NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

MA3700 – AIRCRAFT STRUCTURES I

November/December 2015 | Time allowed : 2½ hours

INSTRUCTIONS

1. This paper contains **THREE**(3) questions and comprises **FOUR**(4) pages.
2. Answer **ALL** questions.
3. Marks for each question are as indicated.
3. This is a **CLOSED BOOK** examination.
4. Useful equations are found in Appendix A.

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1 (a) (i) Why are the structures around cutouts in an aircraft fuselage required to be strengthened by stiffeners? Explain concisely.

(2 marks)

(ii) How can the spars in an aircraft wing be designed to resist bending loads effectively? What are the functions of the ribs in the wing? Answer these questions concisely.

(5 marks)

(iii) An aircraft has a total weight $W = 1{,}500{,}000$N and a wing area $S = 280$ m^2. The slope of the lift coefficient curve is $dC_L/d\alpha = 1.5$/rad. During a steady level flight at the speed $V = 200$ m/s at an altitude where the air density is $\rho = 0.526$ kg/m^3, the aircraft meets a downward gust of equivalent "sharp-edged" speed 8 m/s. What are the lift and the load factor?

(6 marks)

(b) An aircraft has a total weight $W = 160{,}000$N, a wing area $S = 50$m^2, and a mean chord $c = 3$ m. The drag coefficient is related to the lift coefficient by $C_D = 0.02+0.04C_L^2$. The pitch moment coefficient about aerodynamic centre (AC) is $C_{M,AC} = -0.035$ (pitch up positive). In a steady level flight of $V = 200$ m/s at sea level, the relative positions of the centre of gravity (CG), aerodynamic centre of the whole aircraft less tailplane, and the tailplane centre of pressure (CP) are illustrated in Figure 1. Assume that the air density at sea level is $\rho = 1.223$ kg/m^3. Draw the free-body diagram, and calculate the lift, drag and tail load during the steady level flight.

(10 marks)

Note: Question 1 continues on page 2.
Figure 1 appears on page 2.

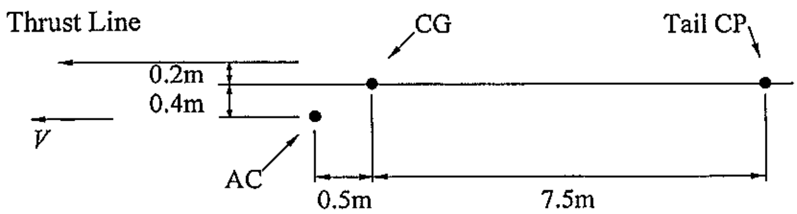

Figure 1

(c) An aircraft has a total weight $W = 60{,}000$ N and a wing area $S = 18\ \text{m}^2$. The maximum lift coefficient is $C_{Lmax} = 1.40$. The maximum manoeuver load factor is 7.0. Assume that the air density is $\rho = 1.223\ \text{kg/m}^3$ for the following two questions :

(i) If the aircraft flies in a correctly banked turn at a constant speed of 140 m/s, what are the maximum angle of bank and the corresponding radius of turn?

(5 marks)

(ii) If the aircraft begins a pull-out from a dive inclined at 60° to the horizontal ground with a constant diving speed of 200 m/s and a constant radius of the flight path, what is the minimum radius of the flight path?

(5 marks)

2. Figure 2 shows an idealized cantilever beam structure carrying a 50 kN load at the free end. The beam is made of aluminium alloy (E=70 GPa, G=28 GPa) and consists of multi-cell section shown. The boom areas $B_1=B_3=B_4=B_6=100\ \text{mm}^2$ and $B_2=B_5=200\ \text{mm}^2$ and the thickness of all walls is 2 mm. The usual assumptions for idealized structures apply here.

