frac{1}{0.6}a_G\right)$ — ②

Along ∡ 120°, $\Sigma F = 0$ ∴ $R = mg\cos 30°$ — ③

now, consider plank

FBD




since plank is in equilibrium

$\Sigma F_y = 0$:

$A_y - f\sin 30° - Mg - R\cos 30° + B\cos 30° = 0$

④ $A_y - \frac{1}{2}f - 0.866R + 0.866B = 196.2$

$\Sigma F_x = 0$

⑤ $A_x - f\left(\frac{\sqrt{3}}{2}\right) + R\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)B = 0$

$\Sigma M_{(midpoint)} = 0$

⑥ $B = \frac{\sqrt{3}}{2}A_y - \frac{1}{2}A_x$

6 equations, 6 unknowns: $f, a_G, R, A_y, A_x, B$

All the Best!!

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**NANYANG TECHNOLOGICAL UNIVERSITY**

**SEMESTER 2 EXAMINATION 2018-2019**

**MA1001 – DYNAMICS**

April/May 2019 Time Allowed: $2^1/_2$ hours

**MATRICULATION NUMBER:**

**SEAT NUMBER:**

**INSTRUCTIONS**

1. This question and answer booklet contains **FOUR (4)** questions. It consists of **SEVENTEEN (17)** pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided A4 size reference sheet is allowed.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

For examiners:

| Questions | 1 (25) | 2 (25) | 3 (25) | 4 (25) | Total (100) |
|---|---|---|---|---|---|
| Marks | | | | | |



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**Q1 (25 marks)** /25

(a) A particle $A$ is projected vertically upward with an initial speed of 30 m/s. One second later, another particle, $B$, is projected with a speed of $v$ m/s, at 60° to the horizontal, as shown in Figure Q1(a). Assume that the effect of air resistance is negligible.

(i) Calculate the height of particle $A$ from the ground after 1 s. (2 marks)

(ii) Calculate the speed of particle $A$ after 1 s. (2 marks)

(iii) By considering the velocity of $B$ relative to $A$, or otherwise, calculate the speed, $v$, of particle $B$ if the two particles were to collide at some point during their motion. (8 marks)

***Solution:***

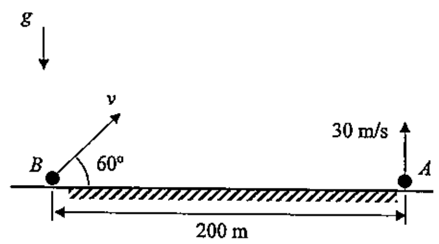


Figure Q1(a)

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**Q1**

(b) Figure Q1(b) shows a straight link $AC$ rotating at a constant rate of 5 rad/s counter-clockwise about a fixed pivot at $A$. At the instant shown, $\theta = 75°$. Two small collars, connected by a pivot joint, are each free to slide, one along the link $AC$ and the other along a circular ring of radius 400 m. At the instant shown, the collars are at position $B$.

(i) Show that the speed of the collars is constant. Determine the velocity (in terms of magnitude and direction) of the pivot joint of the collars at the instant shown. (4 marks)

(ii) Determine the acceleration (in terms of magnitude and direction) of the pivot joint of the collars at the instant shown. (2 marks)

(iii) Determine the sliding acceleration, $\ddot{r}$, of the pivot joint of the collars relative to the link $AC$ at the instant shown. (7 marks)

***Solution:***

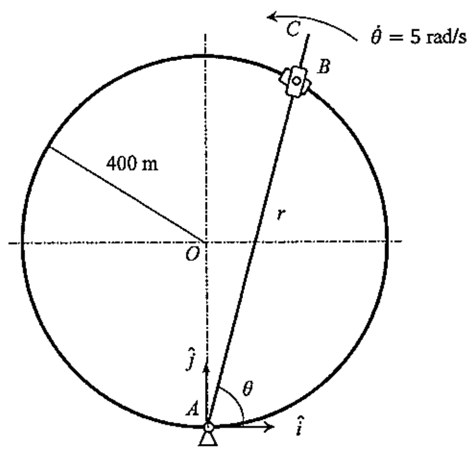


Figure Q1(b)



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**Q2 (25 marks)** /25

A rigid body consists of a triangular part, $ABD$, and a semi-circular part, $BCD$, of radius $GC = 0.5$ m. At the instant shown in Figure Q2, it is rolling without slip at a constant angular rate of 1 rad/s counter-clockwise, and in contact with the horizontal ground at point $C$. Point $A$ of the body is constrained to slide along the slotted link $OE$. Link $OE$ rotates about a fixed pivot at $O$. At the instant shown,

(i) determine the velocity of point $A$, (3 marks)

(ii) using $\vec{a}_A = \vec{a}_G + \vec{a}_{A/G}$ or otherwise, determine the acceleration of point $A$, (5 marks)

(iii) using $\vec{v}_A = \vec{v}_{A'} + \vec{v}_{A/f}$ or otherwise, determine the angular velocity of link $OE$ and the sliding velocity of $A$ relative to the slotted link $OE$, (7 marks)

(iv) using $\vec{a}_A = \vec{a}_{A'} + \vec{a}_{A/f} + \vec{a}_A^{\text{Coriolis}}$ or otherwise, determine the angular acceleration of the slotted link $OE$ and the sliding acceleration of $A$ relative to the slotted link $OE$. (10 marks)

Express your answers as vectors in the form $(\ )\hat{\imath} + (\ )\hat{\jmath} + (\ )\hat{k}$.

