NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

MA4002– FLUID DYNAMICS

April/May 2017 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR** (4) questions and comprises **EIGHT** (8) pages including **THREE** (3) pages of Appendix.
2. Answer **ALL FOUR** questions.
3. All questions carry equal marks.
4. This is a **CLOSED-BOOK** examination.

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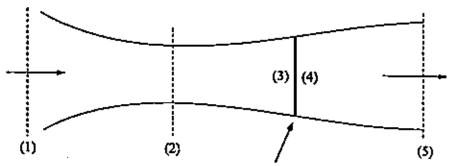
1. The nozzle is fed by air from a large tank where the pressure is $P_1 = 100\,kPa$. A stationary normal shock stands in the diverging section of a converging-diverging nozzle. The Mach number ahead of the shock is $M_3 = 2.0$. The nozzle area at the shock is $A_3 = A_4 = 100\,\text{mm}^2$ and the nozzle exit area is $A_5 = 200\,\text{mm}^2$. For air, the gas constant $R = 287\,J/kg\,K$.



Shock
Figure 1

Calculate

(a) the nozzle throat area A_2, (5 marks)

(b) the stagnation pressure P_{03}, and static pressure P_3 before the shock, (5 marks)

(c) the Mach number M_4, stagnation pressure P_{04}, and static pressure P_4 after the shock, (5 marks)

(d) the entropy change $s_4 - s_3$ across the shock, and (5 marks)

(e) the exit Mach number M_5. (5 marks)

2 (a) Consider a laminar boundary-layer flow over a flat plate aligned with the direction of a uniform oncoming free stream. The velocity profile in the boundary-layer is approximated by the parabolic distribution $\frac{u}{U} = A + B\frac{y}{\delta} + C\left(\frac{y}{\delta}\right)^2$ for y between 0 and the disturbance thickness δ. Here three boundary conditions (no slip condition, continuity with freestream and no shear stress at freestream) are applicable to the laminar boundary-layer velocity profile.

Evaluate

(i) the constants A, B and C, (5 marks)

(ii) the ratio $\frac{\delta^*}{\delta}$ of the displacement thickness $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$ to the disturbance thickness δ, and (5 marks)

(iii) the ratio $\frac{\theta}{\delta}$ of the momentum thickness $\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$ to the disturbance thickness δ. (5 marks)

(b) A small sphere is observed to fall through air at a terminal speed of 10 m/s. Given that the drag coefficient of the sphere is $C_D = 0.4$, the acceleration of gravity is $g = 9.8$ m/s^2, the density of air is $\rho = 1.225$ kg/m^3 and the density of the sphere is $\rho_s = 1000$ kg/m^3,

determine

(i) the diameter D of the sphere, and (5 marks)

(ii) the drag force F_D of the air acting on the sphere. (5 marks)

3 (a) In a two-dimensional incompressible flow field, the velocity vector is given by $\vec{V}=(Ax-By)\vec{i}-Ay\vec{j}$; where $A=2s^{-1}$, $B=1s^{-1}$, x and y are in metres. The density of the fluid is $\rho=1000$ kg/m^3 and $\vec{g}=-g\vec{j}$.

(i) Find the stream function of the flow.

(ii) Sketch the stream lines where $\psi=0$.

(iii) Find $\vec{\omega}$, is this an irrotational flow field? If the answer is yes, determine the potential function $\phi(x,y)$, if no, explain why.

(iv) If the pressure at point $C(4,1)$ is 200 kPa, find the pressure at point $D(2,5)$.

Justify the equation used.

Note: $\vec{\omega}=0.5\nabla\times\vec{V}$, $u=\dfrac{\partial\psi}{\partial y}, v=-\dfrac{\partial\psi}{\partial x}$; $u=\dfrac{\partial\phi}{\partial x}, v=\dfrac{\partial\phi}{\partial y}$, and

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+y_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+y_2.$$

(11 marks)

(b) Engine oil at 60 °C (with density, ρ = 863.9 kg/m^3 and dynamic viscosity, μ = 0.07399 kg/(m·s)) is forced to flow between two very large, stationary, parallel flat plates separated by a thin gap height h =3.50 mm as shown in Figure 2. The plate dimensions are L = 1.50 m and W = 0.90 m. Since $W >> h$ and $L>>h$, it can be assumed that $u=u(y)$, and $v=w=0$. Solve the Navier Stokes equation and show that the fluid velocity distribution $u=\dfrac{1}{2\mu}\dfrac{\partial p}{\partial x}\left(y^2-hy\right)$ with the following two boundary conditions, (i) $u=0$, at $y=0$; (ii) $u=0$, at $y=h$, under no slip condition. State clearly the reasons in reducing the Navier Stokes equations to a single differential equation. If the outlet pressure is atmospheric, and the inlet pressure is 2 atmospheric gage pressure (Note that absolute pressure = gage pressure + atmospheric pressure, you may assume atmospheric pressure as 101 kPa), estimate the volume flow rate ($q=\int_0^h uWdy$) of oil.

(7 marks)

Note: Question 3 continues on page 4.
Figure 2 appears on page 4.

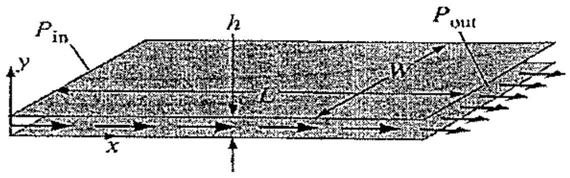


Figure 2

Note that $\rho\left(\dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}+w\dfrac{\partial u}{\partial z}\right)=-\dfrac{\partial p}{\partial x}+\rho g_x+\mu\left(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}\right)$

(c) A flow field consists of a uniform flow of 4 m/s and inclined 30° with the positive x-axis measured in anticlockwise direction, a sink of strength 25 m^2/s is located at A(5, 0), and a free vortex of strength K = 20 m^2/s is rotating in clockwise direction with its center located at B(-4,0). Find the resultant velocity (both magnitude and angle with x-axis) at point C(0, -6).

Note: For source and sink, the velocity components are $v_r=\frac{m}{2\pi r}$ and $v_\theta=0$ where $m > 0$ for source and $m < 0$ for sink. For free vortex, the velocity components are $v_r=0$ and $v_\theta=\dfrac{K}{r}$.

(7 marks)

4 (a) A vertical and truncated conical shaped tank has a height of 6 m. The diametres of the cone at the bottom and top of the tank are 1.5 m and 4.5 m respectively. The tank is filled to a depth of 5.0 m with oil of density 810 kg/m^3. Determine the time to reduce the depth of oil in the tank by 3.5 m if it is drained through a 4.5 m long, 25 mm diameter pipe discharging 3.0 m below the base of the tank and having a friction factor of 0.05 and the total loss coefficient is 2.0.

Note that $\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+f\frac{L}{D}\frac{V^2}{2g}+k\frac{V^2}{2g}$ and $t=\int_{z_1}^{z_2}\frac{A_s}{Q_i-Q_o}dz$

(12 marks)

(b) A centrifugal pump is being designed to pump liquid refrigerant R-134a with density of 1226 kg/m^3 at room temperature and atmospheric pressure. The pump with backward-facing impeller has inlet and outlet radii of 100 mm and 180 mm respectively. The corresponding impeller widths of inlet and outlet are 50 mm and 30 mm respectively. The pump is to deliver 0.25 m^3/s of the liquid at an ideal head of 14.5 m when the impeller rotates at 1720 rpm, and the flow enters the pump axially.

(i) Draw the velocity diagram at both the inlet and outlet of the pump, and indicate clearly all the velocity components and their direction, including the rotating direction and blade shape.

(ii) Evaluate the blade inlet and outlet angles, i.e. β_1 and β_2.

(iii) Find the torque and power supply to the shaft of the pump.

(13 marks)

Appendix

Isentropic Flow Functions (one-dimensional flow, ideal gas, k = 1.4)

M	*T/T₀*	*p/p₀*	*ρ/ρ₀*	*A/A**
0.00	1.0000	1.0000	1.0000	∞
0.01	1.0000	0.9999	1.0000	57.8738
0.02	0.9999	0.9997	0.9998	28.9421
0.03	0.9998	0.9994	0.9996	19.3005
0.04	0.9997	0.9989	0.9992	14.4815
0.05	0.9995	0.9983	0.9988	11.5914
0.06	0.9993	0.9975	0.9982	9.6659
0.07	0.9990	0.9966	0.9976	8.2915
0.08	0.9987	0.9955	0.9968	7.2616
0.09	0.9984	0.9944	0.9960	6.4613
0.10	0.9980	0.9930	0.9950	5.8218
0.11	0.9976	0.9916	0.9940	5.2992
0.12	0.9971	0.9900	0.9928	4.8643
0.13	0.9966	0.9883	0.9916	4.4969
0.14	0.9961	0.9864	0.9903	4.1824
0.15	0.9955	0.9844	0.9888	3.9103
0.16	0.9949	0.9823	0.9873	3.6727
0.17	0.9943	0.9800	0.9857	3.4635
0.18	0.9936	0.9776	0.9840	3.2779
0.19	0.9928	0.9751	0.9822	3.1123
0.20	0.9921	0.9725	0.9803	2.9635
0.21	0.9913	0.9697	0.9783	2.8293
0.22	0.9904	0.9668	0.9762	2.7076
0.23	0.9895	0.9638	0.9740	2.5968
0.24	0.9886	0.9607	0.9718	2.4956
0.25	0.9877	0.9575	0.9694	2.4027
0.26	0.9867	0.9541	0.9670	2.3173
0.27	0.9856	0.9506	0.9645	2.2385
0.28	0.9846	0.9470	0.9619	2.1656
0.29	0.9835	0.9433	0.9592	2.0979
0.30	0.9823	0.9395	0.9564	2.0351
0.31	0.9811	0.9355	0.9535	1.9765
0.32	0.9799	0.9315	0.9506	1.9219
0.33	0.9787	0.9274	0.9476	1.8707
0.34	0.9774	0.9231	0.9445	1.8229
0.35	0.9761	0.9188	0.9413	1.7780
0.36	0.9747	0.9143	0.9380	1.7358
0.37	0.9733	0.9098	0.9347	1.6961
0.38	0.9719	0.9052	0.9313	1.6587
0.39	0.9705	0.9004	0.9278	1.6234
0.40	0.9690	0.8956	0.9243	1.5901
0.41	0.9675	0.8907	0.9207	1.5587
0.42	0.9659	0.8857	0.9170	1.5289
0.43	0.9643	0.8807	0.9132	1.5007
0.44	0.9627	0.8755	0.9094	1.4740
0.45	0.9611	0.8703	0.9055	1.4487
0.46	0.9594	0.8650	0.9016	1.4246
0.47	0.9577	0.8596	0.8976	1.4018
0.48	0.9559	0.8541	0.8935	1.3801
0.49	0.9542	0.8486	0.8894	1.3595
0.50	0.9524	0.8430	0.8852	1.3398
0.51	0.9506	0.8374	0.8809	1.3212
0.52	0.9487	0.8317	0.8766	1.3034
0.53	0.9468	0.8259	0.8723	1.2865
0.54	0.9449	0.8201	0.8679	1.2703
0.55	0.9430	0.8142	0.8634	1.2549
0.56	0.9410	0.8082	0.8589	1.2403
0.57	0.9390	0.8022	0.8544	1.2263
0.58	0.9370	0.7962	0.8498	1.2130
0.59	0.9349	0.7901	0.8451	1.2003
0.60	0.9328	0.7840	0.8405	1.1882
0.61	0.9307	0.7778	0.8357	1.1767
0.62	0.9286	0.7716	0.8310	1.1656
0.63	0.9265	0.7654	0.8262	1.1552
0.64	0.9243	0.7591	0.8213	1.1451
0.65	0.9221	0.7528	0.8164	1.1356
0.66	0.9199	0.7465	0.8115	1.1265
0.67	0.9176	0.7401	0.8066	1.1179
0.68	0.9153	0.7338	0.8016	1.1097
0.69	0.9131	0.7274	0.7966	1.1018
0.70	0.9107	0.7209	0.7916	1.0944
0.71	0.9084	0.7145	0.7865	1.0873
0.72	0.9061	0.7080	0.7814	1.0806
0.73	0.9037	0.7016	0.7763	1.0742
0.74	0.9013	0.6951	0.7712	1.0681
0.75	0.8989	0.6886	0.7660	1.0624
0.76	0.8964	0.6821	0.7609	1.0570
0.77	0.8940	0.6756	0.7557	1.0519
0.78	0.8915	0.6691	0.7505	1.0471
0.79	0.8890	0.6625	0.7452	1.0425
0.80	0.8865	0.6560	0.7400	1.0382
0.81	0.8840	0.6495	0.7347	1.0342
0.82	0.8815	0.6430	0.7295	1.0305
0.83	0.8789	0.6365	0.7242	1.0270
0.84	0.8763	0.6300	0.7189	1.0237
0.85	0.8737	0.6235	0.7136	1.0207
0.86	0.8711	0.6170	0.7083	1.0179
0.87	0.8685	0.6106	0.7030	1.0153
0.88	0.8659	0.6041	0.6977	1.0129
0.89	0.8632	0.5977	0.6924	1.0108
0.90	0.8606	0.5913	0.6870	1.0089
0.91	0.8579	0.5849	0.6817	1.0071
0.92	0.8552	0.5785	0.6764	1.0056
0.93	0.8525	0.5721	0.6711	1.0043
0.94	0.8498	0.5658	0.6658	1.0031
0.95	0.8471	0.5595	0.6604	1.0021
0.96	0.8444	0.5532	0.6551	1.0014
0.97	0.8416	0.5469	0.6498	1.0008
0.98	0.8389	0.5407	0.6445	1.0003
0.99	0.8361	0.5345	0.6392	1.0001
1.00	0.8333	0.5283	0.6339	1.0000
1.01	0.8306	0.5221	0.6287	1.0001
1.02	0.8278	0.5160	0.6234	1.0003
1.03	0.8250	0.5099	0.6181	1.0007
1.04	0.8222	0.5039	0.6129	1.0013
1.05	0.8193	0.4979	0.6077	1.0020