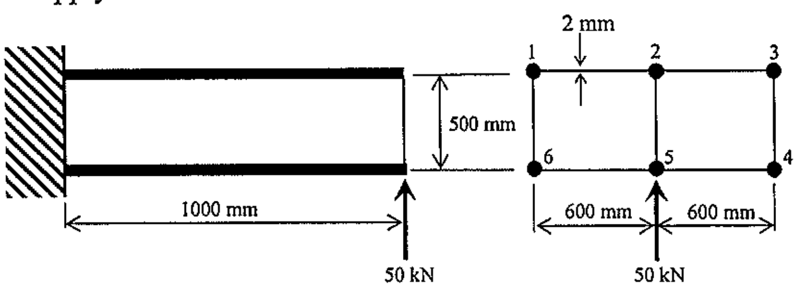

Figure 2

(a) Determine the distribution of the shear flow in the walls of the section.

(14 marks)

(b) Determine the bending deflection at the free end of the beam.

(6 marks)

(c) Determine the shear deflection at the free end of the beam.

(10 marks)

(d) Discuss the parameters that contribute significantly towards shear deflection.

(4 marks)

3 (a) Aircraft structural components are mainly thin skins stiffened with long and slender stiffeners. These structural components are susceptible to failure by buckling. Describe the phenomenon of buckling with respect to aircraft structural design. You may use simple illustrations in your description.

(10 marks)

(b) As an aircraft engineer, you are asked to determine the buckling load of a column shown in Figure 3. This column has a length L and is of uniform EI with one end clamped and the other simply supported.

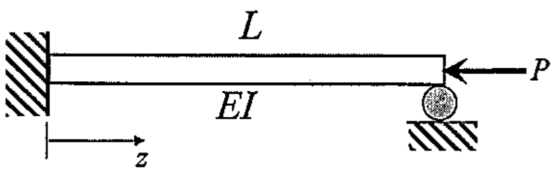

Figure 3

To satisfy these boundary conditions, a deflection form for the column was postulated as :

$$v(z) = A\left(\sin\frac{\pi z}{L} + C\sin\frac{2\pi z}{L}\right)$$

where A and C are arbitrary coefficients.

(i) Check for the admissibility of this function.

(4 marks)

(ii) Determine constant C. (Hint: Consider the clamped end boundary condition).

(4 marks)

(iii) Using the energy approach, derive the expression for the buckling load of this column in the form $P_{CR} = K\frac{\pi^2}{L^2}EI$, where K is the buckling coefficient to be determined.

(12 marks)

(iv) The exact value of K is 2.05 for this column, compare the value K you obtained in part (iii) with the exact value and comment on the difference if any.

(3 marks)

APPENDIX A

(1) Equations of Lift, Drag and Pitching Moment :

$$L = \frac{1}{2}\rho V_\infty^2 S C_L \qquad D = \frac{1}{2}\rho V_\infty^2 S C_D \qquad M = \frac{1}{2}\rho V_\infty^2 S \bar{c} C_{MAC}$$

(2) Bredt-Batho equation for a closed section :

$$T = \oint q\, r ds = 2A_0 q$$

(3) The stress at any point in the cross section due to bending is :

$$\sigma_z = M_x\left(\bar{I}_{yy}y - \bar{I}_{xy}x\right) + M_y\left(\bar{I}_{xx}x - \bar{I}_{xy}y\right)$$

$$\text{where } \bar{I}_{xx} = \frac{I_{xx}}{I_{xx}I_{yy} - I_{xy}^2}; \quad \bar{I}_{yy} = \frac{I_{yy}}{I_{xx}I_{yy} - I_{xy}^2}; \quad \bar{I}_{xy} = \frac{I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}$$

(4) The shear flow in the thin-walled cross section due to applied shear forces :

$$q_s = -\left(S_y\bar{I}_{yy} - S_x\bar{I}_{xy}\right)\sum_{r=1}^{n} B_r y_r - \left(S_x\bar{I}_{xx} - S_y\bar{I}_{xy}\right)\sum_{r=1}^{n} B_r x_r + q_0$$

