NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2014-2015

MA3003 – HEAT TRANSFER

November/December 2014 Time Allowed: $2^1/_2$ hours

INSTRUCTIONS

1. This paper contains **FOUR** (4) questions and comprises **FIVE** (5) pages.
2. Answer **ALL FOUR** (4) questions.
3. All questions carry equal marks.
4. This is a **CLOSED BOOK** examination.
5. Candidates may use the list of formulae provided on pages 4 and 5.

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1 (a) How does the science of heat transfer differ from the science of thermodynamics?

(3 marks)

(b) What are the mechanisms of energy transfer to a closed system? How is heat transfer distinguished from other forms of energy transfer?

(4 marks)

(c) What are the mechanisms of heat transfer? How are they distinguished from each other?

(6 marks)

(d) Consider a stainless steel spoon (k = 15.1 W/m·K) partially immersed in boiling water at 95°C in a kitchen at 25°C. The handle of the spoon has a cross section of 0.2 cm × 1.3 cm, and extends 18 cm in the air from the free surface of the water. If the heat transfer coefficient at the exposed surfaces of the spoon handle is h = 17 W/m^2·K, determine the temperature difference between the tip of the spoon handle and the part of the handle at the water level.

(12 marks)

2. Glycerin (c_p = 2400 J/kg·K) at 20°C and 0.5 kg/s is to be heated by ethylene glycol (c_p = 2500 J/kg·K) at 60°C in a thin-walled double-pipe parallel-flow heat exchanger. The temperature difference between the two fluids is 15°C at the outlet of the heat exchanger. If the overall heat transfer coefficient is U = 240 W/m^2·K and the heat transfer surface area is A = 3.2 m^2, determine

   (a) the rate of heat transfer,

   (14 marks)

   (b) the outlet temperature of the glycerine, and

   (4 marks)

   (c) the mass flow rate of the ethylene glycol.

   (7 marks)

3 (a) Hot-wire anemometry depends on the convective heat transfer from forced flow across a small diameter wire.

A hot-wire anemometer with diameter D = 0.1 mm and length L = 10 mm is used to measure the velocity of air flowing in a duct. The free stream air temperature is 20°C. What is the velocity of the air if an electric energy input of 12 W/m is required to maintain a surface temperature of 90°C? The properties of the air are: Pr = 0.701, k = 0.0282 W/m·K and ν = 1.807 × 10^{-5} m^2/s. The average Nusselt number may be calculated from the following correlation. State all assumptions made.

$$\overline{Nu}_D = 0.75\,Re_D^{0.4}\,Pr^{0.37}$$

(12 marks)

(b) Hot fluid at 90°C with a mass flow rate of 0.005 kg/s flows through a 2-m long thin-walled tube (D = 10 mm). Air at 25°C is blown across the tube so that a convection heat transfer coefficient of 25 W/m^2·K is maintained over the entire surface.
What is the outlet temperature of the fluid leaving the tube and the rate of heat loss from the tube to the air?
The thermophysical properties of fluid are c_p = 1200 J/kg·K, k = 0.06 W/m·K, Pr = 0.8 and μ = 40 × 10^{-6} kg/s·m

(13 marks)

4. Define the terms "emissivity" and "absorptivity" used in radiation heat transfer calculations.

(5 marks)

In Figure 1, a long cylindrical rod of 10 mm diameter (Surface 1) and thermal conductivity of 1 W/m·K is heated by passing electric current through it. The rod is positioned coaxially in an evacuated long cylindrical enclosure of 50-mm diameter (Surface 2). Surface 2 is maintained at 330 K, and when steady operating condition is reached, it is observed that the rod dissipates 100 W/m. Both Surfaces 1 and 2 have an emissivity of 0.2. Determine

(i) the surface temperature of the rod,
(ii) the volumetric heat generation within the rod, and
(iii) the centre temperature of the rod.

(20 marks)

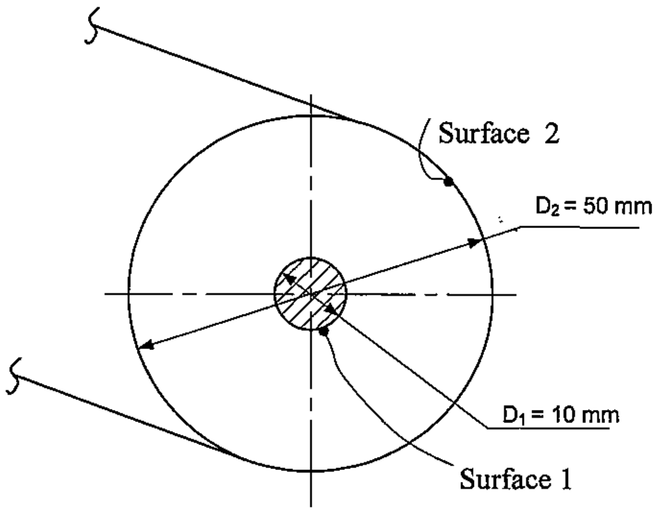

Figure 1

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LIST OF FORMULAE

General Heat Conduction Equations

Cartesian coordinates:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{e}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

Cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{e}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

Spherical coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{e}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

where $\dot{e}_{gen}$ is the heat generation rate per unit volume.

Temperature distribution in a plane wall with uniform heat generation, symmetric boundary conditions and the origin at the midplane

$$T(x)=\frac{\dot{e}_{gen}L^2}{2k}\left(1-\frac{x^2}{L^2}\right)+T_s$$

Temperature distribution in a solid cylinder with uniform heat generation

$$T(r)=\frac{\dot{e}_{gen}r_o^2}{4k}\left(1-\frac{r^2}{r_o^2}\right)+T_s$$

where $\dot{e}_{gen}$ is the heat generation rate per unit volume, L is the half-thickness of the plane wall, r_o is the radius of the cylinder, k is the thermal conductivity and T_s is the boundary temperature.

Temperature distribution along a fin of uniform cross section

Convective tip: $\frac{\theta}{\theta_b}=\frac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$ **Adiabatic tip:** $\frac{\theta}{\theta_b}=\frac{\cosh m(L-x)}{\cosh mL}$ **Prescribed fin tip temperature:** $\frac{\theta}{\theta_b}=\frac{(\theta_L/\theta_b)\sinh mx+\sinh m(L-x)}{\sinh mL}$ **Infinite fin:** $\frac{\theta}{\theta_b}=e^{-mx}$	**Nomenclature:** $\theta=T(x)-T_\infty$ $\theta_b=T_b-T_\infty$ T_b = fin base temperature T_∞ = fluid temperature $m=\sqrt{\frac{hP}{kA_c}}$ L = fin length x = distance along the fin measured from fin base h = convective heat transfer coefficient P = fin perimeter k = thermal conductivity of fin material A_c = fin cross-sectional area

Basic definitions

Nusselt number: $\mathrm{Nu}_x \equiv \dfrac{hx}{k}$

Reynolds number: $\mathrm{Re}_x \equiv \dfrac{\rho u x}{\mu} = \dfrac{ux}{\nu}$

Reynolds number (circular pipe): $\mathrm{Re}_D \equiv \dfrac{\rho u_m D}{\mu} = \dfrac{4\dot{m}}{\pi D \mu}$

Prandtl number: $\mathrm{Pr} \equiv \dfrac{\mu c_p}{k}$

Grashof number: $\mathrm{Gr}_{l_c} \equiv \dfrac{g\beta\,|T_w - T_\infty|\,l_c^3}{\nu^2}$

Volume expansion coefficient: $\beta = \dfrac{1}{T_f}$ for an ideal gas where T_f is in kelvins

Rayleigh number: $\mathrm{Ra}_{l_c} = \mathrm{Gr}_{l_c}\,\mathrm{Pr}$

Flow over a flat plate

The average Nusselt numbers for an isothermal plate are:

Laminar flow: $\overline{\mathrm{Nu}}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$

Mixed flow: $\overline{\mathrm{Nu}}_L = (0.037\,\mathrm{Re}_L^{4/5} - 871)\,\mathrm{Pr}^{1/3}$ for $\mathrm{Re}_{x_c} = 5\times10^5$