Isentropic Flow Functions (one-dimensional flow, ideal gas, k = 1.4)

M	T/T_0	p/p_0	ρ/ρ_0	A/A^*
1.06	0.8165	0.4919	0.6024	1.0029
1.07	0.8137	0.4860	0.5972	1.0039
1.08	0.8108	0.4800	0.5920	1.0051
1.09	0.8080	0.4742	0.5869	1.0064
1.10	0.8052	0.4684	0.5817	1.0079
1.11	0.8023	0.4626	0.5766	1.0095
1.12	0.7994	0.4568	0.5714	1.0113
1.13	0.7966	0.4511	0.5663	1.0132
1.14	0.7937	0.4455	0.5612	1.0153
1.15	0.7908	0.4398	0.5562	1.0175
1.16	0.7879	0.4343	0.5511	1.0198
1.17	0.7851	0.4287	0.5461	1.0222
1.18	0.7822	0.4232	0.5411	1.0248
1.19	0.7793	0.4178	0.5361	1.0276
1.20	0.7764	0.4124	0.5311	1.0304
1.21	0.7735	0.4070	0.5262	1.0334
1.22	0.7706	0.4017	0.5213	1.0366
1.23	0.7677	0.3964	0.5164	1.0398
1.24	0.7648	0.3912	0.5115	1.0432
1.25	0.7619	0.3861	0.5067	1.0468
1.26	0.7590	0.3809	0.5019	1.0504
1.27	0.7561	0.3759	0.4971	1.0542
1.28	0.7532	0.3708	0.4923	1.0581
1.29	0.7503	0.3658	0.4876	1.0621
1.30	0.7474	0.3609	0.4829	1.0663
1.31	0.7445	0.3560	0.4782	1.0706
1.32	0.7416	0.3512	0.4736	1.0750
1.33	0.7387	0.3464	0.4690	1.0796
1.34	0.7358	0.3417	0.4644	1.0842
1.35	0.7329	0.3370	0.4598	1.0890
1.36	0.7300	0.3323	0.4553	1.0940
1.37	0.7271	0.3277	0.4508	1.0990
1.38	0.7242	0.3232	0.4463	1.1042
1.39	0.7213	0.3187	0.4418	1.1095
1.40	0.7184	0.3142	0.4374	1.1149
1.41	0.7155	0.3098	0.4330	1.1205
1.42	0.7126	0.3055	0.4287	1.1262
1.43	0.7097	0.3012	0.4244	1.1320
1.44	0.7069	0.2969	0.4201	1.1379
1.45	0.7040	0.2927	0.4158	1.1440
1.46	0.7011	0.2886	0.4116	1.1501
1.47	0.6982	0.2845	0.4074	1.1565
1.48	0.6954	0.2804	0.4032	1.1629
1.49	0.6925	0.2764	0.3991	1.1695
1.50	0.6897	0.2724	0.3950	1.1762
1.51	0.6868	0.2685	0.3909	1.1830
1.52	0.6840	0.2646	0.3869	1.1899
1.53	0.6811	0.2608	0.3829	1.1970
1.54	0.6783	0.2570	0.3789	1.2042
1.55	0.6754	0.2533	0.3750	1.2116
1.56	0.6726	0.2496	0.3710	1.2190
1.57	0.6698	0.2459	0.3672	1.2266
1.58	0.6670	0.2423	0.3633	1.2344

M	T/T_0	p/p_0	ρ/ρ_0	A/A^*
1.59	0.6642	0.2388	0.3595	1.2422
1.60	0.6614	0.2353	0.3557	1.2502
1.61	0.6586	0.2318	0.3520	1.2584
1.62	0.6558	0.2284	0.3483	1.2666
1.63	0.6530	0.2250	0.3446	1.2750
1.64	0.6502	0.2217	0.3409	1.2836
1.65	0.6475	0.2184	0.3373	1.2922
1.66	0.6447	0.2151	0.3337	1.3010
1.67	0.6419	0.2119	0.3302	1.3100
1.68	0.6392	0.2088	0.3266	1.3190
1.69	0.6364	0.2057	0.3232	1.3283
1.70	0.6337	0.2026	0.3197	1.3376
1.71	0.6310	0.1996	0.3163	1.3471
1.72	0.6283	0.1966	0.3129	1.3567
1.73	0.6256	0.1936	0.3095	1.3665
1.74	0.6229	0.1907	0.3062	1.3764
1.75	0.6202	0.1878	0.3029	1.3865
1.76	0.6175	0.1850	0.2996	1.3967
1.77	0.6148	0.1822	0.2964	1.4070
1.78	0.6121	0.1794	0.2931	1.4175
1.79	0.6095	0.1767	0.2900	1.4282
1.80	0.6068	0.1740	0.2868	1.4390
1.81	0.6041	0.1714	0.2837	1.4499
1.82	0.6015	0.1688	0.2806	1.4610
1.83	0.5989	0.1662	0.2776	1.4723
1.84	0.5963	0.1637	0.2745	1.4836
1.85	0.5936	0.1612	0.2715	1.4952
1.86	0.5910	0.1587	0.2686	1.5069
1.87	0.5884	0.1563	0.2656	1.5187
1.88	0.5859	0.1539	0.2627	1.5308
1.89	0.5833	0.1516	0.2598	1.5429
1.90	0.5807	0.1492	0.2570	1.5553
1.91	0.5782	0.1470	0.2542	1.5677
1.92	0.5756	0.1447	0.2514	1.5804
1.93	0.5731	0.1425	0.2486	1.5932
1.94	0.5705	0.1403	0.2459	1.6062
1.95	0.5680	0.1381	0.2432	1.6193
1.96	0.5655	0.1360	0.2405	1.6326
1.97	0.5630	0.1339	0.2378	1.6461
1.98	0.5605	0.1318	0.2352	1.6597
1.99	0.5580	0.1298	0.2326	1.6735
2.00	0.5556	0.1278	0.2300	1.6875
2.01	0.5531	0.1258	0.2275	1.7016
2.02	0.5506	0.1239	0.2250	1.7160
2.03	0.5482	0.1220	0.2225	1.7305
2.04	0.5458	0.1201	0.2200	1.7451
2.05	0.5433	0.1182	0.2176	1.7600
2.06	0.5409	0.1164	0.2152	1.7750
2.07	0.5385	0.1146	0.2128	1.7902
2.08	0.5361	0.1128	0.2104	1.8056
2.09	0.5337	0.1111	0.2081	1.8212
2.10	0.5313	0.1094	0.2058	1.8369
2.11	0.5290	0.1077	0.2035	1.8529

Normal-Shock Flow Functions (one-dimensional, ideal gas, k = 1.4)