***Solution:***

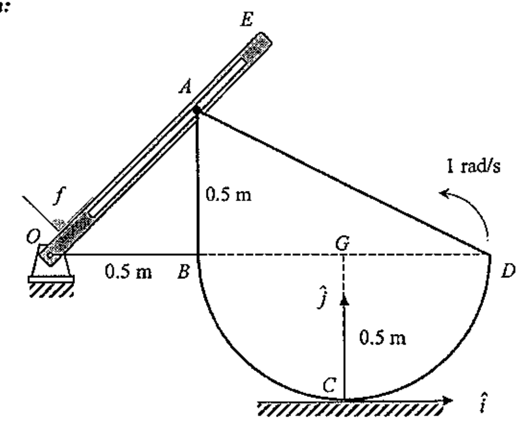


Figure Q2

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**Q3 (25 marks)** /25

(a) As shown in Figure Q3(a), the 1.0 [kg] particle is attached to the light rod $OA$ which rotates about a horizontal axis through point $O$. The system is released from rest while in the position $\theta = 0$ where the spring is unstretched.

(i) If the particle is observed to stop momentarily in the position $\theta = 50°$, determine the spring constant $k$. (6 marks)

(ii) For the $k$ calculated in (i), determine the particle speed at the position $\theta = 25°$. (7 marks)

***Solution:***

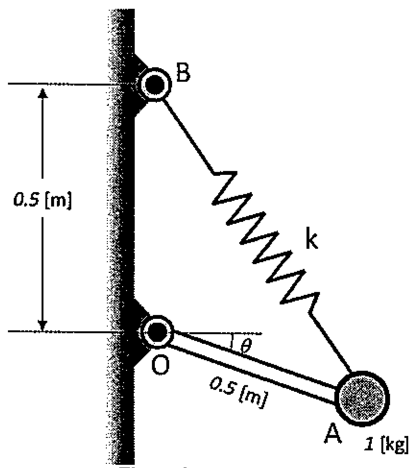


Figure Q3(a)



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**Q3**

(b) As shown in Figure Q3(b), the force $F$ is applied to the 10 [kg] block initially at rest. Knowing that the coefficients of static and kinetic friction between the block and the horizontal surface are 0.6 and 0.4, respectively, determine the velocities of the block when $t = 3$ [s] and $t = 10$ [s].

(12 marks)

***Solution:***

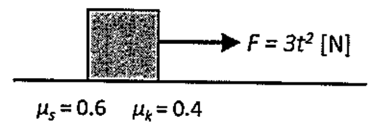


Figure Q3(b)



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**Q4 (25 marks)** /25

(a) If the motor $M$ exerts a constant force of $P = 300$ N on the cable wrapped around the reel's outer rim, determine the velocity of the 50-kg cylinder after it has traveled a distance of 2 m. Initially, the system is at rest. The reel has a mass of 25 kg, and the radius of gyration about its centre of mass, $A$, is $k_A = 125$ mm. Assume no frictional moment on the rotation of the reel.

(10 marks)

***Solution:***

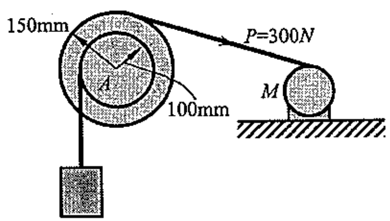


Figure Q4(a)



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**Q4**

(b) A 10-kg semi-circular disk is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. Determine the normal force and friction force exerted on the ground at $A$ at this instant. Assume the disk does not slip as it rolls.
Hint: the distance between the centre of mass of the semi-circular disk and point $O$ is $4r/3\pi$. The moment of inertia about $O$ is $I_O = mr^2/2$, where $r$ is the radius and $m$ is the mass.

(15 marks)

***Solution:***

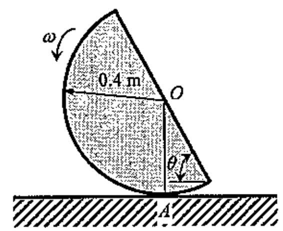


Figure Q4(b)

MA1001

1·a·i) $S = S_0 + V_0 t + \frac{1}{2}at^2$

$h_A = 30(1) + \frac{1}{2}(-9.81)(1)^2$

$= 25.095m$

1·a·ii) $V = V_0 + at$

$V_A = 30 + (-9.81)(1)$

$= 20.19\ m/s$

1·a·iii) $V_{B/A} = V\cos60\ \hat{i} + V\sin60\ \hat{j} - 20.19\ \hat{j}$

Relative acceleration = 0 because both particles experience the same acceleration.

$\hat{i}: 200 = V\cos60 . t$ — ①

$\hat{j}: 25.095 = (V\sin60 - 20.19)t$ — ②

From ①, $t = \frac{200}{V\cos60}$ — ③

Sub. ③ into ②, $25.095 = \frac{200\sin60}{\cos60} - \frac{20.19 \times 200}{V\cos60}$

$25.095V = 200\tan60 . V - \frac{20.19 \times 200}{\cos60}$

$V = 25.1342\ m/s$

1·b·i) Let $\angle AOB = \mu$.

$\mu = 2\theta$. $\therefore \dot{\mu} = 2\dot{\theta} = 10 rad/s$

$\vec{V}_B = 400\dot{\mu}\hat{e}_\mu = 4000\ \hat{e}_\mu$

$\therefore$ Velocity of B is constant in the direction of $\hat{e}_\mu$, thus collar speed is constant.