Turbulent flow: $\overline{\mathrm{Nu}}_L = 0.037\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3}$

The local Nusselt numbers for an isothermal plate are:

Laminar flow: $\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$

Turbulent flow: $\mathrm{Nu}_x = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$

Free convection over a vertical plate

$\overline{\mathrm{Nu}}_L \equiv \dfrac{\overline{h}_L L}{k} = C\,\mathrm{Ra}_L^n$

Flow	Ra_L	C	n
Laminar	10^4 - 10^9	0.59	1/4
Turbulent	10^9 - 10^{13}	0.10	1/3

Internal flow

Laminar flow: ($\mathrm{Re}_D < 2300$)

Uniform heat flux: $\overline{\mathrm{Nu}}_D = 4.36$

Uniform wall temperature: $\overline{\mathrm{Nu}}_D = 3.66$

Turbulent flow: ($\mathrm{Re}_D > 2300$)

Heating ($T_w > T_m$): $\overline{\mathrm{Nu}}_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{0.4}$

Cooling ($T_w < T_m$): $\overline{\mathrm{Nu}}_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{0.3}$

Radiation heat transfer

Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4

Resistances are of the following forms:

Surface: $(1 - \varepsilon)/(\varepsilon A)$

Space: $1/(A_1 F_{1\text{-}2})$

End of Paper

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MA3003. 2014/2015 SEM I.

1.(a). — Thermodynamics deals with equilibrium states and changes from one equilibrium state to another.
— Primary interest in heat, which is the form of energy that can be transferred from one system to another as a result of temperature difference.
— Heat transfer is a science that deals with determining the rates of such energy transfer.
— Deals with systems that lack thermal equilibrium, and is a nonequilibrium phenomenon.

(b) — Heat and work are energy transfer mechanisms between a system and its surroundings.
— Heat transfer involves thermal energy in transit due to a temperature difference.
— Work is the energy transfer associated with a force acting through a distance.

(c) — Conduction is the ~~thermal~~ energy transfer from more energetic particles of a substance to less energetic one as a result of interactions between the particles.
— Can occur in solids and stationary fluids.
— Convection is the energy transfer due to random molecular motion (diffusion) and bulk fluid motion (advection).
— Occurs in presence of temperature gradient in fluid and relative motion between fluid and surface.
— Radiation is the energy emitted by matter in the form of electromagnetic waves or photons as a result of the changes in electronic configurations of the atoms or molecules.
— Occurs in any medium between 2 or more surfaces.

(d) Assumptions:
— One dimensional heat conduction
— Steady state heat transfer.
— Constant thermal properties
— Negligible radiation
— Part of handle at water level is at same temperature as water (95°C)

Consider the extended part of the spoon handle as a fin

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{(17\,W/m^2K)[2(0.002m + 0.013m)]}{(15.1\,W/mK)(0.002m)(0.013m)}} = 36.04\,m^{-1}$$

__DISCLAIMER:__ The solutions are done by students who scored A or above in this subject. The MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.

$\tanh mL = \tanh[(36.04m^{-1})(0.18m)] > 0.99 \Rightarrow$ assume infinite fin.

$\frac{\theta_t}{\theta_b} = e^{-mx}$

$\frac{T_t - 25°C}{95°C - 25°C} = e^{-(36.04m^{-1})(0.18m)}$

$T_t = 25.11°C$

$\therefore$ Temperature difference $= T_b - T_t = 95°C - 25.11°C = 69.9°C$

2.(a). Assumption.

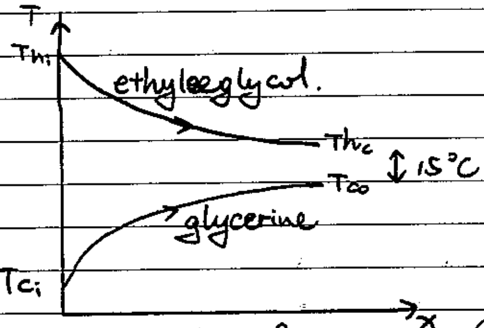
— steady state conditions

— thin-walled double-pipe parallel-flow heat exchanger

— negligible fouling resistance

— negligible radiation.

— negligible heat loss to surroundings



$\Delta T_1 = T_{hi} - T_{ci} = 60°C - 20°C = 40°C$

$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$

$= \frac{15-40}{\ln(15/40)}$

$= 25.49°C$

Note: $T_{ho} > T_{co}$ for parallel flow

$\dot{Q} = UA_s\Delta T_{lm}$

$= (240W/m^2k)(3.2m^2)(25.49°C)$

$= 19575W$

$= 19.58kW$

Note: Fouling factor $F=1$ for pure double-pipe heat exchanger

(b). $\dot{Q} = \dot{m}_c C_{p,c}(T_{co} - T_{ci})$

$19575W = (0.5kg/s)(2400J/kg.k)(T_{co} - 20°C)$

$\therefore T_{co} = 36.3°C$

(c). $\Delta T_2 = T_{ho} - T_{co}$

$15°C = T_{ho} - 36.3°C$

$T_{ho} = 51.3°C$

$\dot{Q} = \dot{m}_h C_{p,h}(T_{hi} - T_{ho})$

$19575W = \dot{m}_h(2500J/kg.k)(60°C - 51.3°C)$

$\therefore \dot{m}_h = 0.90kg/s$

3.(a) Assumptions:

— Steady state conditions

— Constant thermal properties

— Negligible radiation.

Energy balance for anemometer

$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{system}$ Note: " ' " indicates per unit length.

$\dot{E}_{in} = \bar{h} A_s (T_s - T_\infty)$

$\dot{E}_{in}' = \bar{h} \cdot \pi D \cdot (T_s - T_\infty)$

$12\,W/m = \bar{h} \cdot \pi (0.0001\,m)(90^\circ C - 20^\circ C)$

$\bar{h} = 545.7\,W/m^2K$

$\overline{Nu}_D = \frac{\bar{h}D}{k} = \frac{(545.7\,W/m^2k)(0.0001\,m)}{0.0282\,W/mk} = 1.935$

$\overline{Nu}_D = 0.75\,Re_D^{0.4}\,Pr^{0.37}$

$1.935 = 0.75\,Re_D^{0.4}\,(0.701)^{0.37}$

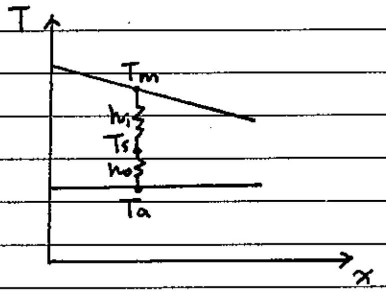
$Re_D = 14.85$

$Re_D = \frac{uD}{\nu}$

$14.85 = \frac{V(0.0001\,m)}{1.807\times10^{-5}\,m^2/s}$ # $\therefore u = 2.68\,m/s$

(b) Assumptions:

- Steady state conditions
- Constant thermal properties
- negligible radiation
- thin-walled tube.
- Constant surface heat flux
- fully developed flow conditions



$Re_D = \frac{4\dot{m}}{\pi D \mu}$

$= \frac{4(0.005\,kg/s)}{\pi(0.010\,m)(40\times10^{-5}\,kg/ms)}$

$= 15915 > 2300 \Rightarrow$ Turbulant flow

For cooling condition i.e. $T_o < T_m$.