M_1	M_2	p_{02}/p_{01}	T_2/T_1	p_2/p_1	ρ_2/ρ_1
1.00	1.0000	1.0000	1.0000	1.0000	1.0000
1.01	0.9901	1.0000	1.0066	1.0235	1.0167
1.02	0.9805	1.0000	1.0132	1.0471	1.0334
1.03	0.9712	1.0000	1.0198	1.0711	1.0502
1.04	0.9620	0.9999	1.0263	1.0952	1.0671
1.05	0.9531	0.9999	1.0328	1.1196	1.0840
1.06	0.9444	0.9998	1.0393	1.1442	1.1009
1.07	0.9360	0.9996	1.0458	1.1691	1.1179
1.08	0.9277	0.9994	1.0522	1.1941	1.1349
1.09	0.9196	0.9992	1.0586	1.2195	1.1520
1.10	0.9118	0.9989	1.0649	1.2450	1.1691
1.11	0.9041	0.9986	1.0713	1.2708	1.1862
1.12	0.8966	0.9982	1.0776	1.2968	1.2034
1.13	0.8892	0.9978	1.0840	1.3231	1.2206
1.14	0.8820	0.9973	1.0903	1.3495	1.2378
1.15	0.8750	0.9967	1.0966	1.3763	1.2550
1.16	0.8682	0.9961	1.1029	1.4032	1.2723
1.17	0.8615	0.9953	1.1092	1.4304	1.2896
1.18	0.8549	0.9946	1.1154	1.4578	1.3069
1.19	0.8485	0.9937	1.1217	1.4855	1.3243
1.20	0.8422	0.9928	1.1280	1.5133	1.3416
1.21	0.8360	0.9918	1.1343	1.5415	1.3590
1.22	0.8300	0.9907	1.1405	1.5698	1.3764
1.23	0.8241	0.9896	1.1468	1.5984	1.3938
1.24	0.8183	0.9884	1.1531	1.6272	1.4112
1.25	0.8126	0.9871	1.1594	1.6563	1.4286
1.26	0.8071	0.9857	1.1657	1.6855	1.4460
1.27	0.8016	0.9842	1.1720	1.7151	1.4634
1.28	0.7963	0.9827	1.1783	1.7448	1.4808
1.29	0.7911	0.9811	1.1846	1.7748	1.4983
1.30	0.7860	0.9794	1.1909	1.8050	1.5157
1.31	0.7809	0.9776	1.1972	1.8355	1.5331
1.32	0.7760	0.9758	1.2035	1.8661	1.5505
1.33	0.7712	0.9738	1.2099	1.8971	1.5680
1.34	0.7664	0.9718	1.2162	1.9282	1.5854
1.35	0.7618	0.9697	1.2226	1.9596	1.6028
1.36	0.7572	0.9676	1.2290	1.9912	1.6202
1.37	0.7527	0.9653	1.2354	2.0231	1.6376
1.38	0.7483	0.9630	1.2418	2.0551	1.6549
1.39	0.7440	0.9607	1.2482	2.0875	1.6723
1.40	0.7397	0.9582	1.2547	2.1200	1.6897
1.41	0.7355	0.9557	1.2612	2.1528	1.7070
1.42	0.7314	0.9531	1.2676	2.1858	1.7243
1.43	0.7274	0.9504	1.2741	2.2191	1.7416
1.44	0.7235	0.9476	1.2807	2.2525	1.7589
1.45	0.7196	0.9448	1.2872	2.2863	1.7761
1.46	0.7157	0.9420	1.2938	2.3202	1.7934
1.47	0.7120	0.9390	1.3003	2.3544	1.8106
1.48	0.7083	0.9360	1.3069	2.3888	1.8278
1.49	0.7047	0.9329	1.3136	2.4235	1.8449
1.50	0.7011	0.9298	1.3202	2.4583	1.8621
1.51	0.6976	0.9266	1.3269	2.4935	1.8792
1.52	0.6941	0.9233	1.3336	2.5288	1.8963
1.53	0.6907	0.9200	1.3403	2.5644	1.9133
1.54	0.6874	0.9166	1.3470	2.6002	1.9303
1.55	0.6841	0.9132	1.3538	2.6363	1.9473
1.56	0.6809	0.9097	1.3606	2.6725	1.9643
1.57	0.6777	0.9062	1.3674	2.7091	1.9812
1.58	0.6746	0.9026	1.3742	2.7458	1.9981
1.59	0.6715	0.8989	1.3811	2.7828	2.0149
1.60	0.6684	0.8952	1.3880	2.8200	2.0317
1.61	0.6655	0.8915	1.3949	2.8575	2.0485
1.62	0.6625	0.8877	1.4018	2.8951	2.0653
1.63	0.6596	0.8838	1.4088	2.9331	2.0820
1.64	0.6568	0.8799	1.4158	2.9712	2.0986
1.65	0.6540	0.8760	1.4228	3.0096	2.1152
1.66	0.6512	0.8720	1.4299	3.0482	2.1318
1.67	0.6485	0.8680	1.4369	3.0871	2.1484
1.68	0.6458	0.8639	1.4440	3.1261	2.1649
1.69	0.6431	0.8599	1.4512	3.1655	2.1813
1.70	0.6405	0.8557	1.4583	3.2050	2.1977
1.71	0.6380	0.8516	1.4655	3.2448	2.2141
1.72	0.6355	0.8474	1.4727	3.2848	2.2304
1.73	0.6330	0.8431	1.4800	3.3251	2.2467
1.74	0.6305	0.8389	1.4873	3.3655	2.2629
1.75	0.6281	0.8346	1.4946	3.4063	2.2791
1.76	0.6257	0.8302	1.5019	3.4472	2.2952
1.77	0.6234	0.8259	1.5093	3.4884	2.3113
1.78	0.6210	0.8215	1.5167	3.5298	2.3273
1.79	0.6188	0.8171	1.5241	3.5715	2.3433
1.80	0.6165	0.8127	1.5316	3.6133	2.3592
1.81	0.6143	0.8082	1.5391	3.6555	2.3751
1.82	0.6121	0.8038	1.5466	3.6978	2.3909
1.83	0.6099	0.7993	1.5541	3.7404	2.4067
1.84	0.6078	0.7948	1.5617	3.7832	2.4224
1.85	0.6057	0.7902	1.5693	3.8263	2.4381
1.86	0.6036	0.7857	1.5770	3.8695	2.4537
1.87	0.6016	0.7811	1.5847	3.9131	2.4693
1.88	0.5996	0.7765	1.5924	3.9568	2.4848
1.89	0.5976	0.7720	1.6001	4.0008	2.5003
1.90	0.5956	0.7674	1.6079	4.0450	2.5157
1.91	0.5937	0.7627	1.6157	4.0895	2.5310
1.92	0.5918	0.7581	1.6236	4.1341	2.5463
1.93	0.5899	0.7535	1.6314	4.1791	2.5616
1.94	0.5880	0.7488	1.6394	4.2242	2.5767
1.95	0.5862	0.7442	1.6473	4.2696	2.5919
1.96	0.5844	0.7395	1.6553	4.3152	2.6069
1.97	0.5826	0.7349	1.6633	4.3611	2.6220
1.98	0.5808	0.7302	1.6713	4.4071	2.6369
1.99	0.5791	0.7255	1.6794	4.4535	2.6518
2.00	0.5774	0.7209	1.6875	4.5000	2.6667
2.01	0.5757	0.7162	1.6956	4.5468	2.6815
2.02	0.5740	0.7115	1.7038	4.5938	2.6962
2.03	0.5723	0.7069	1.7120	4.6411	2.7109
2.04	0.5707	0.7022	1.7203	4.6885	2.7255
2.05	0.5691	0.6975	1.7285	4.7363	2.7400

END OF PAPER

$\boxed{1}$ (a) $P_1 = 100\ kPa$ $\quad M_2 = 1$ $\quad A_3 = A_4 = 100\ mm^2$

$M_1 = 0$ $\quad M_3 = 2$ $\quad A_5 = 200\ mm^2$

1 → 3 isentropic

$\frac{A_3}{A_3^*} = 1.6875$ $\qquad A_2 = A_3^* = 59.26\ mm^2$

$A_3^* = 59.26\ mm^2$

(b) $P_1 = P_{01} = P_{03}$

$\frac{P_3}{P_{03}} = 0.1278$

$P_3 = 12.78\ kPa$

(c) $M_3 = 2 \rightarrow M_4 = 0.5774$ from normal shock table

$\frac{P_4}{P_3} = 4.5$ $\qquad \frac{P_4}{P_{04}} = 0.7962$

$P_4 = 57.51\ kPa$ $\qquad P_{04} = 72.23\ kPa$

(d) $\frac{T_4}{T_3} = 1.6875$

$S_4 - S_3 = c_p \ln \frac{T_4}{T_3} - R \ln \frac{P_4}{P_3} = 1004 \ln(1.6875) - 287 \ln(4.5) = 93.67$

(e) $\frac{A_4}{A_4^*} = 1.213$

$A_4^* = 82.44$

$\frac{A_5}{A_4^*} = 2.426 \rightarrow M_5 = 0.25$

$\boxed{2}$ (a) (i) Boundary condition:

① $u = 0$ when $y = 0$

② $u = U$ when $y = \delta$

③ $\frac{du}{dy} = 0$ when $y = \delta$

let $\eta = y/\delta$

$dy = \delta\, d\eta$

$$\frac{u}{U} = A + B\frac{y}{\delta} + C\left(\frac{y}{\delta}\right)^2$$

① $A = 0$

② $1 = B + C$

③ $\frac{du}{dy} = U\left[\frac{B}{\delta} + C\frac{2y}{\delta^2}\right]$

$0 = \frac{B}{\delta} + C\frac{2}{\delta}$

$0 = B + 2C$

$B = -2C$

substitute ③ to ②

$1 = -2C + C$

$C = -1$

$B = 2$

(ii) $\delta^* = \delta \int_0^1 \left(1 - \frac{u}{U}\right) d\eta$, $\frac{u}{U} = 2\eta - \eta^2$

$\frac{\delta^*}{\delta} = \int_0^1 (1 - 2\eta + \eta^2)\, d\eta$

$= \left[\eta - \eta^2 + \frac{1}{3}\eta^3\right]_0^1$

$= 1 - 1 + \frac{1}{3} = \frac{1}{3}$

(iii) $\theta = \delta \int_0^1 (2\eta - \eta^2)(1 - 2\eta + \eta^2)\, d\eta$

$= \delta \int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4)\, d\eta$

$= \delta\left[\eta^2 - \frac{5}{2}\eta^3 + \eta^4 - \frac{1}{5}\eta^5\right]_0^1$

$\frac{\theta}{\delta} = 1 - \frac{5}{3} + 1 - \frac{1}{5} = \frac{2}{15}$

$\boxed{2}$ (b) (i) $F_D = \frac{1}{2} C_D \rho_a V^2 A$

$$mg = F_D$$

$$\rho \forall g = \frac{1}{2} C_D \rho_a V^2 A$$

$$\rho\left(\frac{4}{3}\pi \frac{D^3}{8}\right) g = \frac{1}{2} C_D \rho_a V^2 \left(\frac{\pi D^2}{4}\right)$$

$$D = \frac{3}{4} \frac{C_D \rho_a V^2}{\rho g} = \frac{3}{4} \frac{0.4(1.225)(10)^2}{1000(9.81)} = 3.746 \times 10^{-3} \text{ m}$$

(ii) $F_D = \frac{1}{2} C_D \rho V^2 A = \frac{1}{2}(0.4)(1.225)(10)^2\left(\frac{\pi}{4}(3.746 \times 10^{-3})^2\right)$

$= 2.7 \times 10^{-4}$ N

$\boxed{3}$ (a) (i) $\vec{V} = (Ax - By)i - Ay j$

$u = 2x - y$ $\quad\quad v = -2y$

$u = \partial\psi / \partial y$ $\quad\quad v = -\partial\psi / \partial x$

$\psi = 2xy - \frac{1}{2}y^2 + f(x)$ $\quad\quad \psi = 2xy + f(y)$

$$\psi = 2xy - \frac{1}{2}y^2$$

(ii) $\psi = 0$

$0 = 2xy - \frac{1}{2}y^2$

$4xy = y^2$

$0 = y(y - 4x)$

$y = 4x$ or $y = 0$

$v = -2y$

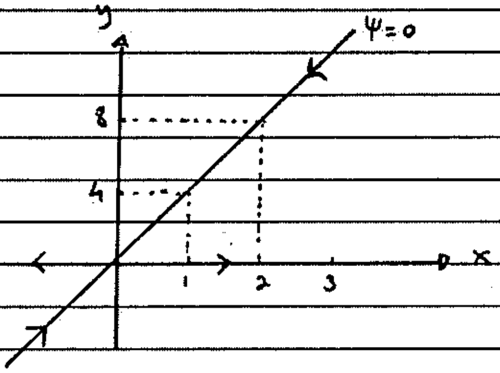
1st Quadrant $y = +ve, v = -ve$

3rd " $y = -ve, v = +ve$

$u = 2x - y$, $y = 0$

$x = +ve$, $u = +ve$

$x = -ve$, $u = -ve$



(iii) $\vec{\omega} = \frac{1}{2} \nabla \times \vec{V} = \frac{1}{2} \left[\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ u & v & w \end{vmatrix} \right] = \frac{1}{2} \left(\frac{dv}{dx} - \frac{du}{dy} \right) k$

$= \frac{1}{2} (0 - (-1)) \hat{k} = \frac{1}{2} \hat{k}$ → rotational flow

3 (a) (iv) C(4,1)

$\psi_C = 2(4)(1) - \frac{1}{2}(1) = 7.5$

$P_C = 200\ kPa$

$y_C = 1$

$\vec{V} = 7i - 2j$

$|\vec{V}_C| = 7.28$

$V_C^2 = 53$

D(2,5)

$\psi_D = 2(2)(5) - \frac{1}{2}(5)^2 = 7.5$

$y_D = 5$

$\vec{V}_D = -1i - 10j$

$|\vec{V}_D| = 10.05$

$V_D^2 = 101$

assumption :

- same streamline
- inviscid
- incompressible

  $\frac{du}{dx} + \frac{dv}{dy} = 2 - 2 = 0$
- steady flow

} Bernalli equation valid

$$\frac{P_C}{\rho g} + \frac{V_C^2}{2g} + y_C = \frac{P_D}{\rho g} + \frac{V_D^2}{2g} + y_D$$

$$\frac{P_D}{\rho g} = 20.34 + 2.7 + 1 - 5.15 - 5 = 13.89$$

$$P_D = 136.26\ KPa$$

$\boxed{3}$ (b)

$$\rho\left(\underset{①}{\cancel{\frac{\partial u}{\partial t}}}+\underset{②}{\cancel{u\frac{\partial u}{\partial x}}}+\underset{③}{\cancel{v}}\frac{\partial u}{\partial y}+\underset{③}{\cancel{w}}\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\underset{④}{\cancel{\rho g_x}}+\mu\left(\underset{②}{\cancel{\frac{\partial^2 u}{\partial x^2}}}+\frac{\partial^2 u}{\partial y^2}+\underset{②}{\cancel{\frac{\partial^2 u}{\partial z^2}}}\right)$$

① steady flow

② $u = f(y)$

③ $v = 0,\ w = 0$

④ $g = g_y j$

$$0 = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{1}{\mu}\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2}$$

$$\frac{1}{\mu}\frac{\partial p}{\partial x}y + C = \frac{\partial u}{\partial y}$$

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}y^2 + Cy + D$$

B.C.