$\hat{e}_\mu = \cos2\theta\ \hat{i} + \sin2\theta\ \hat{j}$

$\vec{V}_c = \vec{V}_B = 4000\cos2\theta\ \hat{i} + 4000\sin2\theta\ \hat{j}$ $(\theta = 75°)$

$= -3464.1016\ \hat{i} + 2000\ \hat{j}$

$= 4000 \angle 150°$

1·b·ii) $\vec{a}_B = \ddot{\mu} r \hat{e}_\mu - \dot{\mu}^2 r \hat{e}_r$ $(\ddot{\mu} = 0)$

$= -40\,000\ \hat{e}_r$

$\hat{e}_r = \cos(2\theta - 90)\hat{i} + \sin(2\theta - 90)\hat{j}$

$\vec{a}_B = -40\,000 \angle 60°$

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1·b iii) $\vec{a}_B = \ddot{r}\hat{e}_r + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r + 2\dot{r}\dot{\theta}\hat{e}_\theta = -40\,000 \angle 60^\circ$

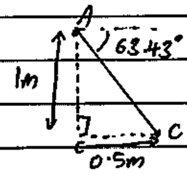
Along $\hat{e}_r$: $\ddot{r} - r\dot{\theta}^2 = 40\,000\cos 15$

$r = 800\sin\theta$.

$\therefore \ddot{r} = 40\,000\cos 15 + (800\sin 75)\times 5^2$

$= 57955.55\ m/s^2$ [pointing towards A].

2·i) $\vec{V}_A = \vec{V}_C + \vec{V}_{A/C} = \vec{V}_{A/C} = \vec{\omega}\times\vec{r}_{A/C} = \omega r\hat{e}_t$




$\hat{e}_t = \cos(-153.43^\circ)\hat{i} + \sin(-153.43^\circ)\hat{j}$

$r = \sqrt{1^2 + 0.5^2} = \sqrt{1.25}$

$\vec{V}_A = \sqrt{1.25}\ \angle -153.43^\circ$

$= -\hat{i} - 0.5\hat{j}$

2·ii) $\vec{a}_A = \vec{a}_G + \vec{a}_{A/G} = \vec{a}_G + \vec{\alpha}\times\vec{r}_{A/G} - \omega^2\vec{r}_{A/G}$

$\alpha = 0,\ \vec{a}_G = 0.5\alpha = 0.$

$\vec{a}_A = -\vec{r}_{A/G} = +0.5\hat{i} + 0.5\hat{j}$

2·iii) $\vec{V}_A = \vec{V}_{A'} + \vec{V}_{A/E}$

$= \dot{r}\hat{e}_r + \vec{\omega}_{OE}\times\vec{r}_{A/O}$

$= \dot{r}\hat{e}_r + \omega_{OE}r_{AO}\hat{e}_\theta$

$\hat{e}_r = \cos 45\hat{i} + \sin 45\hat{j}$, $\hat{e}_\theta = \cos 135\hat{i} + \sin 135\hat{j}$, $r_{AO} = \sqrt{0.5^2 + 0.5^2} = \frac{\sqrt{2}}{2}$

$\vec{V}_A = \dot{r}\frac{\sqrt{2}}{2}\hat{i} + \dot{r}\frac{\sqrt{2}}{2}\hat{j} + \frac{\sqrt{2}}{2}\omega_{OE}\left(-\frac{\sqrt{2}}{2}\right)\hat{i} + \frac{\sqrt{2}}{2}\omega_{OE}\left(\frac{\sqrt{2}}{2}\right)\hat{j}$

$= \left(\frac{\sqrt{2}}{2}\dot{r} - \frac{1}{2}\omega_{OE}\right)\hat{i} + \left(\frac{\sqrt{2}}{2}\dot{r} + \frac{1}{2}\omega_{OE}\right)\hat{j}$

$= -\hat{i} - 0.5\hat{j}$

$\therefore \frac{\sqrt{2}}{2}\dot{r} - \frac{1}{2}\omega_{OE} = -1 \Rightarrow \frac{1}{2}\omega_{OE} = \frac{\sqrt{2}}{2}\dot{r} + 1$ — ①

$\frac{\sqrt{2}}{2}\dot{r} + \frac{1}{2}\omega_{OE} = -0.5$ — ②

sub. ① into ②, $\frac{\sqrt{2}}{2}\dot{r} + \frac{\sqrt{2}}{2}\dot{r} + 1 = -0.5$

$\sqrt{2}\dot{r} = -1.5$

$\dot{r} = -1.06066$

$\therefore \omega_{OE} = 0.5$

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2.iv) $\vec{a}_A = \vec{a}_{A'} + \vec{a}_{A/f} + \vec{a}_A^c$

$= \ddot{r}\hat{e}_r + \vec{\alpha}_f \times \vec{r}_{A/o} - \omega_f^2 r_{A/o}\hat{e}_r + 2\vec{\omega}_f \times \vec{v}_{A/f}$

$= \ddot{r}\hat{e}_r + \alpha_f r_{Ao}\hat{e}_\theta - \omega_f^2 r_{Ao}\hat{e}_r + 2\omega_f \dot{r}\hat{e}_\theta$

$\hat{e}_r = \cos 45\hat{i} + \sin 45\hat{j}$ , $\hat{e}_\theta = \cos 135\hat{i} + \sin 135\hat{j}$ $r_{A0} = \sqrt{2}/2$ $\omega_f = 0.5$