$\overline{Nu}_D = 0.023\,Re_D^{4/5}\,Pr^{0.3}$

$= 0.023\,(15915)^{4/5}\,(0.8)^{0.3}$

$= 49.44$

$\overline{Nu}_D = \frac{\bar{h}D}{k}$

$49.44 = \frac{\bar{h}_i\,(0.010\,m)}{0.06\,W/mK}$

$\bar{h}_i = 296.7\,W/m^2k$

$\frac{1}{U} = \frac{1}{\bar{h}_i} + \frac{1}{\bar{h}_o}$

$= \frac{1}{296.67\,W/m^2k} + \frac{1}{25\,W/m^2k}$

$U = 23.06\,W/m^2k$

$\dot{Q}_{conv} = \dot{m} C_p (T_{mi} - T_{mo}) = U A_s \Delta T_{lm}$

$\dot{m} C_p (\Delta T_i - \Delta T_o) = U A_s \frac{\Delta T_i - \Delta T_o}{\ln(\Delta T_i/\Delta T_o)}$

$(0.005\,kg/s)(1200\,J/kg\cdot k) = \frac{(23.06\,W/m^2k)\cdot\pi(0.010\,m)(2\,m)}{\ln\frac{90^\circ C - 25^\circ C}{T_{mo} - 25^\circ C}}$

$\therefore T_{mo} = 76.06°C = 76.1°C$

$\therefore \dot{Q}_{conv} = \dot{m}C_p(T_{mi} - T_{mo})$

$= (0.005kg/s)(1200J/kg \cdot k)(90°C - 76.1°C)$

$= 83.7W$

4 — Emissivity ε of a surface is defined as the ratio of radiation emitted by the surface to radiation emitted by a blackbody at the same temperature.

— Measure of how closely a surface opportunity a blackbody ($\varepsilon = 1$) and has value $0 \le \varepsilon \le 1$ ie radiation emitted $= \varepsilon E_b = \varepsilon \sigma T^4$

— Gray and diffuse surfaces imply that radiation properties are independent of wavelength and direction

— ~~Absolutety~~ Absorptivity α is the fraction of radiation energy incident upon a surface which is absorbed by the surface.

— Radiation absorbed $= \alpha G = \alpha \sigma T^4$

Assumptions:

— Steady state conditions

— One dimensional heat conduction

— Constant thermal properties.

— Uniform heat generation.

— gray and diffuse surfaces.

(i) $F_{12} = 1$

$R_1' = \frac{1-\varepsilon_1}{A_1'\varepsilon_1} = \frac{1-0.2}{\pi(0.010m)(0.2)} = 127.3/m$

$R_2' = \frac{1-\varepsilon_2}{A_2'\varepsilon_2} = \frac{1-0.2}{\pi(0.050m)(0.2)} = 25.46/m$

$R_{12}' = \frac{1}{A_1' F_{12}} = \frac{1}{\pi(0.010m)(1)} = 31.83/m$

$R'_{total} = R_1' + R_{12}' + R_2' = 184.6/m$

$\dot{Q}'_{rad} = \frac{E_{b1} - E_{b2}}{R'_{total}} = \frac{\sigma(T_1^4 - T_2^4)}{R'_{total}}$

$100W/m = \frac{(5.67 \times 10^{-8} W/m^2K^4)[T_1^4 - (330K)^4]}{184.6/m}$

$\therefore T_1 = 762.2K = 489.2°C$

Energy balance for rod surface

$\dot{E}_{in}' - \dot{E}_{out}' + \dot{E}_{gen}' = \dot{E}_{system}' \Rightarrow \dot{E}_{gen}' = \dot{Q}_{rad}' = 100W/m$

(ii) $\dot{e}_{gen} = \dot{E}_{gen}' \div \frac{1}{4}\pi d_1^2 = 100W/m \div \frac{1}{4}\pi(0.010m)^2 = 1273kW/m^3$

(iii) $T(r_i) = \frac{\dot{e}_{gen} r_o^2}{4k}\left(1 - \frac{r_i^2}{r_o^2}\right) + T_s = \frac{(1273kW/m^3)(0.005m)^2}{4(1W/mK)}\left[1 - \left(\frac{0m}{0.005m}\right)^2\right] + 489.2°C = 497.1°C$

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SEMESTER 1 EXAMINATION 2016-2017

MA3003 – HEAT TRANSFER

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INSTRUCTIONS

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5. Candidates may use the list of formulae provided on pages 5 to 7.

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1. Consider a simple heat exchanger made of a long rectangular enclosure containing a number of aluminium rods running from one passage of fluid 1 at T_{f1} through a thin adiabatic membrane into another passage of fluid 2 at T_{f2}. The cross section of the enclosure is shown in Figure 1. The enclosure is made of an insulation material, and the membrane is located at the middle of the rod. Both ends of the rod of thermal conductivity k are in contact with the enclosure with contact resistance R''_{ct}. It is estimated that the heat transfer coefficients between the two fluids and the rod are h_1 and h_2 corresponding to fluid 1 and fluid 2, respectively. The diameter and the total length of the rod are denoted by D and L, respectively.

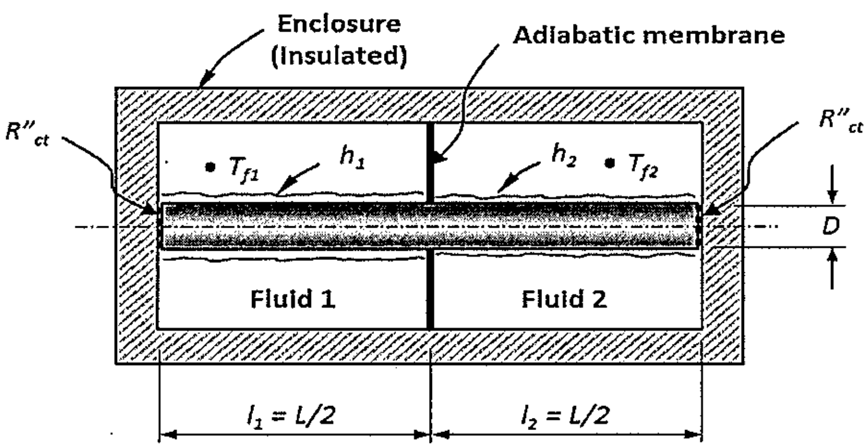

Figure 1

Note: Question 1 continues on page 2.

For the following conditions:-

- $T_{f1} = 125$ °C, $T_{f2} = 65$ °C
- $h_1 = 20$ W/m^2·K, $h_2 = 250$ W/m^2·K
- $k = 185$ W/m·K, $D = 25$ mm, $L = 60$ cm
- $R''_{ct} = 2.8$ cm^2· °C/W

(a) Draw a schematic of the problem.

(b) Provide a list of assumptions complete with validation whenever applicable.

(c) Provide a corresponding thermal resistance network, and clearly indicate all the relevant notations.

(d) Compute the equivalent total resistance of the thermal network.

(e) Compute the total heat transfer rate through each rod.

(25 marks)

2. The heat transfer coefficient for air flowing over a sphere may be determined by observing the temperature-time history of a sphere. A sphere of 1 cm in diameter fabricated from pure aluminium is inserted into an air stream. The air temperature is 27 °C, and the temperature measurements at the surface of the sphere taken after 10 and 60 seconds in the air stream indicate 66 °C and 55 °C, respectively.

(a) Draw a schematic of the problem.

(b) Provide a list of relevant assumptions complete with validation whenever applicable.

(c) Determine the convective heat transfer coefficient.

(d) Determine the amount of heat transfer from the sphere during this time period of 50 seconds.

Property values of pure aluminium are:

$\rho = 2702$ kg/m^3 $c = 903$ J/kg·K $k = 237$ W/m·K $\alpha = 97.1 \times 10^{-6}$ m^2/s

(25 marks)

3 (a) A thin polished-aluminium radiation shield with emissivity of 0.05 on both sides is placed between two very large parallel plates. The plates are maintained at temperatures T_1 and T_2 with emissivities 0.2 and 0.8, respectively.

(i) Find the net rate of radiation heat transfer (in term of T_1 and T_2) per unit surface area between the two plates without the radiation shield.

(ii) Find the percentage reduction in heat transfer rate with the radiation shield.