- $u = 0,\ y = 0$

  $D = 0$

- $u = 0,\ y = h$

  $0 = \frac{1}{2\mu}\frac{\partial p}{\partial x}h^2 + Ch$

  $C = -\frac{1}{2\mu}\frac{\partial p}{\partial x}h$

$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - hy)$

$P_{in} = 2\ P_{atm}$ (gage)

$P_{in}(abs) = 3$ atm

$\frac{\partial p}{\partial x} = \frac{(101) - 3(101)}{1.5} = -134.67$ kPa/m

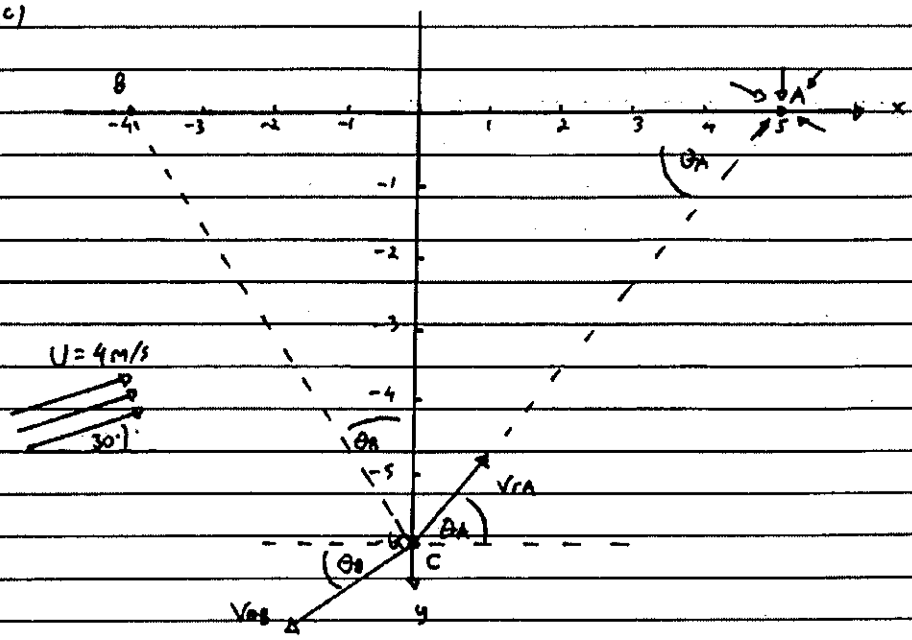
$$q = \frac{1}{2\mu}\frac{\partial p}{\partial x}W\int_0^h (y^2 - hy)\,dy$$

$$= \frac{1}{2(0.074)}(-134666)(0.9)\left[\frac{1}{3}y^3 - \frac{h}{2}y^2\right]_0^h$$

$$= -818918.9\left[\frac{1}{3}(0.0035)^3 - \frac{1}{2}(0.0035)^3\right]$$

$$= 5.85\times10^{-3}\ m^3/s$$

$\boxed{3}$ (c)



$r_B = \sqrt{4^2 + 6^2} = 7.21$

$\theta_B = \tan^{-1}\left(\frac{4}{6}\right) = 33.7°$

$r_A = \sqrt{5^2 + 6^2} = 7.81$

$\theta_A = \tan^{-1}\left(\frac{6}{5}\right) = 50.2°$

$V_{\theta B} = \frac{K}{r_B} = \frac{20}{7.21} = 2.77\ m/s$

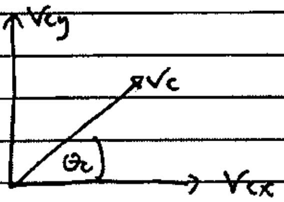
$V_{rA} = \frac{25}{2\pi r_A} = 0.51\ m/s$

$V_{cx} = U\cos 30 + V_{rA}\cos\theta_A - V_{\theta B}\cos\theta_B$

$= 4\cos 30 + 0.51\cos 50.2 - 2.77\cos 33.7$

$= 1.486\ m/s$

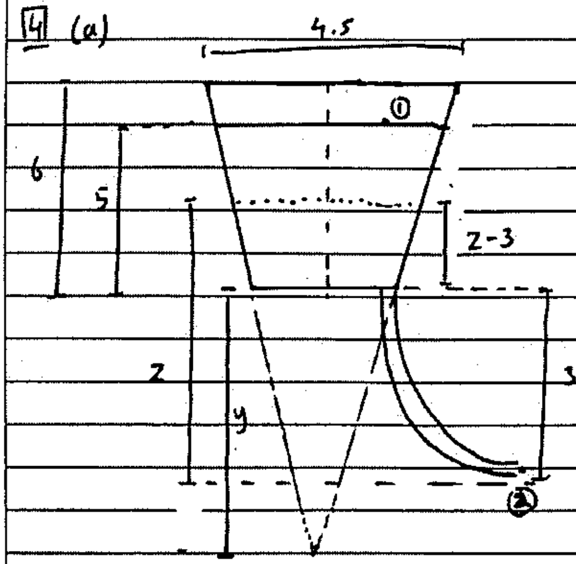
$V_{cy} = U\sin 30 + V_{rA}\sin\theta_A - V_{\theta B}\sin\theta_B = 0.855\ m/s$



$V_c = 1.71\ m/s$

$\theta_c = \tan^{-1}\left(\frac{V_{cy}}{V_{cx}}\right) = 29.9°$

$\boxed{4}$ (a)



$\rho = 810$ $\qquad A_s = \frac{\pi}{4} D^2$

$z_1 = 8$

$z_1' = 4.5$

$z_2 = 0$

$L = 4.5$

$d = 0.025$

$f = 0.05$

$k = 2$

by similar triangle,

$$\frac{6+y}{4.5} = \frac{y}{1.5} \qquad \frac{6+y}{4.5} = \frac{z-3+y}{D}$$

$$y = 3 \qquad D = 0.5z$$

$$A_s = 0.196 z^2$$

$$\cancelto{0}{\frac{P_1}{\rho g}} + \cancelto{0}{\frac{V_1^2}{2g}} + z_1 = \cancelto{0}{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + f\frac{L}{D}\frac{V_2^2}{2g} + k\frac{V_2^2}{2g} + \cancelto{0}{z_2}$$

$$z = \left(1 + f\frac{L}{D} + k\right)\frac{V_2^2}{2g}$$

$$V_2^2 = \frac{2 \times 9.81}{12} z = 1.635 z$$

$$Q_0 = V_2 A_2 = \sqrt{1.635 z}\ \frac{\pi}{4} d^2 = 6.27 \times 10^{-4} \sqrt{z}$$

$$t = \int_{z_1}^{z_1'} \frac{0.196 z^2}{-6.27 \times 10^{-4} \sqrt{z}}\, dz = -312.6 \int_{z_1}^{z_1'} z^{3/2}\, dz$$

$$= -312.6 \left[\tfrac{2}{5} z^{5/2}\right]_8^{4.5}$$

$$= -125.04\,(42.96 - 181.02)$$

$$= 17263.02\ \text{s}$$

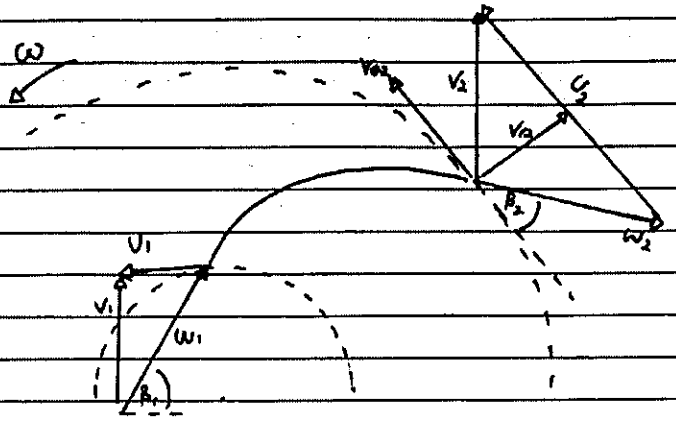
$$= 4.79\ \text{hour}$$

$\boxed{4}$ $\rho = 1226$ $Q = 0.25$

$r_1 = 100mm$ $b_1 = 50mm$ $h_i = 19.5$

$r_2 = 180\ mm$ $b_2 = 30mm$ $\omega = 1720\ rpm = 180.12\ rad/s$

(i)



(ii) $U_1 = \omega r_1 = 18.012$

$U_2 = \omega r_2 = 32.42$

$$V_{r2} = \frac{Q}{2\pi r_2 b_2} = \frac{0.25}{2\pi(0.18)(0.03)} = 7.37$$

$$V_{r1} = V_1 = \frac{Q}{2\pi r_1 b_1} = \frac{0.25}{2\pi(0.1)(0.05)} = 7.96$$

$$h_i = \frac{\dot{W}}{\dot{m} g} = \frac{\dot{m}(U_2 V_{\theta 2} - U_1 \cancelto{0}{V_{\theta 1}})}{\dot{m} g}$$

$$V_{\theta 2} = \frac{g h_i}{U_2} = 4.387$$

$$\beta_2 = \tan^{-1}\left(\frac{V_{r2}}{U_2 - V_{\theta 2}}\right) = \tan^{-1}\left(\frac{7.37}{32.42 - 4.387}\right) = 14.73^\circ$$

$$\beta_1 = \tan^{-1}\left(\frac{V_1}{U_1}\right) = \tan^{-1}(0.442) = 23.84^\circ$$

(iii) $T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) = \rho Q(r_2 V_{\theta 2}) = 1226(0.25)(0.18)(4.387) = 242.03\ Nm$

$\dot{W} = T\omega = 43.6\ kW$

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*
Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: goo.gl/eg192A

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

MA4002 – FLUID DYNAMICS

Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR** (4) questions and comprises **FOUR**(4) pages.
2. Answer **ALL FOUR** questions.
3. All questions carry equal marks.
4. This is a **CLOSED-BOOK** examination.

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1 (a) At some point upstream of the throat of a converging-diverging nozzle, air flows at a speed of 13.75 m/s, with pressure and temperature of 103.4 kPa and 294 K, respectively. If the throat area is 0.09 m^2, and the flow at the end of the nozzle is supersonic (Mach number >1), find the mass flow rate of air, assuming that the flow in the nozzle is isentropic.
For air, the specific heat ratio $k = 1.4$, specific heat at constant pressure $c_p = 1004$ J/kg.K, and gas constant $R = 287$ J/kg.K.

(10 marks)

(b) Air flows through a converging-diverging nozzle (see Figure 1) with an area ratio $A_e/A_t = 2.5$, where A_e and A_t are the areas of the nozzle outlet and the throat, respectively. At the nozzle inlet, the stagnation pressure is $p_0 = 1$ MPa and the stagnation temperature is $T_0 = 320$ K. A constant-area, adiabatic duct is attached to the nozzle outlet. The duct has length L and diameter D satisfying $L/D = 10$. The averaged friction factor of the duct is $f = 0.03$.

Suppose that the flow in the nozzle is isentropic, and there is a normal shock at the nozzle outlet. Calculate the stagnation temperature, the Mach number, and the pressure at the end of the duct.

For air, the specific heat ratio $k = 1.4$, specific heat at constant pressure $c_p = 1004$ J/kg.K, and gas constant $R = 287$ J/kg.K.

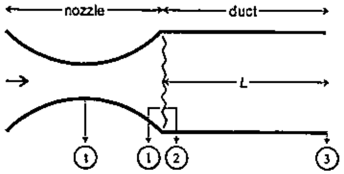


Figure 1

(15 marks)

2 (a) A rectangular plate with length L and width W moves in water at a constant velocity U (see Figure 2 for the orientation of the plate relative to its velocity). Water has dynamic viscosity μ and density ρ. Assume that a laminar boundary layer forms on surfaces of the plate and the velocity profile within the boundary layer is approximated using a sinusoidal function:

$$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right),$$

where u is the boundary layer velocity, y is the vertical distance from the surface, and δ is the boundary layer thickness. It is also assumed that the boundary layer thickness $\delta = 0$ at $x = 0$ (the leading edge of the plate). Find the expression for the total drag F_D on one surface of the plate.

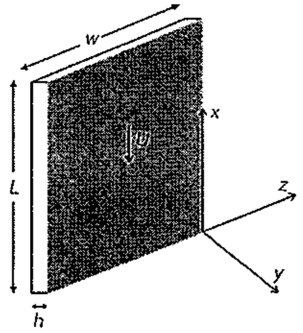


Figure 2

Note that the momentum integral equation for zero-pressure gradient flow is:

$$\tau_w = \rho U^2 \frac{d\theta}{dx}.$$

Here θ is the momentum thickness and is defined as

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy.$$

The wall shear stress is given by

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0}.$$

The drag is related to the wall shear stress as:

$$F_D = \int_0^L \tau_w W dx.$$

(15 marks)

(b) The above plate has a thickness h (see Figure 2) and is made of a material with density $\rho_p > \rho$. If the plate falls freely and vertically in water, find an expression for the maximum velocity of the plate.