(5) Rate of twist of the Rth cell in a multi-cell section in shear:

$$\frac{d\theta}{dz} = \frac{1}{2A_R G}\left[-q_{0,R-1}\delta_{R-1,R} + q_{0R}\delta_R - q_{0,R+1}\delta_{R+1,R} + \oint_R q_b \frac{ds}{t}\right]$$

(6) Rate of twist of a closed section : $\phi = \frac{d\theta}{dz} = \frac{1}{2A_0}\oint \frac{q}{Gt} ds$

(7) Equation for combined deflection : $\Delta_{total} = \int_0^L \left\langle \frac{Mm}{EI} \right\rangle dz + \int_0^L \left(\int_s \frac{Q_s q_s}{Gt} ds \right) dz$

(8) Elastic strain energy and total potential energy for a beam :

$$U = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v}{\partial z^2}\right)^2 dz \qquad W = \frac{1}{2}\int_0^L \left\{2qv + P\left(\frac{\partial v}{\partial z}\right)^2\right\} dz$$

(9) Integration of Orthogonal functions :

$$\int_0^a \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{p\pi x}{a}\right)dx = \begin{cases} 0 & m \neq p \\ a/2 & m = p \end{cases}$$

$$\int_0^b \cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{q\pi y}{b}\right)dy = \begin{cases} 0 & n \neq q \\ b/2 & n = q \end{cases}$$

End of Paper

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MA3700

1) (a) (i) Cutouts impose high stress concentration → Susceptible to crack and failure → needs to be strengthened by stiffeners.

(ii) Spars can be designed such that:
- it maximises second moment of inertia.
- it runs from tip to root of wing.
- multiple spars

Functions of ribs:
- provide aerofoil shape to wing and support the skin.
- compartmental walls for fuel tanks.
- stiffen wing structure against torsion.

(iii) $\Delta L = \frac{dL}{d\alpha}\Delta\alpha$

$= \frac{d}{d\alpha}\left(\frac{1}{2}\rho_\infty V_\infty^2 S C_L\right)\Delta\alpha$

$= \frac{1}{2}\rho_\infty V_\infty^2 S C_{L\alpha}\frac{u}{V_\infty}$ — downgust is negative

$= \frac{1}{2}\rho_\infty V_\infty S C_{L\alpha} u$

$= 0.5(0.526)(200)(280)(1.5)(-8)$

$= -176736\ N$ (decrease in lift).

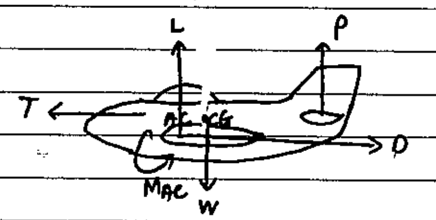
Resultant lift $= 1500000 - \Delta L$

$= 1323264\ N$

$n = \frac{L}{W} = 1323264/1500000$

$= 0.882176$

(b)



$M_0 = \frac{1}{2}\rho_\infty V_\infty^2 S\bar{c}\, C_{M,ac}$

$= 0.5(1.223)(200)^2(50)(3)(-0.035)$

$= -128415\ Nm$

$\Sigma F = 0$

$L + P - 160000 = 0$ —— ①

Since $P \ll L$: $L \simeq 160000\ N$

$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$

$160000 = 0.5(1.223)(200)^2(50)C_L$

$C_L = 100/1223$

$C_D = 0.02 + 0.04\, C_L^2$

$= 0.02 + 0.04\,(100/1223)^2$

$= 0.020684616$

$D = \frac{1}{2}\rho_\infty V_\infty^2 S C_D$

$= 0.5(1.223)(200)^2(50)(0.020684616)$

$= 25297.28537\ N$

$\Sigma M_{cg} = 0$

$0.5L - 7.5P - 0.6D - 128415 = 0$

$0.5L - 7.5P - 0.6(25297.28537) - 128415 = 0$

$0.5L - 7.5P - 143593.3712 = 0$ —— ②

Solving simultaneously for ① & ②:

~~P = 127186.7424 N~~

~~L = 287186.7424 N~~

~~P = 9084.767314 N~~

~~L = 169084.7673 N~~

$P = -7949.1714\ N$

$L = 167949.1714\ N$

c) (i) $n = \left[1+\left(\frac{v^2}{gR}\right)^2\right]^{\frac{1}{2}} = 7$

$7^2 = 1 + \left(\frac{140^2}{9.81R}\right)^2$

$R = 288.38\ m$

(ii) $n = \cos\phi + \frac{v^2}{gR}$

$7 = \cos 60° + \frac{200^2}{9.81R}$

$R = 627.3 m$

$\tan\phi = \frac{v^2}{gR}$

$= 140^2 / 9.81(288.38)$

$\phi = 81.8°$

2) (a) $q_b = -\left(\frac{S_y I_{yy} - \cancel{S_x I_{xy}}^{=0}}{I_{xx}I_{yy} - \cancel{I_{xy}^2}^{=0}}\right)\sum B_r y_r - \left(\frac{\cancel{S_x I_{xx} - S_x I_{xy}}^{=0}}{I_{xx}I_{yy} - \cancel{I_{xy}^2}^{=0}}\right)\sum B_r x_r + q_0$

Symmetric : $I_{xy} = 0$; $S_x = 0$; $I_{xx} = \sum B_r y_r^2$

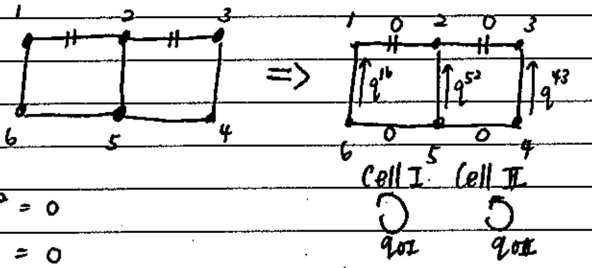
$= 4(100\times10^{-6})(0.25)^2 + 2(200\times10^{-6})(0.25)^2$

$= 0.00005\ m^4$

$\Rightarrow q_b = -\frac{S_y}{I_{xx}}\sum B_r y_r$

$= -\frac{50000}{0.00005}\sum B_r y_r$

$= -1\times10^9 \sum B_r y_r$



$q^{32} = 0$

$q^{21} = 0$

$q^{25} = -1\times10^9(200\times10^{-6})(0.25) = -50000\ Nm^{-1}$

$q^{16} = -1\times10^9(100\times10^{-6})(0.25) = -25000\ Nm^{-1}$

$q^{65} = q^{16} + (-1\times10^9)(100\times10^{-6})(-0.25) = 0$

$q^{54} = q^{25} + (-1\times10^9)(200\times10^{-6})(-0.25) = 0$

$q^{43} = q^{54} + (-1\times10^9)(100\times10^{-6})(-0.25) = 25000\ Nm^{-1}$

50kN is applied at a location such that $\frac{d\theta}{dz} = 0 = \frac{1}{2A_R G}\left[-q_{0,R-1}\delta_{R-1,R} + q_{0R}\delta_R - q_{0,R+1}\delta_{R+1,R} + \oint_R \frac{q_b}{t^*}ds\right]$

Cell I: $0 = q_{0I}(2\times0.6 + 2\times0.5) - 0.5\, q_{0II} + 50000(0.5) - 25000(0.5)$

$= 2.2\, q_{0I} - 0.5\, q_{0II} + 12500$ ——— ①

Cell II: $0 = q_{0II}(2\times0.6 + 2\times0.5) - 0.5\, q_{0I} + 25000(0.5) - 50000(0.5)$