(15 marks)

(b) An electrical power transmission line of 2 cm diameter carries a current of 200 amps and has a resistance of 5×10^{-4} Ω/m. Determine the surface temperature of the transmission line if air at 28 °C and a free stream velocity of 10 m/s flows across the line. The properties of air are: $\nu = 15.7 \times 10^{-6}$ m²/s, $k = 0.0262$ W/m·K and Pr = 0.71. The average Nusselt number for forced convection over a cylinder may be calculated from the following correlation.

$$\overline{\mathrm{Nu}}_D = \frac{\bar{h}D}{k} = C\,\mathrm{Re}_D{}^m\,\mathrm{Pr}^{1/3}$$

Range of Re	Nusselt number
0.4 – 4	$\overline{\mathrm{Nu}}_D = 0.989\,\mathrm{Re}_D{}^{0.330}\,\mathrm{Pr}^{1/3}$
4 – 40	$\overline{\mathrm{Nu}}_D = 0.911\,\mathrm{Re}_D{}^{0.385}\,\mathrm{Pr}^{1/3}$
40 – 4000	$\overline{\mathrm{Nu}}_D = 0.683\,\mathrm{Re}_D{}^{0.466}\,\mathrm{Pr}^{1/3}$
4000 – 40,000	$\overline{\mathrm{Nu}}_D = 0.193\,\mathrm{Re}_D{}^{0.618}\,\mathrm{Pr}^{1/3}$
40,000 – 400,000	$\overline{\mathrm{Nu}}_D = 0.027\,\mathrm{Re}_D{}^{0.805}\,\mathrm{Pr}^{1/3}$

(10 marks)

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4 (a) What is the physical significance of the number of transfer units NTU = $hA/\dot{m}c_p$? What do a small and a large NTU tell the heat transfer engineer about a heat transfer system?

(5 marks)

(b) The surface of a long, horizontal wire of 5 mm diameter and thermal conductivity of 1 W/m·K is maintained at 160 °C by an electric current. The wire is exposed to ambient air and the surrounding walls at 30 °C. The emissivity of the wire surface is 0.8.

Determine

(i) the electric power necessary to maintain the wire temperature if the length is 1 m,

(ii) the rate of the volumetric heat generation within the wire,

(iii) the maximum temperature in the wire and draw the temperature profile along the radius within the wire.

State all assumptions made.

The properties of air are: $\nu = 2.32 \times 10^{-5}$ m^2/s, $k = 0.032$ W/m·K, $\rho = 0.94$ kg/m^3 and Pr = 0.692.

(20 marks)

LIST OF FORMULAE

General Heat Conduction Equations

Cartesian coordinates:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{e}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

Cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{e}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

Spherical coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{e}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

where $\dot{e}_{gen}$ is the heat generation rate per unit volume.

Temperature distribution in a plane wall with uniform heat generation, symmetric boundary conditions and the origin at the midplane

$$T(x)=\frac{\dot{e}_{gen}L^2}{2k}\left(1-\frac{x^2}{L^2}\right)+T_s$$

Temperature distribution in a solid cylinder with uniform heat generation

$$T(r)=\frac{\dot{e}_{gen}r_o^2}{4k}\left(1-\frac{r^2}{r_o^2}\right)+T_s$$

where $\dot{e}_{gen}$ is the heat generation rate per unit volume, L is the half-thickness of the plane wall, r_o is the radius of the cylinder, k is the thermal conductivity and T_s is the boundary temperature.

Temperature distribution along a fin of uniform cross section

Convective tip: $\dfrac{\theta}{\theta_b}=\dfrac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$ **Adiabatic tip:** $\dfrac{\theta}{\theta_b}=\dfrac{\cosh m(L-x)}{\cosh mL}$ **Prescribed fin tip temperature:** $\dfrac{\theta}{\theta_b}=\dfrac{(\theta_L/\theta_b)\sinh mx+\sinh m(L-x)}{\sinh mL}$ **Infinite fin:** $\dfrac{\theta}{\theta_b}=e^{-mx}$	**Nomenclature:** $\theta=T(x)-T_\infty$ $\theta_b=T_b-T_\infty$ T_b = fin base temperature T_∞ = fluid temperature $m=\sqrt{\dfrac{hP}{kA_c}}$ L = fin length x = distance along the fin measured from fin base h = convective heat transfer coefficient P = fin perimeter k = thermal conductivity of fin material A_c = fin cross-sectional area

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Basic definitions

Nusselt number: $\mathrm{Nu}_x \equiv \dfrac{hx}{k}$

Reynolds number: $\mathrm{Re}_x \equiv \dfrac{\rho u x}{\mu} = \dfrac{ux}{\nu}$

Reynolds number (circular pipe): $\mathrm{Re}_D \equiv \dfrac{\rho u_m D}{\mu} = \dfrac{4\dot{m}}{\pi D \mu}$

Prandtl number: $\mathrm{Pr} \equiv \dfrac{\mu c_p}{k}$

Grashof number: $\mathrm{Gr}_{l_c} \equiv \dfrac{g\beta\,|T_w - T_\infty|\,l_c^3}{\nu^2}$

Volume expansion coefficient: $\beta = \dfrac{1}{T_f}$ for an ideal gas where T_f is in kelvins

Rayleigh number: $\mathrm{Ra}_{l_c} = \mathrm{Gr}_{l_c}\,\mathrm{Pr}$

Flow over a flat plate

The average Nusselt numbers for an isothermal plate are:

Laminar flow: $\overline{\mathrm{Nu}}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$

Mixed flow: $\overline{\mathrm{Nu}}_L = (0.037\,\mathrm{Re}_L^{4/5} - 871)\,\mathrm{Pr}^{1/3}$ for $\mathrm{Re}_{x_c} = 5\times10^5$

Turbulent flow: $\overline{\mathrm{Nu}}_L = 0.037\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3}$

The local Nusselt numbers for an isothermal plate are:

Laminar flow: $\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$

Turbulent flow: $\mathrm{Nu}_x = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$

Free convection over a horizontal cylinder

$\overline{\mathrm{Nu}}_D = C\,\mathrm{Ra}_D^n$

Rayleigh number (Ra)	C	n
$10^{-10} - 10^{-2}$	0.675	0.058
$10^{-2} - 10^{2}$	1.02	0.148
$10^{2} - 10^{4}$	0.85	0.188
$10^{4} - 10^{7}$	0.48	0.25
$10^{7} - 10^{12}$	0.125	0.333

Internal flow

Laminar flow: ($Re_D < 2300$)

Uniform heat flux:	$\overline{Nu}_D = 4.36$
Uniform wall temperature:	$\overline{Nu}_D = 3.66$

Turbulent flow: ($Re_D > 2300$)

Heating ($T_w > T_m$):	$\overline{Nu}_D = 0.023 Re_D^{4/5} Pr^{0.4}$
Cooling ($T_w < T_m$):	$\overline{Nu}_D = 0.023 Re_D^{4/5} Pr^{0.3}$

Radiation heat transfer

Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4
Resistances are of the following forms:

Surface:	$(1 - \varepsilon) / (\varepsilon A)$
Space:	$1 / (A_1 F_{1\text{-}2})$

END OF PAPER

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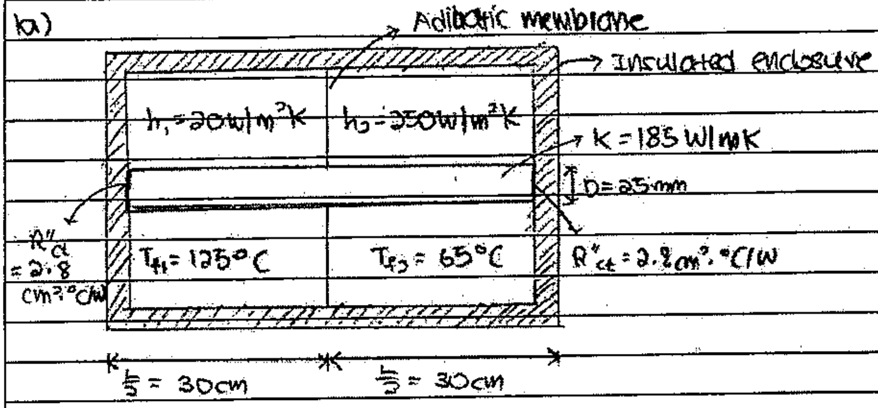
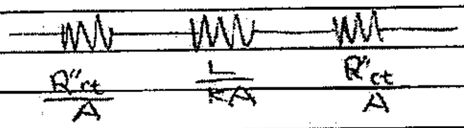
a)



b) Constant properties
1 dimensional
Negligible radiation

c) By assuming that the heat transfer occurs only in the horizontal direction, there will be no heat transfer radially. Thus heat transfer only occurs through the aluminium rods.