(10 marks)

3 (a) Consider a two-dimensional steady flow field with fluid density of 1030 kg/m^3 and $\vec{g} = -g\vec{j}$. The velocity distribution of the flow field is

$$\vec{V} = (3x^2y - y^3)\vec{i} + (-3xy^2 + x^3)\vec{j}$$

(i) Show that this is an incompressible flow field and find its corresponding stream function, ψ.

(ii) Find the vorticity, $\vec{\omega} = 0.5\nabla \times \vec{V}$. Is it an irrotational flow? If no, explain why and if yes, find the velocity potential, ϕ.

(iii) Find the stream function values at points: $A(0, 1)$, $B(1, 0)$ and $C(1, 1)$.

(iv) If the pressure at point A is 150 kPa, find the pressure at point B. Justify the equation used.

(v) Can you find the pressure at point $C(1, 1)$ with the pressure of point A as given in Part (iv)? If your answer is yes, justify the equation used. If your answer is no, explain why. **Note that you do not have to find pressure at point $C(1, 1)$.**

Note: $u = \dfrac{\partial \psi}{\partial y}; v = -\dfrac{\partial \psi}{\partial x}; u = \dfrac{\partial \phi}{\partial x}; v = \dfrac{\partial \phi}{\partial y}$ and $\dfrac{p_1}{\rho g} + \dfrac{V_1^2}{2g} + y_1 = \dfrac{p_2}{\rho g} + \dfrac{V_2^2}{2g} + y_2$

(11 marks)

(b) The viscous oil in Figure 3 is set into steady motion by an outer cylinder moving axially at constant velocity V axially in negative z direction, with a concentric inner cylinder that remains stationary. Assuming constant pressure and density and a purely axial fluid motion i.e. $v_r = v_\theta = 0$, solve the Navier-Stokes equation and show that the fluid velocity distribution $v_z = V\left(\dfrac{\ln(r/a)}{\ln(a/b)}\right)$. State clearly the reasons for reducing the Navier-Stokes equation to a single differential equation.

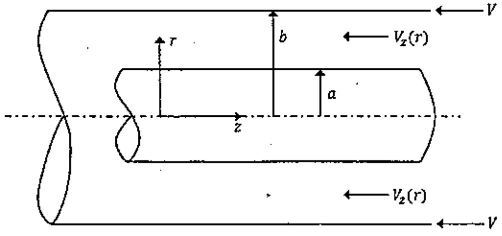


Figure 3

Note: Question 3 continues on page 4.

Note that

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right)$$

(7 marks)

(c) A flow field consisting of a free vortex of strength $K = 20$ m^2/s is rotating clockwise with its center at $A(2, 2)$, and another free vortex of strength $K = 30$ m^2/s is also rotating clockwise with its center being located at $B(2, -6)$.

(i) Find the location of stagnation point, S(x,y) and express it clearly in x and y coordinates.

(ii) Determine the resultant velocity at point $C(-4, 0)$ with both the magnitude and included angle with positive x-axis.

Note: For free vortex, the velocity components are $v_r = 0$ and $v_\theta = \dfrac{K}{r}$.

(7 marks)

4 (a) A spherical tank with diameter 6.0 m, is to discharge oil of density 870 kg/m^3 through a 8 m long, 40 mm diameter pipe with a height of 6.0 m below the base of the tank. The pipe has a friction factor of 0.026 and the total minor losees due to exit of pipe, bending of the pipe and etc. amount to 1.2. The tank is initially filled to a depth of 3 m with oil. (i) Calculate the time to reduce the depth in the tank by 1.5 m, and (ii) Suggest a method to fasten the process in order to reduce the time required.

Note: The equation for a circle with center (a,b) and radius r is $(x-a)^2 + (y-b)^2 = r^2$.

$\dfrac{p_1}{\rho g} + \dfrac{V_1^2}{2g} + y_1 = \dfrac{p_2}{\rho g} + \dfrac{V_2^2}{2g} + y_2 + f\dfrac{L}{D}\dfrac{V_2^2}{2g} + k\dfrac{V_2^2}{2g}$ and $t = \displaystyle\int_{y_1}^{y_2} \frac{A_s}{Q_i - Q_o}dy$

(12 marks)

(b) A centrifugal fan delivers 2.5 m^3/s when running at 1260 rpm. The impeller diameter is 78 cm and the diameter at blade inlet is 56 cm. The impeller width at inlet is 18 cm and at outlet 12.5 cm. The air enters the impeller with a small whirl component in the direction of impeller rotation, but the relative velocity meets the blade tangentially. The blades are backward inclined making angles of 25 ° and 52.5 ° with the tangents at the inlet and outlet respectively. Assuming the air with density of 1.23 kg/m^3.

(i) Sketch a complete inlet and outlet velocities diagram, and label clearly all the velocity components, shape of blade and the pump rotating direction.

(ii) Evaluate the absolute and relative velocities at both the inlet and outlet.

(iii) Determine torque required by the pump to drive the air and the ideal head rise h_i.

(13 marks)

END OF PAPER

MA4002 Fluid Dynamics 17/18 S1 Paper

$\boxed{1}$ (a) $A^* = 0.09\ m^2$ $V_1 = 13.75\ m/s$

$M_t = 1$ $T_1 = 294\ K$

$P_1 = 103.4\ kPa$

$$M_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{13.75}{\sqrt{1.4 \times 287 \times 294}} = 0.04$$

at $M = 0.04$,

$$\frac{A_1}{A^*} = 14.4815$$

$$A_1 = 1.303\ m^2$$

$$P_1 = \rho_1 R T_1$$

$$\rho_1 = \frac{103400}{287 \times 294} = 1.225$$

$$\dot{m} = \rho_1 A_1 V_1 = 1.225 \times 1.303 \times 13.75 = 21.95\ kg/s$$

(b) $M_t = 1$ $P_{01} = 1\ MPa$ $f\frac{L}{D} = 0.3$

at outlet, $T_{01} = 320\ K$

$$\frac{A_1}{A^*} = 2.5$$

$$M_1 = 2.44$$

$$\frac{T_1}{T_{01}} = 0.4505 \rightarrow T_1 = 146.08\ K$$

from normal shock function, $M_2 = 0.5189$

$$\frac{P_{02}}{P_{01}} = 0.5234 \rightarrow P_{02} = 0.5234\ MPa$$

$$\frac{T_2}{T_1} = 2.0788 \rightarrow T_2 = 303.67\ K$$

$$\frac{P_2}{P_{02}} = 0.8317 \rightarrow P_2 = 435.3\ kPa$$

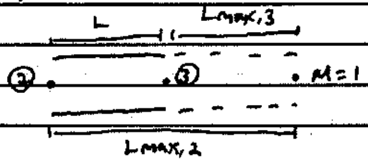
$\boxed{II}$ (b) cont.

at $M_2 = 0.52$

$\frac{fL_{max}}{D} = 0.9174$

$\frac{T_2}{T_{02}} = 0.9487 \rightarrow T_{02} = 320.09\ K$

$\frac{P_2}{P^*} = 2.0519 \rightarrow P^* = 212.14\ kPa$



since $\frac{fL}{D} < \frac{fL_{max}}{D}$, $M_3 < 1$

$\frac{fL_{max,3}}{D} = \frac{f(L_{max,2} - L)}{D} = 0.9174 - 0.3 = 0.6174$

from table, $M_3 \approx 0.57$

$T_{03} = T_{02} = 320.09\ K$

$\frac{P_3}{P^*} = 1.8623 \rightarrow P_3 = 395.07\ kPa$

$\boxed{2}$(a) let $\eta = \frac{y}{\delta} \rightarrow dy = \delta\, d\eta$

$$\theta = \delta \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta$$

$$= \delta \int_0^1 \sin\left(\frac{\pi\eta}{2}\right)\left(1 - \sin\left(\frac{\pi\eta}{2}\right)\right) d\eta$$

$$= \delta \int_0^1 \sin\left(\frac{\pi\eta}{2}\right) - \sin^2\left(\frac{\pi\eta}{2}\right)\, d\eta$$

$$= \delta \int_0^1 \sin\left(\frac{\pi\eta}{2}\right) - \left(\frac{1}{2} - \frac{1}{2}\cos \pi\eta\right) d\eta$$

$$= \delta \left[-\frac{2}{\pi}\cos\left(\frac{\pi\eta}{2}\right) - \frac{1}{2}\eta + \frac{\sin \pi\eta}{2\pi}\right]_0^1$$

$$= \delta\left[-\frac{1}{2} - \left(-\frac{2}{\pi}\right)\right]$$

$$= 0.1366\,\delta$$

$$\tau_w = \rho U^2 \frac{d\theta}{dx}$$
$$= \rho U^2 \frac{d}{dx}(0.137\,\delta)$$
$$= 0.137\,\rho U^2 \frac{d\delta}{dx}$$

$$\tau_w = \mu \frac{du}{dy}\Big|_{y=0}$$
$$= \mu\left[U \frac{\pi}{2\delta} \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]_{y=0}$$
$$= \frac{\pi\mu U}{2\delta}$$

hence,

$$\frac{\pi\mu U}{2\delta} = 0.137\,\rho U^2 \frac{d\delta}{dx}$$

$$\frac{\pi\mu}{2\rho U}\, dx = 0.137\,\delta\, d\delta$$

$$\frac{\pi\mu}{2\rho U}\, x + C = 0.137\, \frac{\delta^2}{2}$$

at $x = 0$, $\delta = 0$, so $C = 0$

$$\frac{\pi\mu x}{2\rho U} = 0.137\, \frac{\delta^2}{2}$$

$$\delta = 4.79\sqrt{\frac{\mu x}{\rho U}}$$

$\boxed{12}$ (a) cont.

$$\delta(x=L) = 4.79\sqrt{\frac{\mu L}{\rho U}}, \quad \theta(x=L) = 0.656\sqrt{\frac{\mu L}{\rho U}}$$

$$F_D = \int_0^L \tau_w W\,dx = \int_0^L \rho U^2 \frac{d\theta}{dx} W\,dx = \int_0^{\theta(x=L)} \rho U^2 W\,d\theta$$

$$= \rho W U^2 \theta(x=L)$$

$$= 0.656\,\rho U^2 W\sqrt{\frac{\mu L}{\rho U}}$$

$$= 0.656\,W\sqrt{\mu \rho U^3 L}$$

(b) buoyant force = drag force (both sides)

$$(\rho_p-\rho)gWLh = 2\left(0.656\,W\sqrt{\mu\rho U^3 L}\right)$$

$$(\rho_p-\rho)^2 g^2 L h^2 = 1.72\,\mu\rho\,U_{max}^3$$

$$U_{max} = \left[\frac{[(\rho_p-\rho)gh]^2 L}{1.72\,\mu\rho}\right]^{1/3}$$

$\boxed{3}$ (a) $u = 3x^2y - y^3 \quad ; \quad v = -3xy^2 + x^3$

(i) $\nabla.\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

$\nabla.\vec{V} = (6xy) + (-6xy) + 0 = 0$

$u = \frac{\partial \psi}{\partial y}$ $\qquad\qquad$ $v = -\frac{\partial \psi}{\partial x}$

$\partial \psi = u \, \partial y$ $\qquad\qquad$ $\partial \psi = 3xy^2 - x^3 \, \partial x$

$\psi = \frac{3}{2}x^2y^2 - \frac{1}{4}y^4 + f(x)$ $\qquad\qquad$ $\psi = \frac{3}{2}x^2y^2 - \frac{1}{4}x^4 + f(y)$

$\psi = \frac{3}{2}x^2y^2 - \frac{1}{4}(y^4 + x^4)$

(ii)

$\vec{\omega} = 0.5 \, \nabla \times \vec{V} = 0.5 \left[\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} \right]$

$= 0.5 \left[0\vec{i} + 0\vec{j} + \left((-3y^2 + 3x^2) - (3x^2 - 3y^2)\right)\vec{k} \right]$

$= 0$

the flow is irrotational

$u = \frac{\partial \phi}{\partial x}$ $\qquad\qquad$ $v = \frac{\partial \phi}{\partial y}$

$\partial \phi = 3x^2y - y^3 \, \partial x$ $\qquad\qquad$ $\partial \phi = -3xy^2 + x^3 \, \partial y$

$\phi = x^3y - xy^3 + f(y)$ $\qquad\qquad$ $\phi = -xy^3 + x^3y + f(x)$

$\phi = x^3y - xy^3$

$\boxed{3}$ (a) cont.