$\therefore \vec{a}_A = \frac{\sqrt{2}}{2}\ddot{r}\hat{i} + \frac{\sqrt{2}}{2}\ddot{r}\hat{j} + \frac{\sqrt{2}}{2}\alpha_f(-\frac{\sqrt{2}}{2})\hat{i} + \frac{\sqrt{2}}{2}\alpha_f(\frac{\sqrt{2}}{2})\hat{j} - 0.25(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2})\hat{i} - 0.25(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2})\hat{j}$

$+ (-1.06066)(-\frac{\sqrt{2}}{2})\hat{i} + (-1.06066)(\frac{\sqrt{2}}{2})\hat{j}$

$= (\frac{\sqrt{2}}{2}\ddot{r} - \frac{1}{2}\alpha_f - 0.125 + 0.75)\hat{i} + (\frac{\sqrt{2}}{2}\ddot{r} + \frac{1}{2}\alpha_f - 0.125 - 0.75)\hat{j}$

$= +0.5\hat{i} - 0.5\hat{j}$

$\frac{\sqrt{2}}{2}\ddot{r} - \frac{1}{2}\alpha_f + 0.625 = 0.5$ —①

$\frac{\sqrt{2}}{2}\ddot{r} + \frac{1}{2}\alpha_f - 0.875 = -0.5$ —②

② − ①, $\alpha_f - 1.5 = -1$

$\alpha_f = 0.5$

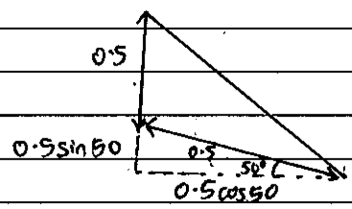
$\ddot{r} = 0.176777$

3.a.i) Height dropped by weight $= 0.5\sin 50 = 0.383022$ m

Initial length of spring $= \sqrt{0.5^2 + 0.5^2} = \sqrt{0.5}$

Final length of spring $= \sqrt{(0.5 + 0.5\sin 50)^2 + (0.5\cos 50)^2}$

$= 0.9396926$




By conservation of energy,

$\frac{1}{2}k(0.9396926 - \sqrt{0.5})^2 = 1(9.81)(0.383022)$

$k = 138.917$ N/m

3.a.ii) Final length $= \sqrt{(0.5\sin 25 + 0.5)^2 + (0.5\cos 25)^2}$

$= 0.8433914$

$1(9.81)(0.5\sin 25) = \frac{1}{2}(1)v^2 + \frac{1}{2}(138.917)(0.8433914 - \sqrt{0.5})^2$

$V = 1.25128$ m/s

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3.b) When $t=3s$, $F=27N$.

$F_f = \mu_s F_N$, when not moving. $F_f = \mu_k F_N$ when moving.

$F_N = 10(9.81) = 98.1N$

$F_f = 98.1(0.6) = 58.86N$

At $t=3s$, $F < F_f$, hence velocity of block $=0$.

Block starts moving when $F = 58.86N$.

$\therefore t = 4.429447s$.

$F = ma = m\frac{dv}{dt}$

$F\,dt = m\,dv$

$[3t^2 - 0.4(98.1)]\,dt = m\,dv$

$\int_{4.429447}^{10} [3t^2 - 0.4(98.1)]\,dt = \int_0^v m\,dv$

$[t^3 - 39.24t]_{4.429447}^{10} = [mv]_0^v$

$694.5057 = mv$

Velocity at $t=10s$ $= 69.45 m/s$.

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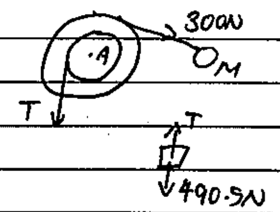
4.a) $T - 490.5 = 50a$ — (1)




$T(0.1) - 300(0.15) = I_A\alpha$

$0.1T - 45 = I_A\alpha$ — (2)

$I_A = 25(0.125)^2$

$-a = \alpha(0.1)$ — (3)

Sub (3) into (1), $T - 490.5 = -5\alpha \Rightarrow T = 490.5 - 5\alpha$ — (4)

Sub. (4) into (2), $49.05 - 0.5\alpha - 45 = 0.390625\alpha$

$\alpha = 4.547368$

$\therefore a = -0.4547368$

After travelling 2m, $v^2 = 2(0.4547368)(2)$

$v = 1.34868$ m/s (downwards)

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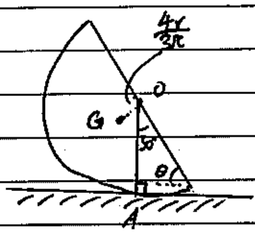



4.b) $GA^2 = \left(\frac{4r}{3\pi}\right)^2 + 0.4^2 - 2\left(\frac{4r}{3\pi}\right)(0.4)\cos 60$

$GA = 0.347727$

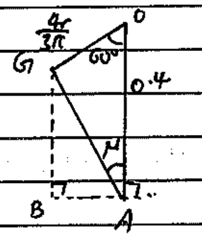
$\frac{\sin \mu}{4r/3\pi} = \frac{\sin 60}{0.347727}$

$\mu = 25.0119°$

$GB = 0.347727 \sin 64.9881$

$= 0.315117$

$BA = 0.347727 \cos 64.9881 = 0.14702$




$I_A = I_o + m r_{GA}^2$

$= \frac{3}{2} m r^2$

$= 2.4$

$10(9.81)(0.14702) = I_A \alpha$

$\alpha = 6.0094425$

$a_0 = 0.4\alpha = 2.403777$

$\vec{a}_G = \vec{a}_o + \vec{a}_{G/o}$

$= -2.403777\hat{i} + \alpha(0.169765)\hat{e}_t - \omega^2(0.169765)\hat{e}_r$

$\hat{e}_t = \angle -60°, \quad \hat{e}_r = \angle -150°$

$\vec{a}_G = -2.403777\hat{i} + 0.5100965\hat{i} - 0.883513\hat{j} + 2.35233\hat{i} + 1.35812\hat{j}$