d) $R_t = \frac{R''_{ct}}{A} + \frac{L}{kA} + \frac{R''_{ct}}{A}$

$= \frac{1}{A}\left(2R''_{ct} + \frac{L}{k}\right)$

$= \left(1 / \frac{\pi \times 0.025^2}{4}\right)\left(2 \times 2.8 \times 10^{-4} + \frac{0.6}{185}\right)$

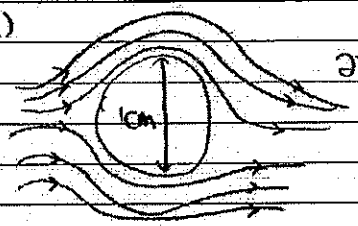
$= 7.75 \ K/W$

e) $= \dot{Q} = \frac{\Delta T}{R_t}$

$= \frac{125-65}{7.75}$

$= 7.74 \ W$

2a)



37°C

Sphere

After 10s : 66°C

After 60s : 55°C

b) 1 Dimensional

Constant properties

Negligible radiation

c) Assuming lumped capacitance method is valid.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (0.5)^3$

$= 0.524 cm^3$

$A_s = 4\pi r^2$

$= 4\pi (0.5)^2$

$= 3.14 cm^2$

$t = \frac{\rho V c}{hA} \ln \frac{\theta_i}{\theta}$

$60 - 10 = \frac{(2702)(0.524 \times 10^{-6})(903)}{h(3.14 \times 10^{-4})} \ln\left(\frac{66-27}{55-27}\right)$

$50 = \frac{1347}{h}$

$h = 26.9 \ W/m^2K$

Check if LCM is valid

$L_c = \frac{V}{A_s}$

$= \frac{0.524}{3.14}$

$= 0.167 cm$

$Bi = \frac{hL_c}{k}$

$= \frac{(26.9)(0.167 \times 10^{-2})}{237}$

$= 1.90 \times 10^{-4} < 0.1$

$\therefore$ LCM is valid.

d) $\tau_t = \frac{\rho V_c}{hA_s}$

$= \frac{(2702)(0.504\times10^{-6})(903)}{(26.9)(3.14\times10^{-4})}$

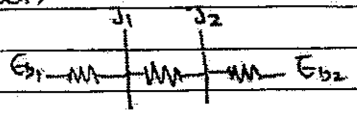
$= 151$

$Q = \rho V_c \theta_i [1 - \exp(-\frac{t}{\tau_t})]$

$= (2702)(0.504\times10^{-6})(903)(66-27)[1 - \exp(-\frac{50}{151})]$

$= 14.1\ W$

3ai)



$$\dot{q}_{,\text{no shield}} = \frac{E_{b_1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

$$= \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$\dot{q}_{,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{(5.67\times10^{-8})(T_1^4 - T_2^4)}{\frac{1}{0.2} + \frac{1}{0.8} - 1}$$

$$= 1.08\times10^{-8}\,(T_1^4 - T_2^4)\ \text{W/m}^2$$

aii)

$$\dot{q}_{,\text{with shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$= \frac{(5.67\times10^{-8})(T_1^4 - T_2^4)}{\left(\frac{1}{0.2} + \frac{1}{0.8} - 1\right) + \left(\frac{1}{0.05} + \frac{1}{0.05} - 1\right)}$$

$$= 1.28\times10^{-9}\,(T_1^4 - T_2^4)\ \text{W/m}^2$$

$$\text{Percentage reduction} = \frac{1.08\times10^{-8}(T_1^4 - T_2^4) - 1.28\times10^{-9}(T_1^4 - T_2^4)}{1.08\times10^{-8}(T_1^4 - T_2^4)} \times 100\%$$

$$= 88.1\%$$

b) $Re_D = \frac{u_\infty D}{\nu}$

$= \frac{(10)(0.02)}{15.7\times10^{-6}}$

$= 12738$

$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.193\, Re_D^{0.618}\, Pr^{1/3}$

$\bar{h} = \frac{k}{D}\, 0.193\, Re_D^{0.618}\, Pr^{1/3}$

$= \frac{0.0262}{0.02}(0.193)(12738)^{0.618}(0.71)^{1/3}$

$= 77.7\ W/m^2K$

$\dot{Q} = I^2R = \bar{h}A(T_s - T_\infty)$

$I^2R'L = \bar{h}(\pi DL)(T_s - T_\infty)$

$I^2R' = \bar{h}(\pi D)(T_s - T_\infty)$

$(200^2)(5\times10^{-4}) = 77.7(\pi\times0.02)(T_s - 28)$

$T_s = 32.1\ °C$

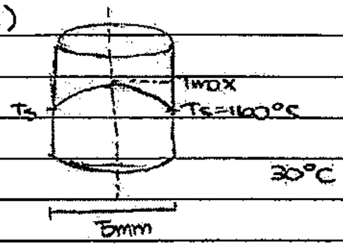
(3)

4a) NTU is a measure of the effectiveness of the heat transfer system.
A small value of NTU indicates more opportunity for heat transfer while a large NTU indicates little opportunity for heat transfer as the limit for heat transfer is reached.

bi) $T_f = \frac{160+30}{2} + 273$
$= 368K$
$\beta = \frac{1}{T_f}$
$= 0.00272$
$Ra_D = Gr_D Pr$
$= \frac{g\beta(T_s - T_\infty)D^3}{v^2} \cdot Pr$
$= \frac{(9.81)(0.002717)(160-30)(0.005)^3}{(2.32\times10^{-5})^2} \times 0.692$
$= 557$
$\overline{Nu_D} = 0.85 Ra_D^{0.188}$
$\bar{h} = \frac{\overline{Nu_D} k}{D}$
$= \frac{k}{D} \times 0.85 Ra_D^{0.188}$
$= \frac{0.032}{0.005} \times 0.85(557)^{0.188}$
$= 17.9 \, W/m^2K$
$A = \pi DL$
$= \pi \times 0.005 \times 1$
$= 0.0157 m^2$
$\dot{Q} = \bar{h}A(T_s - T_\infty) + \varepsilon\sigma A(T_s^4 - T_{sur}^4)$
$= 17.9(0.0157)(160-30) + 0.8(5.67\times10^{-8})(0.0157)[(160+273)^4 - (30+273)^4]$
$= 55.5W$
Electrical power $= 55.5W$

ii) $\dot{e}_{gen} \times V = \dot{Q}$
$\dot{e}_{gen} = \frac{\dot{Q}}{V}$
$= 55.5 / (\frac{\pi \times 0.005^2}{4} \times 1)$
$= 2.83 \times 10^6 \, W/m^3$

b)ii)



$$T(r) = \frac{\dot{e}_{gen} r_o^2}{4k}\left(1-\frac{r^2}{r_o^2}\right) + T_s$$

Max temp occurs at $r = 0$,

$$T_{max} = \frac{\dot{e}_{gen} r_o^2}{4k} + T_s$$

$$= \frac{(2.83\times10^6)(0.0025)^2}{4(1)} + 160$$

$$= 164\,°C$$

Assumptions:

1) Constant properties
2) 1 Dimensional
3) Steady state

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

MA3003 – HEAT TRANSFER

April/May 2019 Time Allowed: $2^1/_2$ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED BOOK** examination.
5. Candidates may use the list of formulae provided on pages 5 and 6.

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1 (a) A thin, 1.5-m-outer diameter spherical tank containing liquid oxygen at 90 K and 1 atm is insulated with a blanket of 10-cm-thick fibreglass insulation (k = 0.022 W/m·K). It is placed in ambient air at 310 K where the outside convective and radiative heat transfer coefficients are 5 W/m^2·K and 3 W/m^2·K, respectively. The tank is vented to the atmosphere.

(i) What is the boil-off rate of the liquid oxygen in kg/h?

(ii) Which thermal resistance controls the rate of heat flow? How would the precisions of the insulation thermal conductivity and heat transfer coefficients affect the accuracy of the heat flow rate?

State all assumptions made. The boiling point of oxygen is 90 K and its enthalpy of vaporization, h_{fg}, is 213 kJ/kg.

(15 marks)

Note: Question 1 continues on page 2.

(b) A two-dimensional solid of constant thermal conductivity k is subjected to thermal conditions at its boundaries as shown in Figure 1. Write down the appropriate form of the steady-state heat conduction equation and the boundary conditions for the determination of $T(x, y)$ in the solid. Do not solve the equation.