(iii) $\psi_A = \frac{3}{2}(0)(1) - \frac{1}{4}(1+0) = -\frac{1}{4}$

$\psi_B = \frac{3}{2}(1)(0) - \frac{1}{4}(0+1) = -1/4$

$\psi_C = \frac{3}{2}(1)(1) - \frac{1}{4}(1+1) = 1$

(iv) Bernoulli equation can be used, because:

- $\psi_A = \psi_B$, A & B are in the same streamline
- incompressible flow
- assume steady & inviscid flow

$\vec{V}_A = (3(0)(1) - 1)\vec{i} + (-3(0)(1) + 0)\vec{j} = -\vec{i}$ $\qquad |\vec{V}_A| = 1$

$\vec{V}_B = (3(1)(0) - 0)\vec{i} + (-3(1)(0) + 1)\vec{j} = \vec{j}$ $\qquad |\vec{V}_B| = 1$

$y_A = 1$

$y_B = 0$

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + y_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + y_B$$

$$\frac{150000}{1050 \times 9.81} + \frac{1}{2g} + 1 = \frac{P_B}{\rho g} + \frac{1}{2g} + 0$$

$$P_B = \rho g\,(15.845) = 160.1\ kPa$$

(v) can, since the flow is irrotational, pressure at any point can be found even if it is in a different streamline from the reference point

$\boxed{3}$ (b)

$$\rho\left(\cancelto{①}{\frac{\partial v_z}{\partial t}} + \cancelto{②}{v_r}\frac{\partial v_z}{\partial r} + \cancelto{②}{\frac{v_\theta}{r}}\frac{\partial v_z}{\partial \theta} + v_z\cancelto{③}{\frac{\partial v_z}{\partial z}}\right) = \cancelto{④}{-\frac{\partial p}{\partial z}} + \cancelto{⑤}{\rho g_z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r}\cancelto{③}{\left(\frac{\partial^2 v_z}{\partial \theta^2}\right)}\right.$$

$$\left. \dots + \cancelto{③}{\frac{\partial^2 v_z}{\partial z^2}}\right)$$

Assumptions:

① steady flow

② $v_r = 0$, $v_\theta = 0$

③ $v_z = f(r)$

④ $\partial p/\partial z = 0$, since along z, the height & velocity are the same, applying bernoulli equation along the same streamline gives the same pressure

⑤ $\vec{g} = -g\vec{j}$

$$0 = \frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$$

$$c = r\frac{\partial v_z}{\partial r}$$

$$\frac{c}{r} = \frac{\partial v_z}{\partial r}$$

$$v_z = c\ln r + D$$

Boundary conditions:

$v_z = 0$ at $r = a$

$0 = c\ln a + D$

$D = -c\ln a$

$v_z = -V$ at $r = b$

$-V = c\ln b + D$

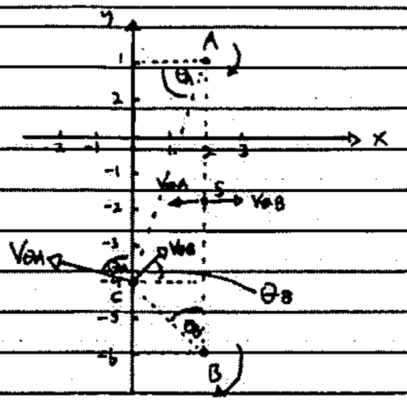
$-V = c\ln b - c\ln a$

$V = c(\ln a - \ln b)$

$c = \dfrac{V}{\ln(a/b)}$

$$v_z = c\ln r - c\ln a = c\ln\left(\frac{r}{a}\right) = V\frac{\ln(r/a)}{\ln(a/b)}$$

$\boxed{3}$ (c)



at point S,

(i) $V_{\theta A} = V_{\theta B}$ $\qquad r_A + r_B = 8$

$\frac{K_A}{r_A} = \frac{K_B}{r_B}$ $\qquad r_A = 8 - r_B$

$\frac{20}{8 - r_B} = \frac{30}{r_B}$

$20 r_B = 240 - 30 r_B$

$r_B = 4.8$

hence, $S(2, -1.2)$

(ii) $r_A = \sqrt{6^2 + 2^2} = 6.32$ $\qquad \theta_A = \tan^{-1}\left(\frac{6}{2}\right) = 71.57°$

$r_B = \sqrt{2^2 + 2^2} = 2.83$ $\qquad \theta_B = \tan^{-1}\left(\frac{2}{2}\right) = 45°$

$V_{\theta A} = \frac{20}{r_A} = 3.16$

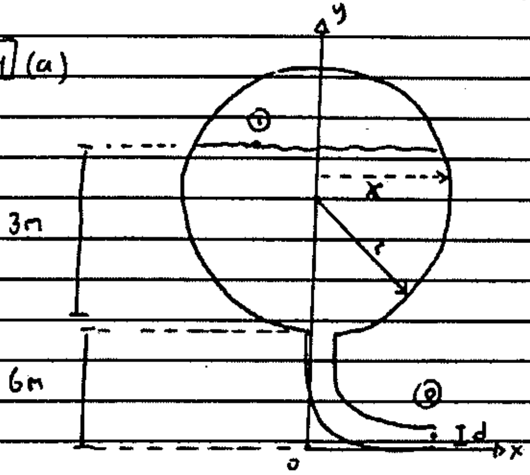
$V_{\theta B} = \frac{30}{r_B} = 10.6$

$$\vec{V_c} = (V_{\theta B} \cos\theta_B - V_{\theta A} \sin\theta_A)\vec{i} + (V_{\theta A} \cos\theta_A + V_{\theta B} \sin\theta_B)\vec{j}$$
$$= (7.49 - 2.998)\vec{i} + (0.999 + 7.49)\vec{j}$$
$$= 4.49\vec{i} + 8.49\vec{j}$$

$|\vec{V_c}| = 9.6$ m/s

$\angle V_c = \tan^{-1}\left(\frac{8.49}{4.49}\right) = 62.13°$

$\boxed{4}$ (a)



$D = 6$ $\quad d = 0.04$
$\rho = 870$ $\quad L = 8$
$y_1 = 9$ $\quad f = 0.026$
$y_2 = 7.5$ $\quad k = 1.2$

$$A_o = \frac{\pi d^2}{4} = 1.26 \times 10^{-3}$$

$$r^2 = (x-0)^2 + (y-9)^2 = 9$$
$$x^2 = 9 - y^2 + 18y - 81$$
$$x^2 = -y^2 + 18y - 72$$

apply bernoulli from ① to ⓪ (surface of tank to end of pipe)

$$\cancel{\frac{P}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + y_1 = \cancel{\frac{P}{\rho g}} + \frac{V_o^2}{2g} + \cancel{y} + \frac{fL}{D}\frac{V_o^2}{2g} + \frac{KV_o^2}{2g}$$

$$y = 0 + \frac{V_o^2}{2g}\left(1 + \frac{fL}{D} + K\right)$$

$$y = \frac{V_o^2}{2g}(1 + 5.2 + 1.2)$$

$$V_o = 1.63\sqrt{y}$$

$$A_s = \pi x^2 = \pi(-y^2 + 18y - 72)$$
$$Q_i = 0$$
$$Q_o = A_o V_o = 2.046 \times 10^{-3}\sqrt{y}$$

4 (a) cont.

$$t = \int_{9}^{7.5} \pi \frac{-y^2+18y-72}{-2.046\times10^{-3}\sqrt{y}}\,dy$$

$$= -1535.5 \int_{9}^{7.5} -y^{3/2} + 18y^{1/2} - 72y^{-1/2}\,dy$$

$$= -1535.5 \left[-\tfrac{2}{5}y^{5/2} + 12y^{3/2} - 144y^{1/2} \right]_{9}^{7.5}$$

$$= -1535.5 \left[(-61.62 + 246.48 - 394.36) - (-97.2 + 324 - 432) \right]$$

$$= 6602.65\ s$$

$$= 1.83\ \text{hours}$$

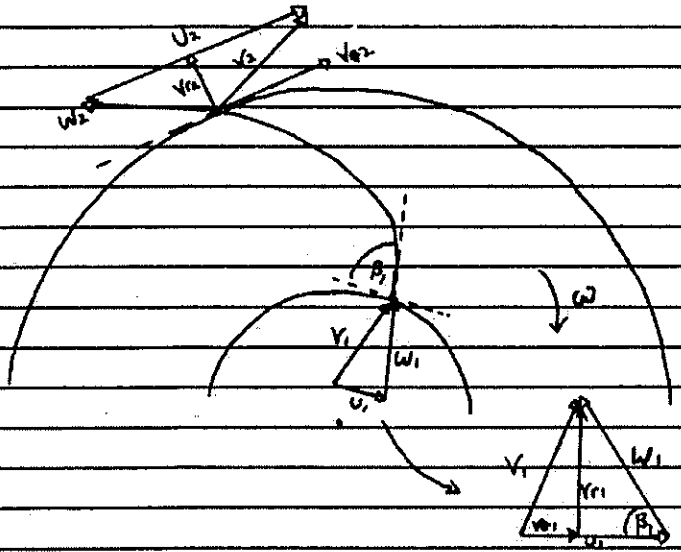
4 (b) $Q = 2.5\ m^3/s$

$\omega = 1260\ rpm = 131.95\ rad/s$

$r_2 = 0.39\ m$ $\quad b_2 = 0.125\ m$ $\quad \beta_2 = 52.5°$

$r_1 = 0.28\ m$ $\quad b_1 = 0.18\ m$ $\quad \beta_1 = 25°$

(i)



$\boxed{4}$ (b) cont.

(ii) $U_1 = \omega r_1 = 36.946$

$U_2 = \omega r_2 = 51.46$

$V_{r1} = \dfrac{Q}{2\pi r_1 b_1} = 7.89$

$V_{r2} = \dfrac{Q}{2\pi r_2 b_2} = 3.16$

$W_1 \sin\beta_1 = V_{r1}$

$W_1 = \dfrac{V_{r1}}{\sin\beta_1} = 18.67$

$W_2 = \dfrac{V_{r2}}{\sin\beta_2} = 10.28$

$V_{\theta 1} = U_1 - W_1 \cos\beta_1 = 20.02$

$V_{\theta 2} = U_2 - W_2 \cos\beta_2 = 45.2$

$V_1 = \sqrt{V_{r1}^2 + V_{\theta 1}^2} = 21.52$

$V_2 = \sqrt{V_{r2}^2 + V_{\theta 2}^2} = 45.93$

(iii) $T = \rho Q (r_2 V_{\theta 2} - r_1 V_{\theta 1}) = 1.23 \times 2.5 (0.39 \times 45.2 - 0.28 \times 20.02)$

$T = 36.97$ Nm

$h_i = \dfrac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g} - 161.7$ m

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

MA4002 – FLUID DYNAMICS

April/May 2018 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED-BOOK** examination.

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1 (a) Air is flowing in an adiabatic nozzle at a mass flow rate of 6.1 kg/s. At the inlet, the pressure is 205.2 kPa, the temperature is 623.5 K, and the area is 32.5 cm^2. At the outlet, the Mach number is 1.3. Sketch the shape of the nozzle and provide your explanation. Find the outlet area provided that the flow is reversible.

If the flow conditions at the inlet are kept fixed, and there is a normal shock in the nozzle, find the Mach number at the outlet if the temperature at the outlet is 1678.9 K.

For air, the specific heat ratio $k = 1.4$, specific heat at constant pressure $c_p = 1004$ J/kg.K, and gas constant $R = 287$ J/kg.K.