$= 2.2\, q_{0II} - 0.5\, q_{0I} - 12500$ ——— ②

Solving simultaneously ① & ② : $q_{0I} = -q_{0II}$

$q_{0I} = 4629.63\ N$

$q_{0II} = -4629.63\ N$

$\Rightarrow q^{32} = -4629.63\ N/m$

$q^{21} = 4629.63\ N/m$

$q^{16} = -25000 + 4629.63 = -20370.37\ N/m$

$q^{65} = 4629.63\ N/m$

$q^{54} = -4629.63\ N/m$

$q^{43} = 25000 - 4629.63 = 20370.37\ N/m$

$q^{25} = -(4629.63)(2) - 50000 = -59259.26\ N/m$

(b) $A_M = \int_0^L \left\langle \frac{Mm}{EI} \right\rangle dz$ $\qquad M = -50000z$, $m = -1z$

$= \int_0^1 \frac{50000z^2}{EI} dz$

$= \frac{50000}{(70\times10^9)(0.00005)} \left[\frac{1}{3}z^3\right]_0^1$

$= 4.76\ mm$

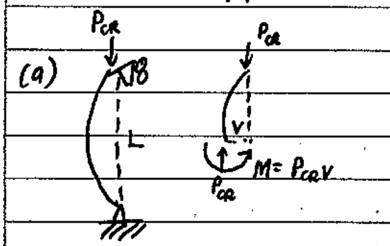
(c) $\Delta_q = \int_0^L \left(\int_s \frac{q_0 q_1}{Gt} ds \right) dz$ $\qquad Q = -\frac{50000}{0.00005} \sum B_r y_r$

$= \frac{1}{Gt} \int_0^1 \left[\int_s \frac{Q^2}{50000} ds \right] dz$ $\qquad q = -\frac{1}{0.00005} \sum B_r y_r = \frac{Q}{50000}$

$= \frac{1}{(28\times10^9)(0.002)(50000)} \int_0^1 \left[2(4629.63)^2(0.6) + 2(20370.37)^2(0.5) + 2(4629.63)^2(0.6) + (59259.26)^2(0.5) \right]$

$= 0.79\ mm$

3) (a)



$EI \frac{\partial^2 v}{\partial z^2} = -M = -P_{CR}v$

Let $\lambda^2 = \frac{P_{CR}}{EI}$

$\therefore EI \frac{\partial^2 v}{\partial z^2} = -P_{CR}v$

$\frac{\partial^2 v}{\partial z^2} + \lambda^2 v = 0$

Form of solution for the above homogeneous ODE:

$v = A \cos \lambda z + B \sin \lambda z$

Applying boundary conditions:

At $z = 0$, $v = 0 \Rightarrow A = 0$

At $z = L$, $v = 0 \Rightarrow \underbrace{B \sin \lambda L = 0}_{B = 0 \text{ or } \sin \lambda L = 0}$

$\sin \lambda L = 0$

$\lambda L = \pi, 2\pi, \ldots, m\pi$

$\lambda L = m\pi$

$\sqrt{\frac{P_{CR}}{EI}} = \frac{m\pi}{L}$

$P_{CR} = \left(\frac{m^2\pi^2}{L^2}\right) EI$

- Long and slender stiffened are susceptible to buckling, which leads to failure
- ~~Wh~~ Buckling occurs even below the yield strength of material.
- That is, when the load reaches a certain number — the critical buckling load P_{CR}, the material starts to buckle.
- The longer the stiffeners, the lower the P_{CR} and the more vulnerable it is to buckling.
- The risk of buckling can be reduced by using material of higher E and I.

b) (i) $v = A\left(\sin\frac{\pi z}{L} + C\sin\frac{2\pi z}{L}\right)$ At $z = L$; $v = 0$

$\frac{\partial v}{\partial z} = A\left(\frac{\pi}{L}\cos\frac{\pi z}{L} + C\frac{2\pi}{L}\cos\frac{2\pi z}{L}\right)$ At $z = L$; $\frac{\partial v}{\partial z} = -A\frac{\pi}{L} + C\frac{2\pi}{L}$

$\frac{\partial^2 v}{\partial z^2} = A\left(-\left(\frac{\pi}{L}\right)^2\sin\frac{\pi z}{L} - C4\left(\frac{\pi}{L}\right)^2\sin\frac{2\pi z}{L}\right)$ At $z = L$; $\frac{\partial^2 v}{\partial z^2} = 0$

Geometric boundary condition is satisfied, but natural boundary condition is not satisfied.