$= 0.4586565\hat{i} + 0.474607\hat{j}$

In $\hat{j}$ direction: $F_N - 10(9.81) = 10(0.474607)$

$F_N = 102.846N$

In $\hat{i}$ direction: $F_t = 10(0.4586505)$

$= 4.586565N$

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19/20 Sem. 1



**Q1 (25 marks)**

/25

(a) A particle travels along a parabolic path $y = bx^2$. If its velocity along the $y$ axis is $v_y = ct^2$, $t$ is time, determine the particle's acceleration along the $x$ and $y$ axis, respectively. Here $b$ and $c$ are constants.

(10 marks)

***Solution:***





**Q1**

(b) If the truck travels at a constant speed of $v_T = 6$ m/s, determine the speed of the crate for any angle $\theta$ of the rope. The rope has a length of 100 m and passes over a pulley of negligible size at A as shown in Figure Q1(b).

(15 marks)

***Solution:***

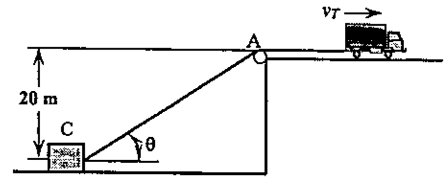


Figure Q1(b)



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**Q2 (25 marks)**

/25

(a) As shown in Figure Q2(a), the point B crosses the horizontal axis through point O. The velocity of B is $v_B = 6$ [m/s] downward. Determine $\vec{\omega}_{BA}$ and $\vec{\omega}_{AO}$, the angular velocities of the rods BA and AO, respectively.

(15 marks)

***Solution:***

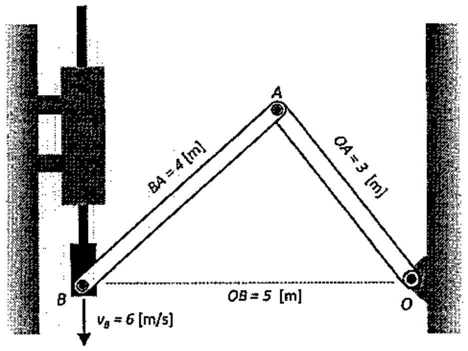


Figure Q2(a)



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**Q2**

(b) The absolute speed of point A on the disk is 12 [m/s] in the position shown in Figure Q2(b). Determine $v_o$, the absolute speed of O, and the angular velocity, ω, of the disk knowing that the direction of $v_o$ is rightward and the disk rolls without slipping.

(10 marks)

***Solution:***

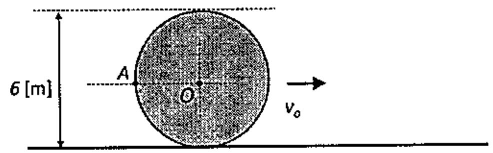


Figure Q2(b)



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**Q3 (25 marks)**

/25

(a) As shown in Figure Q3(a), the sphere A of mass 10 [kg] is suspended via the rod of length 1 [m] from the slider B of mass 2 [kg]. Knowing that the system is released from rest at $\theta = 0°$, determine $v_B$, the velocity of the slider B when $\theta = 90°$. Friction and the mass of the rod are negligible.

(13 marks)

***Solution:***

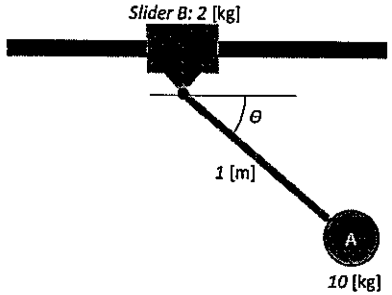


Figure Q3(a)



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**Q3**

(b) As shown in Figure Q3(b), the spring of modulus $k = 200$ [N/m] is compressed a distance 1 [m] and suddenly released from rest. Determine the absolute speeds of both masses of A and B at the instant the spring becomes uncompressed.

(12 marks)

***Solution:***

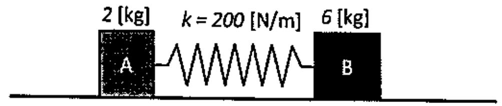


Figure Q3(b)



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**Q4 (25 marks)** /25

(a) The spool and the wire wrapped around its core have a mass of 50 kg and a radius of gyration of $k_G = 235$ mm. If the coefficient of kinetic friction at the surface is $\mu_k = 0.15$, determine the angular acceleration of the spool after it is released from rest (the rope is cut by a scissor).

(10 marks)

***Solution:***

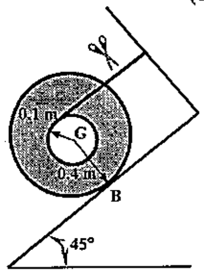


Figure Q4(a)



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**Q4**

(b) The 15-kg disk is pinned at O and is initially at rest. If a 10-g bullet is fired into the disk with a speed of 200 m/s, as shown in Figure Q4(b), determine the maximum angle $\theta$ to which the disk swings. The bullet is embedded in the disk.