(10 marks)

(b) Air flows through a converging nozzle and an insulated duct with constant diameter $D = 6.3$ mm (see Figure 1). The air is supplied from a tank where the temperature is constant at 15°C and the pressure is variable. The outlet of the duct exhausts to atmosphere with atmospheric pressure $P_e = 101.3$ kPa. The flow in the nozzle can be considered isentropic. When the exit flow is just choked, measurements at the duct inlet show that the pressure is $P_2 = 367$ kPa and the Mach number is $M_2 = 0.30$, respectively. Determine the pressure in the tank. Determine the temperature, stagnation pressure at the duct outlet. Determine the mass flow rate.

For air, the specific heat ratio $k = 1.4$, specific heat at constant pressure $c_p = 1004$ J/kg.K, and gas constant $R = 287$ J/kg.K.

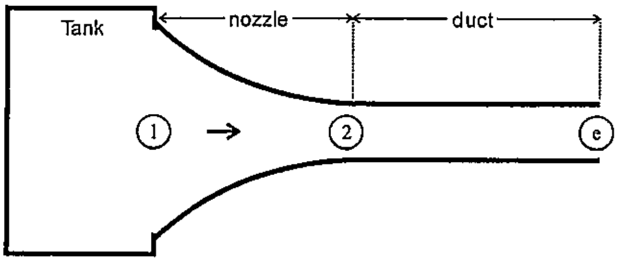

Figure 1

(15 marks)

2 (a) The velocity profile in a turbulent boundary-layer flow at zero pressure gradient and on top of a flat plate is approximated by the 1/6-power profile expression:

$$\frac{u}{U} = \eta^{1/6}, \text{ where } \eta = \frac{y}{\delta}$$

Here u is the boundary layer velocity, U is the freestream velocity, y is the vertical distance from the surface, and δ is the boundary layer thickness. It is also assumed that $\delta = 0$ at $x = 0$ (the leading edge of the plate).

Use the momentum integral equation with this velocity profile to obtain expressions for δ/x and the skin friction C_f.

Note that the momentum integral equation for zero-pressure gradient flow is:

$$\tau_w = \rho U^2 \frac{d\theta}{dx}$$

Here θ is the momentum thickness:

$$\theta \approx \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

The wall shear stress for turbulent flows is given by

$$\tau_w = 0.0233 \rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{0.25}$$

The skin friction is defined as

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$$

(15 marks)

(b) A field hockey ball has diameter $D = 73$ mm and mass $m = 160$ g. When struck well, it leaves the stick with initial speed $U_0 = 50$ m/s in standard air with kinematic viscosity $\nu = 1.45 \times 10^{-5}$ m^2/s and density $\rho = 1.23$ kg/m^3. Assuming that gravity is neglected, the drag coefficient remains constant at $C_D = 0.47$, and the ball flies horizontally, estimate the time and distance it takes for the speed of the ball reduced by 10 percent by aerodynamic drag.

(10 marks)

3 (a) Consider the velocity field $\vec{V} = (Axy - Bx^2)\vec{\imath} + (Axy - By^2)\vec{\jmath}$, in the xy plane, where $A = 2\text{m}^{-1}\text{s}^{-1}$, $B = 1\text{m}^{-1}\text{s}^{-1}$, and the coordinates are measured in meters, such that the velocity is in m/s. The fluid is water with density of 1000 kg/m^3 and $\vec{g} = -g\vec{\jmath}$.

(i) Is this a possible incompressible flow field? If your answer is no, explain why, or otherwise, find the stream function , ψ

(ii) Find the rotation, $\vec{\omega} = 0.5\nabla \times \vec{V}$, is the flow irrotational? If no, explain why and if yes, find the velocity potential, $\emptyset$.

(iii) Sketch the stream function of $\psi = 0$.

(iv) Find the pressure at point $C(1,1)$, if the pressure at point $D(3,3)$ is 200 kPa. Justified the method or equation used. Could you find the pressure at $E(2,4)$? If yes, justify it, if not, explain why.

Note: $u = \dfrac{\partial \psi}{\partial y}; v = -\dfrac{\partial \psi}{\partial x}; u = \dfrac{\partial \phi}{\partial x}; v = \dfrac{\partial \phi}{\partial y}$ and $\dfrac{p_1}{\rho g} + \dfrac{V_1^2}{2g} + y_1 = \dfrac{p_2}{\rho g} + \dfrac{V_2^2}{2g} + y_2$

(11 marks)

(b) An incompressible Newtonian liquid is confined between two concentric circular cylinders of infinite length – a solid inner cylinder of radius R_i and a hollow, stationary outer cylinder of radius R_o as shown in Figure 2. Note that the z axis is out of the paper. The inner cylinder rotates at angular velocity, ω_i. The flow is steady, laminar and two dimensional in the r - θ plane. The flow is also rotational symmetric, so properties do not vary with θ and $\partial / \partial\theta = 0$. The flow is also circular, meaning that $v_r = v_z = 0$ and $v_\theta = f(r)$. Using Navier-Stokes equation, show that expressions for the liquid velocity profile:

$$v_\theta = \frac{\omega R_O}{\left(\frac{R_O}{R_i}\right)^2 - 1}\left[\frac{R_O}{r} - \frac{r}{R_O}\right]$$

With the boundary conditions (i) at $r = R_o$, $v_\theta = 0$; and (ii) at $r = R_i$, $v_\theta = \omega R_i$.

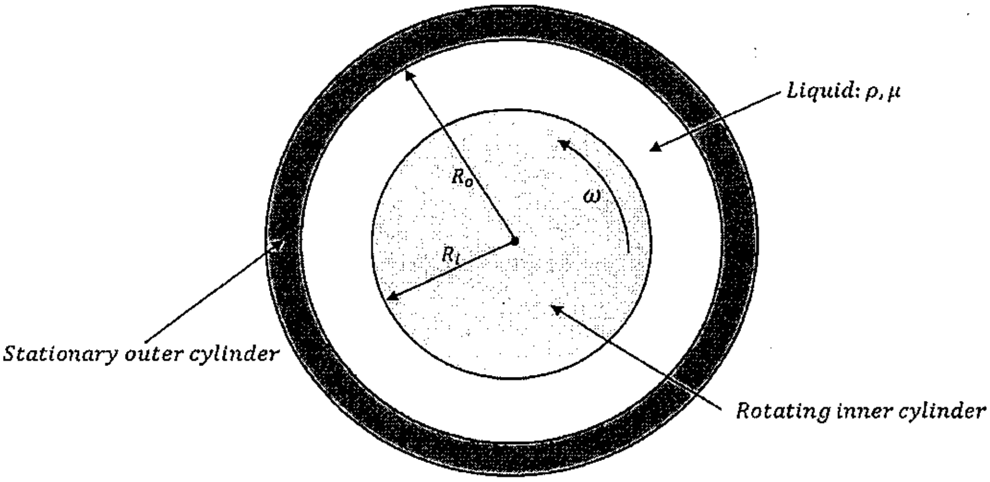

Figure 2

Note: Question 3 continues on page 4.

Note that

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta$$
$$+\mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left[r v_\theta\right]\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right\}$$

(7 marks)

(c) The nose of a solid strut 90 mm wide is to be placed in an infinite two-dimensional air stream of velocity 20 m/s and density 1.23 kg/m^3 and is to be made in the shape of a half-body as shown in Figure 3.

(i) Determine the value of the stream function at the stagnation point of the strut.

(ii) Determine the strength of the source, and the distance between the stagnation point and the source.

(iii) Determine the difference in pressure between the stagnation point and the point on the strut where it is 45 mm wide. Assume the potential energy difference is negligible. Justify the equation used.

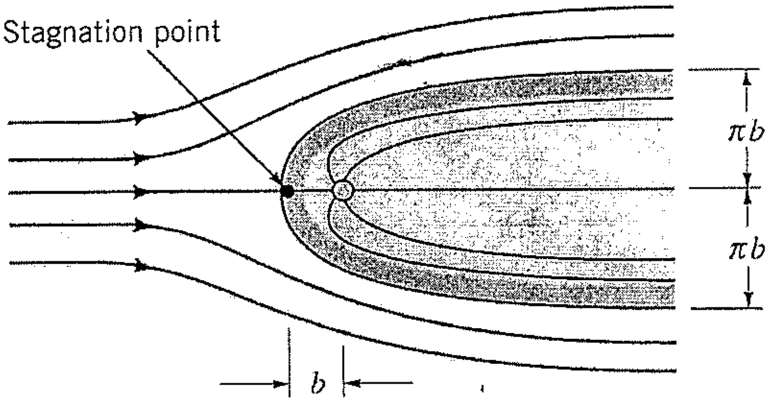

Figure 3.

Note that $\psi = Ur\sin\theta+\frac{m}{2\pi}\theta$, $r=\frac{b(\pi-\theta)}{\sin\theta}$, $V^2=U^2(1+2\frac{b}{r}\cos\theta+\frac{b^2}{r^2})$.

and Bernoulli's equation between points A and B: $\frac{p_A}{\rho g}+\frac{V_A^2}{2g}+z_A=\frac{p_B}{\rho g}+\frac{V_B^2}{2g}+z_B$.

(7 marks)

4 (a) A pump draws water from a river and delivers it at a steady rate of 0.15 m^3/s to a reservoir in which the free surface level is 16 m higher than that in the river. The pipe system consists of 70 m of 300 mm diameter pipe ($f = 0.08$) and 100 m of 150 mm diameter pipe ($f = 0.045$) arranged in series. Neglect minor losses in the pipe connection and in the pump, and assume an incompressible fluid in rigid pipes and f is independent of Reynolds number.

(i) Sketch a clear schematic diagram of the flow and determine the head of pump, h_p, under steady flow condition.

(ii) Find the flow rate 2.5 seconds after a power failure to the pump, assuming that the pump stops instantaneously.

Note that $\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2-h_p+\sum\frac{L}{g}\frac{dV}{dt}+\sum f\frac{L}{D}\frac{V^2}{2g}+\sum k\frac{V^2}{2g}$

and $\int_{Q_1}^{Q_2}\frac{dQ}{a^2+Q^2}=\frac{1}{a}\left[arc\tan\frac{Q}{a}\right]_{Q_1}^{Q_2}$

(12 marks)

(b) A centrifugal pump rotates at 1200 rpm. Water enters the impeller with no whirl ($\alpha_1 = 90°$) and exit at an angle of $\alpha_2 = 50°$. The inlet radius is 12.0 cm, at which the blade width is 9.0 cm. The outlet radius is 24.0 cm at which blade width is 7.0 cm. The volume flow rate is 800 l/s.

(i) Sketch a complete inlet and outlet velocities diagram, and label clearly all the velocities, angles, shape of blade and the pump rotating direction.

(ii) Find the blade angles at both the inlet and outlet.

(iii) Determine the torque required by the pump to drive the water and the ideal head rise h_i.

(13 marks)

END OF PAPER

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MA4002 17/18 S2

1a) $\dot{m} = 6.1 kg/s$ $M_2 = 1.3$

$P_1 = 205.2 kPa$

$T_1 = 623.5 K$

$A_1 = \frac{32.5}{100 \times 100} = 0.00325 m^2$

Note:
The flow is Adiabatic and reversible as described. Hence it must be an isentropic flow.

$\dot{m} = A_1 V_1 \rho_1 = A_2 V_2 \rho_2$

$\rho_1 = \frac{P_1}{RT_1} = 1.147 kg/m^3$

$\frac{\dot{m}}{\rho_1 A_1} = V_1 = 1636 m/s$

$M_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{1636}{\sqrt{(1.4)(287)(623.5)}}$

$= 3.27$

$A_1^* = A_2^*$ (isentropic)

$M_1 = 3.27 \rightarrow \frac{A_1}{A^*} = 5.4715$

$M_2 = 1.3 \rightarrow \frac{A_2}{A^*} = 1.0663$

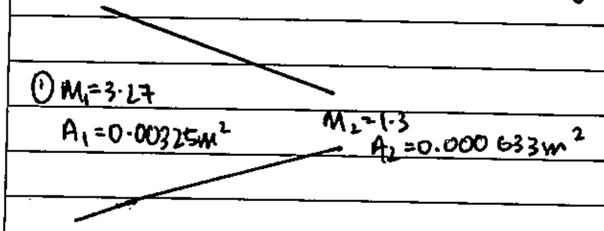
$\frac{A_1}{A^*} \times \frac{A^*}{A_2} = \frac{A_1}{A_2} = 5.4715 \times \left(\frac{1}{1.0663}\right)$

$= 5.131$

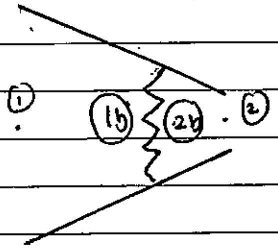
$= \frac{0.00325}{A_2}$

$\therefore A_2 = 0.000633 m^2$

Supersonic Diffuser with coveraging Area



1a) Now with Normal shock..