(ii) At $z = 0$, $\partial v/\partial z = 0$: $0 = A\left(\frac{\pi}{L} + C\frac{2\pi}{L}\right)$

$C = -\frac{1}{2}$

(iii) $\frac{\partial v}{\partial z} = A\left(\frac{\pi}{L}\cos\frac{\pi z}{L} - \frac{\pi}{L}\cos\frac{2\pi z}{L}\right)$

$\frac{\partial^2 v}{\partial z^2} = A\left(-\left(\frac{\pi}{L}\right)^2\sin\frac{\pi z}{L} + 2\left(\frac{\pi}{L}\right)^2\sin\frac{2\pi z}{L}\right)$

$\left(\frac{\partial v}{\partial z}\right)^2 = A^2\left[\left(\frac{\pi}{L}\right)^2\cos^2\frac{\pi z}{L} - 2\left(\frac{\pi}{L}\right)^2\cos\frac{\pi z}{L}\cos\frac{2\pi z}{L} + \left(\frac{\pi}{L}\right)^2\cos^2\frac{2\pi z}{L}\right]$

$\left(\frac{\partial^2 v}{\partial z^2}\right)^2 = A^2\left[\left(\frac{\pi}{L}\right)^4\sin^2\frac{\pi z}{L} - 4\left(\frac{\pi}{L}\right)^4\sin\frac{\pi z}{L}\sin\frac{2\pi z}{L} + 4\left(\frac{\pi}{L}\right)^4\sin^2\frac{2\pi z}{L}\right]$

Total potential energy, $\Pi = U - W_E = \frac{EI}{2}\int_0^L\left(\frac{\partial^2 v}{\partial z^2}\right)dz - \frac{P}{2}\int_0^L\left(\frac{\partial v}{\partial z}\right)^2 dz$

$\int_0^L\left(\frac{\partial^2 v}{\partial z^2}\right)^2 dz = A^2\int_0^L\left[\left(\frac{\pi}{L}\right)^4\sin^2\frac{\pi z}{L} - 4\left(\frac{\pi}{L}\right)^4\sin\frac{\pi z}{L}\sin\frac{2\pi z}{L} + 4\left(\frac{\pi}{L}\right)^4\sin^2\frac{2\pi z}{L}\right]$

$= A^2\left[\left(\frac{\pi}{L}\right)^4\left(\frac{L}{2}\right) + 2L\left(\frac{\pi}{L}\right)^4\right]$

$\int_0^L\left(\frac{\partial v}{\partial z}\right)^2 dz = A^2\int_0^L\left[A^2\left(\frac{\pi}{L}\right)^2\cos^2\frac{\pi z}{L} - 2\left(\frac{\pi}{L}\right)^2\cos\frac{\pi z}{L}\cos\frac{2\pi z}{L} + \left(\frac{\pi}{L}\right)^2\cos^2\frac{2\pi z}{L}\right]$

$= A^2\left[\left(\frac{\pi}{L}\right)^2\left(\frac{L}{2}\right) + \left(\frac{\pi}{L}\right)^2\left(\frac{L}{2}\right)\right]$

$\Pi = \frac{EI}{2}\left[\frac{5}{2}L\left(\frac{\pi}{L}\right)^4\right]A^2 - \frac{P}{2}\left[L\left(\frac{\pi}{L}\right)^2\right]A^2$

$\frac{d\Pi}{dA} = EI\frac{5}{2}L\left(\frac{\pi}{L}\right)^4 - PL\left(\frac{\pi}{L}\right)^2 = 0$

$\frac{5}{2}EI\left(\frac{\pi}{L}\right)^2 = P$

$P_{CR} = 2.5\left(\frac{\pi}{L}\right)^2 EI$

$K = 2.5$ #

(iv) Solution above ($K = 2.5$) is not exact because geometric boundary condition is satisfied, but natural boundary condition is not satisfied.