(15 marks)

***Solution:***

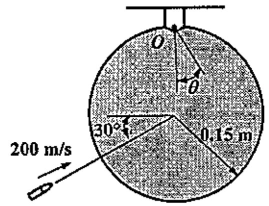


Figure Q4(b)



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MA1007

19/20 S1



Q1 Q1

~~a) $y = bx^2 \rightarrow \dot{y} = 2bx(\dot{x})$~~

a) $y = bx^2$ $\quad \dot{y} = 2bx(\dot{x})$ $\quad \dot{y} = V_y = ct^2$ $\quad \dot{y} = \frac{ct^3}{3}$

$\hookrightarrow \dot{x} = \frac{ct^2}{2bx}$ $\quad \ddot{x} = \frac{2ct}{2bx} + (ct^2)(-1)(2bx)^{-2}(2b)$

$= \frac{ct}{bx} - \frac{ct^2}{2bx^2}$

$\ddot{y}$ = acceleration along y direction

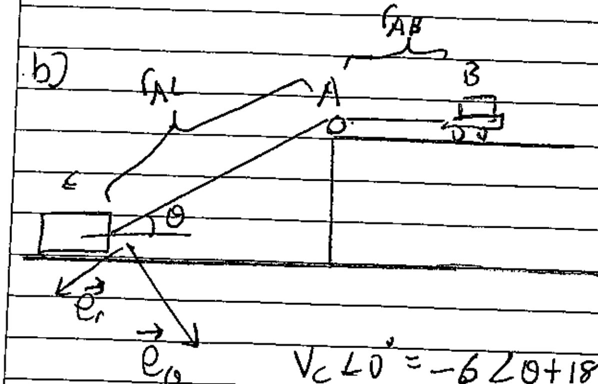
$= \frac{ct^3}{3}$ ※

$\ddot{x}$ = acceleration along x direction

$= \frac{c}{2b}\left(\frac{2t}{x} - \frac{t^2}{x^2}\right)$ ※

* ~~could not represent $\ddot{x}$ interms of t because~~ initial position of x or X when t=0 is unknown

b)




$r_{AB} + r_{AL} = L \quad \dot{r}_{AB} = V_T = 6\,ms^{-1}$

$\dot{r}_{AB} + \dot{r}_{AL} = 0 \rightarrow \dot{r}_{AL} = -6\,ms^{-1}$

$\vec{V}_L = \dot{r}_{AL}\hat{e}_r + (\dot{\omega})(r_{AL})\hat{e}_\theta$

$\hat{e}_r = \angle\theta + 180^\circ \quad \hat{e}_\theta = \angle\theta + 270^\circ$

$V_c \angle 0^\circ = -6\angle\theta + 180^\circ + (\omega)(r_{AL})\angle\theta + 270^\circ$

$\cdot \angle\theta + 180^\circ \quad \cdot \angle\theta + 180^\circ \quad \cdot \angle\theta + 180^\circ \rightarrow$ ~~dot product with~~ $\angle\theta + 180^\circ$

$V_c\cos(\theta + 180^\circ) = -6 + 0 \rightarrow V_L = \frac{-6}{\cos(\theta + 180^\circ)} = \frac{6}{\cos\theta}$ ※

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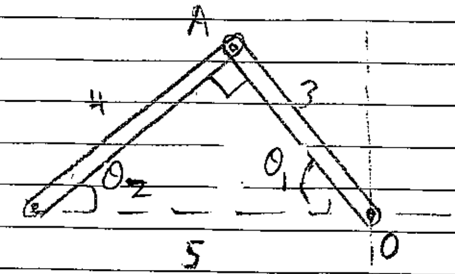
Q2

a) $\vec{V}_A = \vec{V}_{A/B} + \vec{V}_B \qquad \vec{V}_B = 6\angle{-90^\circ}$




$\theta_1 = \tan^{-1}\left(\frac{4}{3}\right) = 53.1301^\circ$

$\theta_2 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$

$\vec{V}_A = W_{AO}(r_{AO}) \angle 270^\circ - \theta_1 = 3W_{AO} \angle 216.87^\circ$

$\vec{V}_{A/B} = W_{AB}(r_{AB}) \angle \theta_2 + 90^\circ = 4W_{AB} \angle 126.87^\circ$

} assume $W_{AO}$ & $W_{AB}$ both CCW

$\vec{V}_A = \vec{V}_{A/B} + \vec{V}_B \rightarrow 3W_{AO}\angle 216.87^\circ = 4W_{AB}\angle 126.87^\circ + 6\angle{-90^\circ}$

dot product with $\angle 216.87^\circ$

$3W_{AO} = 0 + 6\cos(306.87) \rightarrow$

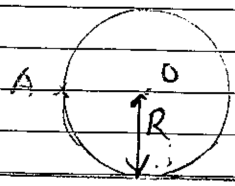
$W_{AO} = 1.2\ \text{rads}^{-1} \longrightarrow \vec{W}_{AO} = 1.2\hat{k}$ #

dot product with $\angle 306.87^\circ$

$0 = 4W_{AB}(-1) + 6\cos(396.87) \rightarrow$

$W_{AB} = 1.2\ \text{rads}^{-1} \longrightarrow \vec{W}_{AB} = 1.2\hat{k}$ #

b)




$\vec{V}_A = \vec{V}_{A/O} + \vec{V}_O \qquad \vec{V}_A = 12\angle\theta$

$\vec{V}_{A/O} = (W)(r_{AO})\angle 90^\circ = 3W\angle 90^\circ \rightarrow$ ~~assume~~ W clockwise