(10 marks)

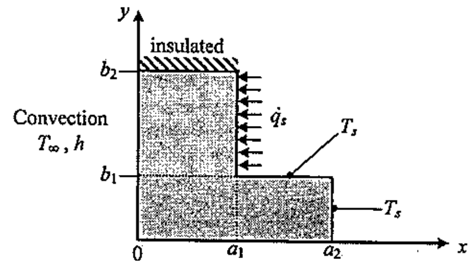

Figure 1

2 (a) A wall has its surface maintained at 200°C and is in contact with air at 30°C. What is the percentage increase in the rate of heat dissipation if an array of very wide triangular fins is added to the surface? The 200-mm-long fins are 6-mm thick at the base and are spaced at a pitch of 15 mm from centre to centre of each fin base.

Assume that the convective heat transfer coefficient is 20 W/m^2·K for both the plain and finned surfaces and that the fin material has a thermal conductivity of 50 W/m·K. The fin efficiency of triangular fins can be obtained from Figure 2. State all assumptions made.

(10 marks)

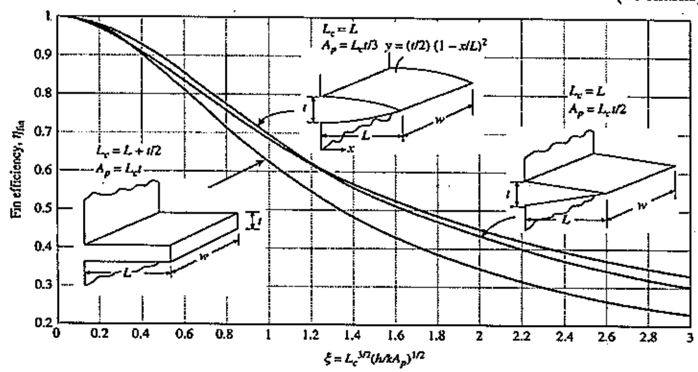

Figure 2

Note: Question 2 continues on page 3.

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(b) An aircraft oil cooler is to be designed as a double-pipe heat exchanger to cool the oil temperature from 120°C to 90°C using air at an inlet temperature of 40°C. The mass flow rate of the oil is 2 kg/s. If the overall heat transfer coefficient U of the heat exchanger is 150 W/m^2·K, determine the required heat transfer surface area for (i) counter-flow, and (ii) parallel-flow using the effectiveness-NTU method. Assume that the heat capacity rates of the oil and air flows are the same, i.e., $C_h = C_c$. The specific heat of the oil may be taken as $c_{p,h}$ = 2240 J/kg·K. The NTU relations for the double-pipe heat exchanger are given in Table 1 together with L'Hôpital's Rule.

(15 marks)

Table 1

NTU relations for heat exchangers: NTU = UA_s/C_{min} and $c = C_{min}/C_{max} = (\dot{m}c_p)_{min}/(\dot{m}c_p)_{max}$

Heat exchanger type	NTU relation
1 *Double-pipe:*	
Parallel-flow	$\text{NTU} = -\dfrac{\ln[1-\varepsilon(1+c)]}{1+c}$
Counter-flow	$\text{NTU} = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon c-1}\right)$

L'Hôpital's Rule for Indeterminate Forms

For $\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{0}{0}$ or $\dfrac{\pm\infty}{\pm\infty}$ where a can be any real number, $+\infty$ or $-\infty$,

$\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{f'(x)}{g'(x)}$ where $f'(x)$ and $g'(x)$ are the derivatives of their functions.

3 (a) Air at 20°C is flowing with a velocity of 4 m/s along the 3-m length of a flat plate maintained at 100°C. Determine the heat transfer rate from

(i) the first $^1/_3$ length of the plate, and

(ii) the next $^2/_3$ length of the plate.

The thermophysical properties of air are as follows:
$\mu = 20 \times 10^{-6}$ kg/s·m, k = 0.03 W/m·K, ρ = 1.02 kg/m^3 and Pr = 0.71

(12 marks)

(b) The rating for a horizontal-plate resistance heater is to be determined. The 1-m square horizontal plate with an emissivity of 0.8 is suspended in a room where the ambient air and surrounding walls are at 20°C. What is the maximum heat dissipation rate allowed if both the top and bottom surfaces are maintained at 190°C?
The properties of air are as follows:
$\nu = 2.32 \times 10^{-5}$ m^2/s, k = 0.03 W/m·K and Pr = 0.69

(13 marks)

4 (a) A thin radiation shield that has the same emissivity on both sides is placed between two very large parallel plates. The plates are maintained at temperatures T_1 and T_2 with emissivities 0.8 and 0.4, respectively. Find the emissivity of the shield to reduce the radiation losses from the system with the radiation shield to one-tenth of that without the shield.

(12 marks)

(b) Process fluid at 20°C flows through a long, electrically heated tube with the tube wall maintained at a constant heat flux. The velocity and temperature profiles are fully developed. The 10-m-long tube has an inside diameter of 15 mm with the fluid flowing at an average velocity of 0.8 m/s. If the fluid exit temperature is 85°C, determine

(i) the average heat transfer coefficient, and

(ii) the temperatures of the tube wall at the inlet and exit.

The properties of fluid are as follows:
c_p = 0.15 kJ/kg·K, μ = 0.008 kg/s·m, ρ = 800 kg/m^3 and k = 0.12 W/m·K.

(13 marks)

LIST OF FORMULAE

General Heat Conduction Equations

Cartesian coordinates:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{e}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

Cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{e}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

Spherical coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{e}_{gen}=\rho c_p\frac{\partial T}{\partial t}$$

where $\dot{e}_{gen}$ is the heat generation rate per unit volume.

Temperature distribution in a plane wall with uniform heat generation, symmetric boundary conditions and the origin at the midplane

$$T(x)=\frac{\dot{e}_{gen}L^2}{2k}\left(1-\frac{x^2}{L^2}\right)+T_s$$

Temperature distribution in a solid cylinder with uniform heat generation

$$T(r)=\frac{\dot{e}_{gen}r_o^2}{4k}\left(1-\frac{r^2}{r_o^2}\right)+T_s$$

where $\dot{e}_{gen}$ is the heat generation rate per unit volume, r_o is the radius of the cylinder, k is the thermal conductivity and T_s is the boundary temperature.

Temperature distribution along a fin of uniform cross section

Convective tip: $\dfrac{\theta}{\theta_b}=\dfrac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$ **Adiabatic tip:** $\dfrac{\theta}{\theta_b}=\dfrac{\cosh m(L-x)}{\cosh mL}$ **Prescribed fin tip temperature:** $\dfrac{\theta}{\theta_b}=\dfrac{(\theta_L/\theta_b)\sinh mx+\sinh m(L-x)}{\sinh mL}$ **Infinite fin:** $\dfrac{\theta}{\theta_b}=e^{-mx}$	**Nomenclature:** $\theta=T(x)-T_\infty$ $\theta_b=T_b-T_\infty$ T_b = fin base temperature T_∞ = fluid temperature $m=\sqrt{\dfrac{hP}{kA_c}}$ L = fin length x = distance along the fin measured from fin base h = convective heat transfer coefficient P = fin perimeter k = thermal conductivity of fin material A_c = fin cross-sectional area

Basic definitions

Nusselt number:	$Nu_x \equiv \dfrac{hx}{k}$
Reynolds number:	$Re_x \equiv \dfrac{\rho u x}{\mu}=\dfrac{ux}{\nu}$
Reynolds number (circular pipe):	$Re_D \equiv \dfrac{\rho u_m D}{\mu}=\dfrac{4\dot{m}}{\pi D\mu}$
Prandtl number:	$Pr \equiv \dfrac{\mu c_p}{k}$
Grashof number:	$Gr_{l_c} \equiv \dfrac{g\beta\lvert T_w-T_\infty\rvert l_c^3}{\nu^2}$
Volume expansion coefficient:	$\beta = \dfrac{1}{T_f}$ for an ideal gas where T_f is in kelvins
Rayleigh number:	$Ra_{l_c}=Gr_{l_c}Pr$