$\vec{V}_O = (R)(W)\angle 0^\circ = 3W\angle 0^\circ$

$\vec{V}_A = \vec{V}_{A/O} + \vec{V}_O \rightarrow 12\angle\theta = 3W\angle 90^\circ + 3W\angle 0^\circ = \sqrt{18}W\angle 45^\circ$

$12\angle\theta = \sqrt{18}W\angle 45^\circ \rightarrow \theta = 45^\circ,\ W = 2.8284\ \text{rads}^{-1}$

$\vec{V}_O = 8.4853\angle 0^\circ$ # $\vec{W} = -2.828\hat{k}$ #

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Q3

a) conservation of energy → $\Delta KE + \Delta GPE = 0$

$$\frac{1}{2}(m_A)(V_A^2) + \frac{1}{2}(m_B)(V_B^2) + (0 - m_A g(h)) = 0 \qquad h = 1$$

$$\frac{1}{2}(10)(V_A^2) + \frac{1}{2}(2)(V_B^2) = (10)(9.81)(1) \rightarrow 98.1 = 5V_A^2 + V_B^2 \quad \text{—(1)}$$

conservation of horizontal momentum: $\sum \vec{H}_1^h = \sum \vec{H}_2^h$

$$0 = m_A V_A \angle 180° + m_B V_B \angle 0° \rightarrow m_A V_A = m_B V_B \rightarrow 5V_A = V_B \quad \text{—(2)}$$

from (1) & (2) → $98.1 = 5V_A^2 + (5V_A)^2 \rightarrow V_A = 1.8083\ ms^{-1}$

$V_B = 9.0416\ ms^{-1} \rightarrow \vec{V}_B = 9.0416 \angle 0°$ #

b) assume no energy lost: → $\Delta KE + \Delta EPE = 0$

$$\frac{1}{2}(m_A)V_A^2 + \frac{1}{2}m_B V_B^2 + \left[0 - \frac{1}{2}(k)(x^2)\right] = 0 \qquad x = 1$$

$$\frac{1}{2}(2)V_A^2 + \frac{1}{2}(6)(V_B^2) - \frac{1}{2}(200)(1) = 0 \rightarrow 100 = V_A^2 + 3V_B^2 \quad \text{—(1)}$$

conservation of momentum: $\sum \vec{H}_1 = \sum \vec{H}_2$

$$0 = m_B V_B \angle 0° + m_A V_A \angle 180° \rightarrow m_B V_B = m_A V_A \rightarrow 3V_B = V_A \quad \text{—(2)}$$

from (1) & (2) → $100 = V_A^2 + 3\left(\frac{V_A}{3}\right)^2 \rightarrow V_A = 75\ ms^{-1}$ #

$V_B = 25\ ms^{-1}$ #

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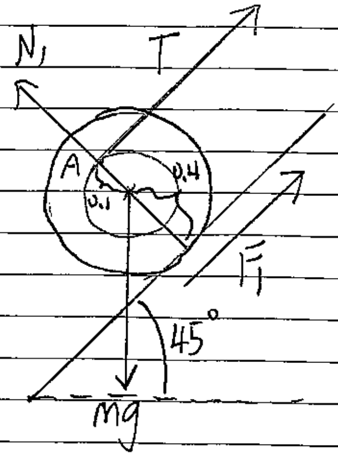
Q4

a)




at static condition:

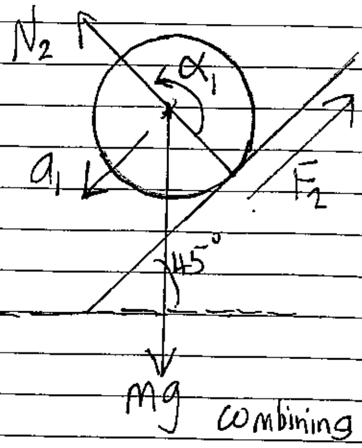
apply summation of moment about A:

$F_1(0.4+0.1) = mg(0.1\sin 45)$

$F_1 = 69.367N$

$\mu_{static\,1} = \frac{F_1}{mg\cos 45} = 0.2$

assume rolling without slipping:




$mg\sin 45 - F_2 = ma$

$346.836 - F_2 = 50a_1$ —①

$a_1 = R\alpha_1$

$a_1 = 0.4\alpha_1$ —②

$F_2(R) = I_G\alpha_1$

$0.4F_2 = (50)(0.235)^2(\alpha_1)$

$\alpha_1 = 0.14486 F_2$ —③

combining ①, ② & ③ → $346.836 - F_2 = 50(0.4)(0.14486F_2)$

$F_2 = 88.99572N$

$\mu_{static\,2} = \frac{F_2}{mg\cos 45} = 0.2566 > \mu_{static\,1} = 0.2$

If we assume static condition is limiting case → static friction not large enough to create rolling without slipping

Hence → rolling with slipping

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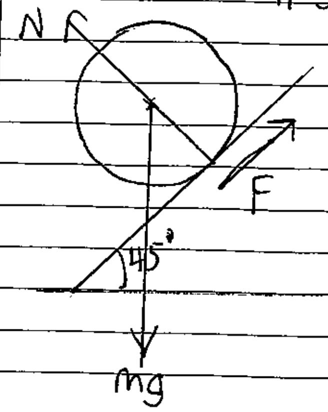
for rolling with slipping




$F = \mu_k N = 0.15(50)(9.81)\cos 45°$

$F = 52.0254 N$

$I_G \alpha = F(R)$

$\alpha = \frac{(52.0254)(0.4)}{(50)(0.235^2)} = 7.5365\ rads^{-2}$ ✱

clockwise

✱ Please take note on the assumption on limiting case as to be honest there is no evidence which suggest so. Please consult professor on question.

$\vec{L}_1^{bullet} = (V_{bullet})(L_1) m_{bullet} \hat{k}$

$= (200)(0.15\cos 30°) m_{bullet} \hat{k}$

$= 0.2598 \hat{k}$

$\vec{L}_1^{disk} = 0$

$\vec{L}_2^{bullet} = (L_2)^2 (W)(m_{bullet}) \hat{k}$

$= (2)(0.15)\cos(30°)(0.010) W \hat{k}$

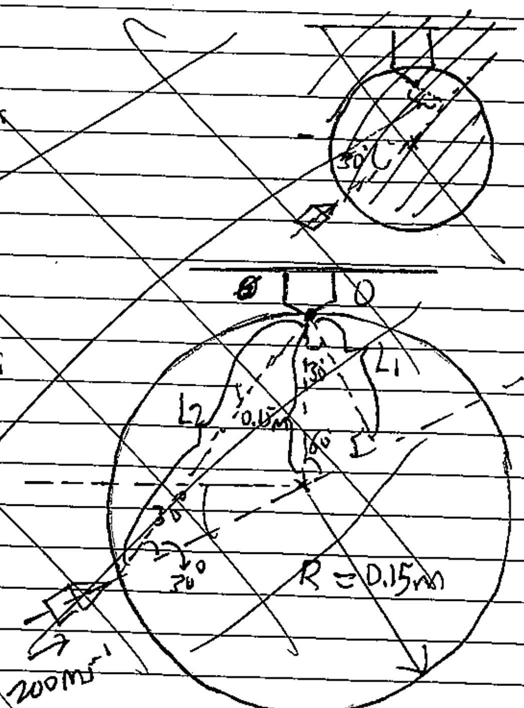
$= 0.0025981 W \hat{k}$

$\vec{L}_2^{disk} = I_0^{disk} (W) \hat{k}$

$= (\frac{1}{2} m_{disk} R^2 + m_{disk} R^2)(W) \hat{k}$

$= (\frac{1}{2}(15)(0.15^2) + 15(0.15^2))(W) \hat{k}$

$= 0.50625 W \hat{k}$

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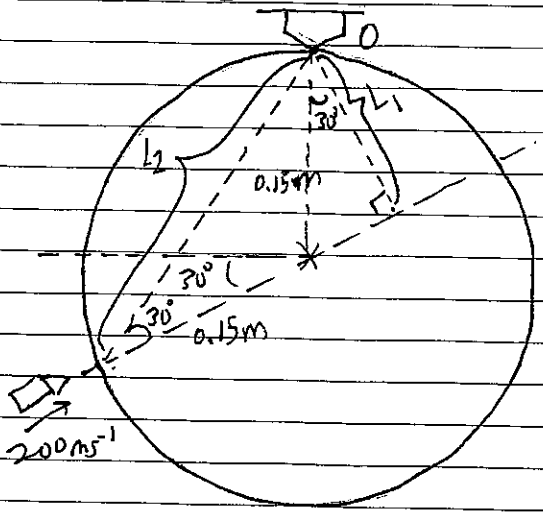
4) b) applying conservation on angular momentum about O before & after collision

$\sum \vec{L_1} = \sum \vec{L_2} \rightarrow \vec{L}_1^{bullet} + \vec{L}_1^{disk} = \vec{L}_2^{bullet} + \vec{L}_2^{disk}$ —(1)




$\vec{L}_1^{bullet} = (m_{bullet})(V_{bullet})(L_1)\hat{k}$

$= (200)(0.01)(0.15\cos 30°)$
$= 0.2598\hat{k}$

$\vec{L}_1^{disk} = 0$

$\vec{L}_2^{bullet} = m_{bullet}(L_2)^2 w \hat{k}$
$= (0.01)(2\times 0.15\cos 30)^2 (w)\hat{k}$
$= 0.000675 w \hat{k}$

where W = angular velocity about O of hybrid object after collision

$\vec{L}_2^{disk} = I_0^{disk} W \hat{k}$

$= (\frac{1}{2} M_{disk}(R^2) + M_{disk}(R^2))(w)\hat{k}$

$= [\frac{1}{2}(15)(0.15)^2 + 15(0.15^2)] w\hat{k} = 0.50625 w\hat{k}$

from (1) → $0.2598\hat{k} + 0 = 0.000675w + 0.50625w$

$w = 0.5125 \text{ rads}^{-1}$ → angular momentum after collision

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applying conservation of energy:

$\Delta KE + \Delta GPE = 0$

$\Delta KE = 0 - \left[\frac{1}{2} I_o^{total} \omega^2\right]$

$I_o^{total} = \frac{1}{2} m_{disk} R^2 + m_{disk} R^2 + m_{bullet} L_2^2$

$= \frac{1}{2}(15)(0.15^2) + 15(0.15^2) + (0.01)(2 \times 0.15 \cos 30)^2$

$= 0.506925$

$= -\frac{1}{2}(0.506925)(0.5125^2)$

$= -0.06657\ J$

$\Delta GPE = (m_{disk} + m_{bullet})(g)(R)(1 - \cos\theta)$
$= (15.01)(9.81)(0.15)(1 - \cos\theta)$
$= 22.0872(1 - \cos\theta)$

} ignore shift in centre of gravity cause by bullet

$\Delta KE + \Delta GPE = 0 \rightarrow 22.0872(1 - \cos\theta) = 0.06657$

$\theta = 4.45°$ #

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