Flow over a flat plate

The average Nusselt numbers for an isothermal plate are:

Laminar flow:	$\overline{Nu}_L=0.664Re_L^{1/2}Pr^{1/3}$	
Mixed flow:	$\overline{Nu}_L=(0.037Re_L^{4/5}-871)Pr^{1/3}$	for $Re_{x,c}=5\times10^5$
Turbulent flow:	$\overline{Nu}_L=0.037Re_L^{4/5}Pr^{1/3}$	

The local Nusselt numbers for an isothermal plate are:

Laminar flow:	$Nu_x=0.332Re_x^{1/2}Pr^{1/3}$
Turbulent flow:	$Nu_x=0.0296Re_x^{4/5}Pr^{1/3}$

External free convection flow over a horizontal plate

Upper surface of hot plate

$\overline{Nu}_L=0.54Ra_L^{1/4}$ for $10^4\le Ra_L\le10^7$

$\overline{Nu}_L=0.15Ra_L^{1/3}$ for $10^7\le Ra_L\le10^{11}$

Upper surface of cold plate

$\overline{Nu}_L=0.27Ra_L^{1/4}$ for $10^5\le Ra_L\le10^{10}$

Internal flow

Laminar flow: ($Re_D<2300$)

Uniform heat flux:	$\overline{Nu}_D=4.36$
Uniform wall temperature:	$\overline{Nu}_D=3.66$

Turbulent flow: ($Re_D>2300$)

Heating ($T_w>T_m$):	$\overline{Nu}_D=0.023Re_D^{4/5}Pr^{0.4}$
Cooling ($T_w<T_m$):	$\overline{Nu}_D=0.023Re_D^{4/5}Pr^{0.3}$

Radiation heat transfer

Stefan-Boltzmann constant, $\sigma=5.67\times10^{-8}$ W/m^2·K^4

Resistances are of the following forms:

Surface:	$(1-\varepsilon)/(\varepsilon A)$
Space:	$1/(A_1F_{1\text{-}2})$

END OF PAPER

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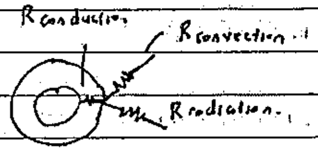
1 a) Sphere

$k = 0.022\ W/m\cdot K$

$T_\infty = 310K$

$h_c = 5\ W/m^2\cdot K \quad h_r = 3\ W/m^2\cdot K$



$$Q = -kA\frac{dT}{dr}$$

$$Q\int_{r_1}^{r_2}\frac{1}{4\pi r^2}dr = -k\int_{T_1}^{T_2}dT$$

$$Q\frac{1}{4\pi}\left(-\frac{1}{r}\right)_{r_1}^{r_2} = k(T_1 - T_2)$$

$$Q = \frac{(T_1 - T_2)}{\frac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi k}} \qquad R_{conduction} = \frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi k} = \frac{\frac{1}{0.75}-\frac{1}{0.85}}{4\pi \times 0.022}$$

$$= 0.567397$$

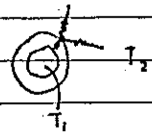
$$R_{convection} = \frac{1}{h_c 4\pi r_2^2} \qquad R_{rad} = \frac{1}{h_R 4\pi r_2^2}$$

$$= \frac{1}{5\times 4\pi \times \left(\frac{1.7}{2}\right)^2} \qquad = \frac{1}{3\times 4\times \pi \times \left(\frac{1.7}{2}\right)^2}$$

$$= 0.022028 \qquad = 0.036714$$

$$\Sigma R = R_{conduction} + \left(\frac{1}{R_{convection}} + \frac{1}{R_{rad}}\right)^{-1}$$

$$= 0.58116$$



$$Q = \frac{(T_1 - T_2)}{\Sigma R} = \frac{90-310}{0.58116} = -378.55\ W$$

(goes in).

i) $Q = \dot{m} h_f \qquad \dot{m} = \frac{Q}{h_f} = \frac{378.55}{213\times 10^3} = 0.00177723\ kg/s$

$$= 6.398\ kg/h$$

$R_{conduction} = 0.567397$

$R_{convection + Radiation} = \left(\frac{1}{R_{convect}} + \frac{1}{R_{rad}}\right)^{-1} = 0.013768$

$Q = \frac{T_1 - T_2}{\Sigma R}$ Since $R_{conduction} > \Sigma R_{convection + Radiation}$

$\therefore$ precision of k will affect $\dot{Q}$ more while precision of $h_{convect}$ $h_{radiation}$ will affect $\dot{Q}$ less.

1 a) Assumption

1) no heat gen
2) 1-D along r

b) Assumption

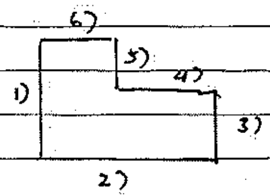
1) 2-D
2) steady state $\cancel{\rho c_p \frac{\partial T}{\partial t}}^{0}$
3) no heat gen $\cancel{\dot{e}_{gen}}^{0}$
4) constant k

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}^{0}+\cancel{\dot{e}_{gen}}^{0}=\cancel{\rho c_p\frac{\partial T}{\partial t}}^{0}$$

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=0$$

4 boundary conditions required to solve.

boundary condition..



1) $-k\frac{\partial T}{\partial y}\Big|_{y=0} = -h(T_s - T_\infty)$

2) $-k\frac{\partial T}{\partial x}\Big|_{x=0} = -h(T_s - T_\infty)$

3) $T_{x=a_2,\ 0<y<b_1} = T_s$

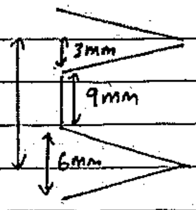
4) $T_{y=b_1,\ a_1<x<a_2} = T_s$

5) $-k\frac{\partial T}{\partial x}\Big|_{x=a_1,\ b_1<y<b_2} = -\dot{q}_s$

6) $-k\frac{\partial T}{\partial y}\Big|_{y=b_2,\ 0<x<a_1} = 0$

$h = 20\ W/m^2\cdot K$ $\quad k = 50\ W/m\cdot K$

2 a) $T_b = 200°C$ $\quad T_\infty = 30°C$



$L = 200mm = 0.2m$

$t = 0.006m$

read graph

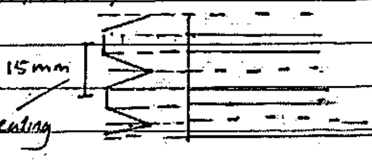
$L_c = L = 0.2$

$A_p = L_c t/2 = 0.2\times0.006/2 = 6\times10^{-4}$

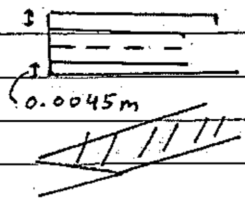
$\xi = L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = 0.2^{3/2}\left(\frac{20}{50\times60\times10^{-4}}\right)^{1/2} = 2.309$

From graph

$\eta = 0.38$



$\varepsilon = \frac{Q_{w\,fin}}{Q_{no\,fin}}$



$= \frac{W\times0.0045\times2\times\cancel{h}\times\cancel{\theta_b} + \eta\times\cancel{h}\times A_{fin}\,\cancel{\theta_b}}{0.015\times W\times\cancel{h}\times\cancel{\theta_b}}$

$A_{fin} = W\times2\times\sqrt{L^2+\left(\frac{t}{2}\right)^2}$

$= W\times0.400045$

$\varepsilon = \frac{\cancel{W}\times0.0045\times2 + \eta\times\cancel{W}\times0.400045}{0.015\cancel{W}}$

$= 10.734 = 1073.45\%$

b) Oil $120°C$ $\quad 90°C$ $\quad \dot{m}_o = 2 kg/s$

Air in $40°C$ $\quad$ out ? $= 70°C$

$U = 150\ W/m^2\cdot K$

Assume

$C_h = C_c$ $\quad C_h = c_o\dot{m}_o = 2240\times2 = 4480$

$Q = C_o(T_{o\,in} - T_{o\,out}) = C_{air}(T_{air\,out} - T_{air\,in})$

$T_{o\,in} - T_{out} = T_{air\,out} - T_{air\,in}$

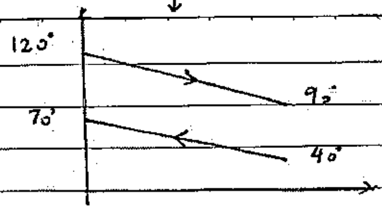
$120 - 90 = T_{air\,out} - 40$

$T_{air\,out} = 70$

NO.: Date:

Not required ↓

i) Counter flow



$$\varepsilon = \frac{Q}{Q_{max}}$$

$$= \frac{\cancel{C_{oil}}\,(T_{Hin} - T_{Hout})}{\cancel{C_{min}}\,(T_{Hin} - T_{Cin})}$$

$$= \frac{120 - 90}{120 - 40} = 0.375$$

$$C = \frac{C_{min}}{C_{max}} = 1$$

Find NTU

$$NTU = \frac{1}{C-1} \ln\left(\frac{\varepsilon - 1}{\varepsilon C - 1}\right) = \frac{0}{0}$$

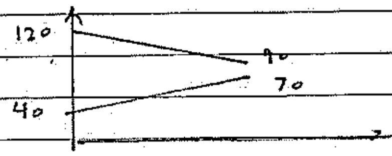
$$= \lim_{C \to 1} \frac{\frac{\partial}{\partial C}\left(\ln\left(\frac{\varepsilon-1}{\varepsilon C-1}\right)\right)}{\frac{\partial}{\partial C}(C-1)} = \lim_{C \to 1} \frac{\frac{1}{\cancel{\frac{\varepsilon-1}{\varepsilon C-1}}} \times \frac{-\cancel{(\varepsilon-1)}}{(\varepsilon C-1)^{\cancel{2}}} \times \varepsilon}{1}$$

$$= \lim_{C \to 1} \frac{-\varepsilon}{\varepsilon C - 1} = \frac{-\varepsilon}{\varepsilon - 1}$$

$$NTU = \frac{-\varepsilon}{\varepsilon - 1} = 0.6$$

$$NTU = \frac{UA}{C_{min}} = 0.6 \Rightarrow A = \frac{0.6\,C_{min}}{U} = \frac{0.6 \times 4480}{150} = 17.92\ m^2 //$$

ii) parallel flow.



$$NTU = -\frac{\ln[1 - \varepsilon(1+C)]}{1+C}$$

$$= -\frac{\ln(1 - 2\varepsilon)}{2} = 0.69315$$

$$NTU = \frac{UA}{C_{min}} \qquad A = \frac{NTU \times C_{min}}{U} = 20.702\ m^2 //$$

3 a). $T_\infty = 20°C$ $V = 4m/s$ $L = 3m$

$T_s = 100°C$

$Re = \frac{\rho V x}{\mu} = \frac{1.02 \times 4 \times 1}{20 \times 10^{-6}} = 204000 < 5 \times 10^5$

(laminar)

$\bar{h} = \frac{\bar{Nu}\, k}{L} = \frac{k}{L}\left(0.664\, Re^{1/2}\, Pr^{1/3}\right)$

$= 8.02646$

i) $\frac{\dot{Q}}{w} = \bar{h} \frac{A}{w} (T_s - T_\infty) = 642.117\ W/m$

$Re = \frac{\rho V x}{\mu} = \frac{1.02 \times 4 \times 3}{20 \times 10^{-6}} = 612000 > 5 \times 10^5$

(mixed).

$\bar{h}_{0-3} = \frac{\bar{Nu}\, k}{L} = \frac{k}{L}\left(0.037\, Re^{4/5} - 871\right) Pr^{1/3}$

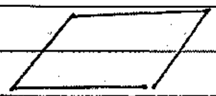
$= 6.2909$

$\bar{h}_{1-3} = \frac{1}{2}\left(3 \times \bar{h}_{0-3} - 1 \times \bar{h}_{0-1}\right)$

$= 5.42312$

ii) $\frac{\dot{Q}}{w} = \bar{h}_{1-3} \frac{A}{w} (T_s - T_\infty) = 867.699\ W/m$

3 b). $L_c = \frac{A}{P} = \frac{1 \times 1}{4} = 0.25$



$Ra_L = Gr \times Pr$

$= \frac{g \beta |T_w - T_\infty| L_c^3}{\nu^2} \times Pr$

$T_f = \frac{190 + 20}{2} = 105$

$\beta = \frac{1}{T_f} = \frac{1}{105}$

$= 318142563.9 > 10^7$

hot on top

$\bar{h} = \frac{Nu\, k}{L} = \frac{k}{L}\left(0.15\, Ra^{1/3}\right)$

$= 12.288$

$\dot{Q}_1 = \bar{h} A (190 - 20) = 2088.96$

hot on bellow

$\bar{h} = \frac{Nu\, k}{L} = \frac{k}{L}\left(0.27\, Ra^{1/4}\right) = 4.3271$

$\dot{Q}_2 = \bar{h} A (190 - 20) = 735.613$

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: http://bit.ly/2lW2C32

$$\dot{Q}_{3\,rad} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon A_1} + \frac{1}{A_1 F_{12}}}$$

$T_s = 190 + 273 = 463$

$T_{surr} = 20 + 273 = 293$

① $A_1 = A$

② $F_{12} = 1$

$$\dot{Q}_{3\,rad} = A_1 \varepsilon \sigma (T_s^4 - T_{surr}^4)$$

$$= 1 \times 1 \times 2 \times 0.8 \times 5.67 \times 10^{-8} (T_s^4 - T_{surr}^4)$$

$$\approx 3500.342\ W$$

$$\dot{Q}_{total} = \dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3$$

$$= 6324.92\ W$$

with sheld.

4 c)

$$\dot{Q} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1s}} + \frac{1-\varepsilon_s}{\varepsilon_s A_s} + \frac{1-\varepsilon_s}{\varepsilon_s A_s} + \frac{1}{A_s F_{s2}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

① $A = A_1 = A_s = A_2$

$F_{1s} = F_{s2} = 1$

$$\dot{Q} = \frac{A(E_{b1} - E_{b2})}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_s} + \frac{1}{\varepsilon_s} - 1\right)} = \frac{A(E_{b1} - E_{b2})}{2.75 + \left(\frac{2}{\varepsilon_s} - 1\right)}$$

no sheld

$$\dot{Q} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

$A_1 = A_2 = A$

$F_{12} = 1$

$$\dot{Q} = \frac{A(E_{b1} - E_{b2})}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)} = \frac{A(E_{b1} - E_{b2})}{2.75}$$

$$\frac{\dot{Q}_{sheld}}{\dot{Q}_{no\ sheld}} = \frac{\frac{1}{2.75 + \left(\frac{2}{\varepsilon_s} - 1\right)}}{\frac{1}{2.75}} = \frac{1}{10}$$

$$27.5 = 2.75 + \frac{2}{\varepsilon_s} - 1$$

$$\varepsilon_s = 0.07767$$

$\dot{q}$ constant.

4 b). $T_{mi} = 20°C$ $\quad l = 10m$

$T_{mo} = 85°C$ $\quad D = 0.015\ m$

$V = 0.8\ m/s$

$Re = \frac{\rho V D}{\mu} = \cancel{\frac{800 \times 0.8 \times 0.015}{0.008} = 1200 < 2300}$

constant $\dot{q}$

$\bar{h} = \frac{\overline{Nu}\,k}{D} = \frac{k}{D}\,4.36 = 34.88\ W/m \cdot K$ //

$\dot{q} = \bar{h}(T_s - T_m)$

$\dot{Q} = \dot{m}\,C_p\,(T_{mo} - T_{mi})$

$= \rho V \times \pi \times \left(\frac{0.015}{2}\right)^2 \times 0.15 \times 10^3\,(85 - 20)$

$= 1102.699\ W$

$\frac{\dot{Q}}{A} = \frac{1102.699}{\pi D \times l} = \cancel{2339.99999}$

$\cancel{T_s = \frac{\dot{q}}{h} + T_m}$

$= 87.0871°C$

$T_s = \frac{\dot{q}}{h} + T_m$

$= 152.087°C$