① Flow from 1b to 2b is irreversible but adiabatic ($T_{01b} = T_{02b}$)

② Flow ① to ①b is isentropic ($T_{01} = T_{01b}$), adiabatic

③ Flow ②b to ② is isentropic ($T_{02b} = T_{02}$), adiabatic

$\therefore \quad T_{01b} = T_{02b} = T_{01} = T_{02} = 1957K$

(From $M_1 = 3.27 \rightarrow \frac{T_1}{T_{01}} = 0.3186 \quad T_{01} = 1957K$)

since $T_2 = 1678.9K \rightarrow \frac{T_2}{T_{02}} = 0.8574 \rightarrow M_2 = 0.91_*$

1b) state 1: (stagnation tank)

$T_1 = 15°C = T_{01} = 288K$

$P_e = 101.3KPa$

state ① to ② is isentropic $\therefore T_{01} = T_{02} = 288K$

$P_2 = 367KPa \quad M_2 = 0.3$

$P_e = P^*$

$P_1 = P_{02} = P_{01}$ (isentropic)

since $M_2 = 0.3 \rightarrow \frac{P_2}{P_{02}} = 0.9395 \rightarrow P_{02} = 390.6KPa$

$P_1 = 390.6KPa_*$

$M_e = M^* = 1.0$ (just choked outlet)

$\hookrightarrow \frac{P_e}{P_{0e}} = 0.5283 \rightarrow P_{0e} = 191.7KPa$

From $M_e = 1.0 \rightarrow \frac{T_e}{T_{0e}} = 0.8333$

1c) since ② → ⓔ is ~~insutated~~ (Adiabatic) and ① → ② also isentropic

$T_{02} = T_{0e} = T_{01} = 288K$.

$T_e = 0.8333(288) = 240K$

$\dot{m} = \rho_e A_e V_e$

$P_e = \rho_e R T_e$

$\rho_e = \frac{P_e}{RT_e} = 1.471 kg/m^3$

$\dot{m} = 1.471 \left(\frac{\pi}{4}\right) D^2 (T_e)$

$= 0.011 kg/s$

2a) $\frac{u}{U} = n^{1/6}$ $\left(n = \frac{y}{\delta}\right) \rightarrow dn = \frac{dy}{\delta}$ $\delta\,dn = dy$

$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$

$= \int_0^\delta n^{1/6}\left(1 - n^{1/6}\right) dy$

$= \delta\int_0^1 n^{1/6}\left(1 - n^{1/6}\right) dn$

$= \delta\int_0^1 \left(n^{1/6} - n^{1/3}\right) du$

$= \delta\left[\frac{n^{7/6}}{7/6} + \frac{n^{4/3}}{4/3}\right]_0^1$

$= \delta\left[\frac{1}{7/6} + \frac{1}{4/3}\right]$

$\theta = 1.607\,\delta$

$0.0233\,\cancel{\rho}\,\cancel{U^2}\left(\frac{\mu}{\rho U\delta}\right)^{0.25} = \cancel{\rho}\,U^2\,\frac{d\theta}{dx}$

$= \frac{d(1.607\delta)}{dx}$

$0.0233\left(\frac{\mu}{\rho U}\right)^{0.25}\frac{1}{\delta^{0.25}} = 1.607\left(\frac{d\delta}{dx}\right)$

$0.0233\left(\frac{\mu}{\rho U}\right)^{0.25} dx = 1.607\,\delta^{0.25}\,d\delta$

$0.0145\left(\frac{\mu}{\rho U}\right)^{0.25} dx = \delta^{0.25}\,d\delta$

$0.0145\left(\frac{\mu}{\rho U}\right)^{0.25}(x + c) = \frac{\delta^{1.25}}{1.25} = 0.8\,\delta^{1.25}$ $(x = 0, \delta = 0$ so $c = 0)$

$0.018125\left(\frac{\mu}{\rho U}\right)^{0.25} x = \delta^{1.25}$

$= \delta^{0.25}\,\delta \rightarrow \frac{\delta}{x} = 0.018125\left(\frac{\mu}{\rho U}\right)^{0.25}\cdot\frac{1}{\delta^{0.25}}$

$\frac{\delta}{x} = 0.018125\left(\frac{\mu}{\rho U\delta}\right)^{0.25}$

$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$

$= \frac{\cancel{\rho U^2}\,d\theta/dx}{\frac{1}{2}\cancel{\rho U^2}}$

$= 2\frac{d\theta}{dx}$

$= \frac{d(1.607\cdot\delta)}{dx} \times 2$

$= 3.214\,\frac{d\delta}{dx}$

$= 3.214\left(\frac{d\delta}{dx}\right)$

$= 3.2414\left[0.03232\left(\frac{\mu}{\rho U}\right)^{0.2} x^{-0.2}\right]$

$C_f = 0.1039\left(\frac{\mu}{\rho U}\right)^{0.2} x^{-0.2}$

$\rightarrow \frac{\delta}{x} = 0.018125\left(\frac{\mu}{\rho U}\right)^{0.25}\cdot\frac{1}{\delta^{0.25}}$

$\delta = \left(0.018125\left(\frac{\mu}{\rho U}\right)^{0.25} x\right)^{\frac{1}{1.25}}$

$= 0.0404\left(\frac{\mu}{\rho U}\right)^{0.25} x^{0.8}$

$\frac{d\delta}{dx} = 0.0404\left(\frac{\mu}{\rho U}\right)^{0.2}(0.8)\,x^{-0.2}$

$= 0.03232\left(\frac{\mu}{\rho U}\right)^{0.2} x^{-0.2}$

③

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2b) $D = 0.073\,m$

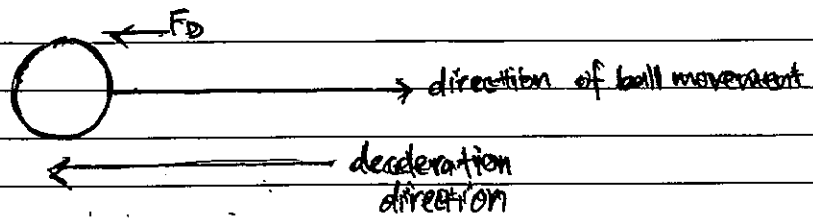
$m = 0.16\,kg$

$U_o = 50\,m/s \quad U_f = 0.9 \times 50 = 45\,m/s$

$\gamma = 1.45 \times 10^{-5}\,m^2/s$

$\rho = 1.23\,kg/m^3$

$C_D = 0.47$



For time:

$-F_D = mA$

$\quad = m\frac{dv}{dt}$ — ①

$F_D = \frac{1}{2}\rho V^2 A\,(C_D)$

$\quad = \frac{1}{2}(1.23)\left[\frac{\pi}{4}(0.073)^2\right](0.47)\;V^2$

$\quad = 0.00121\,V^2$

$-0.00121\,V^2 = 0.16\,\frac{dV}{dt}$ — ①

$-0.00121\,dt = 0.16\left(\frac{1}{V^2}\right)dV$

$\int_0^t -0.00121\,dt = 0.16\int_{50}^{45}\frac{1}{V^2}\,dV$

$\quad = 0.16\left(\frac{1}{45} - \frac{1}{50}\right)$

$-0.00121\,(t_f) = -0.0003556$

$t_f = 0.294$ seconds #

①

2b) $\text{Distance} = \left|\int \frac{1}{2}at\,dt\right| + (U_f \times t)$

$= \left|\int \frac{1}{2}t\frac{dv}{dt}\,dt\right| + 45(0.294)$

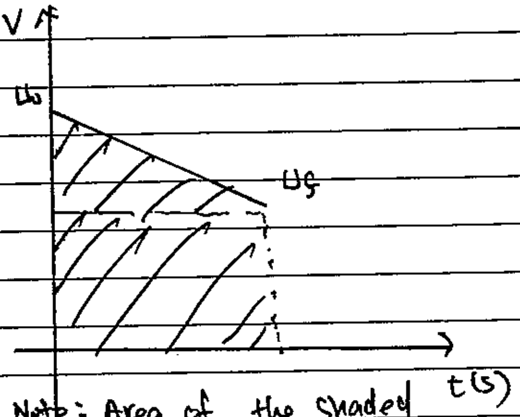
$= \left|\int \frac{1}{2}t\,dV\right| + 13.23$

$= \left|\left(\frac{0.294}{2}\right)\int_{50}^{45} dV\right| + 13.23$

$= 0.147(5) + 13.23$

$= 0.735 + 13.23$

$= 13.965\,m$



Note: Area of the shaded region is the distance covered.

④

3ai) $u = Axy - Bx^2$ $\qquad v = Axy - By^2$

$u = 2xy - x^2$ $\qquad v = 2xy - y^2$

$\frac{du}{dx} = 2y - 2x$ $\qquad \frac{dv}{dy} = 2x - 2y$

$$\frac{du}{dx} + \frac{dv}{dy} = (2y-2x) + (2x-2y) = 0$$

since $\frac{du}{dx} + \frac{dv}{dy} = 0$ (it's a possible incompressible flow)

$u = \frac{d\psi}{dy}$ $\qquad v = -\frac{d\psi}{dx}$

$\psi = \int u\,dy$ $\qquad \psi = \int -v\,dx$

$= \int (2xy - x^2)\,dy$ $\qquad = -\int (2xy - y^2)\,dx$

$= \frac{2xy^2}{2} - x^2y + f(x)$ $\qquad = -\left(\frac{2yx^2}{2} - y^2x\right) + f(y)$

$= xy^2 - x^2y + f(x)$ $\qquad = -(yx^2 - y^2x) + f(y)$

$f(x) = 0 \quad f(y) = 0$

$\qquad = (y^2x - yx^2) + f(y)$

$\therefore \psi = xy^2 - x^2y$

ii) $\vec{\omega} = 0.5(\nabla \times \vec{V})$

$= \frac{1}{2}\left(\frac{dv}{dx} - \frac{du}{dy}\right)\hat{k}$ $\qquad \frac{dv}{dx} = 2y \quad \frac{du}{dy} = 2x$

$= \frac{1}{2}(2y - 2x)\hat{k}$

$= (y - x)\hat{k}$

Flow is not irrotational

⑤

iii) for $\psi = 0$

$0 = xy^2 - x^2y$

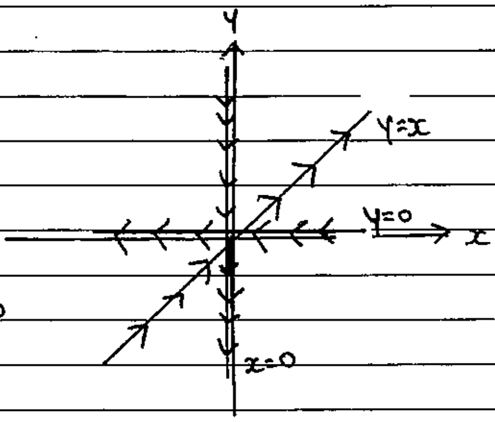
$= y\,(xy - x^2)$

$y = 0$ or $(xy - x^2) = 0$

$x\,(y - x) = 0$

$x = 0$ or $(y - x) = 0$

$y = x = 0$



Three possible solution for $\psi = 0$

① $y = 0$

③ $x = 0$

② $y = x$

$u = 2xy - x^2$ $v = 2xy - y^2$

iv) ① incompressible

② Steady

③ Inviscid

④ Same Streamline

$\psi_c = \psi(1,1) = 0$

$\psi_D = \psi(3,3) = 0$

Can use B.E between point C and point D.

$$\frac{P_C}{\rho g} + \frac{V_C^2}{2g} + y_C = \frac{P_D}{\rho g} + \frac{V_D^2}{2g} + y_D$$

$|\vec{V_C}| = \sqrt{(2-1)^2 + (2-1)^2} = \sqrt{2}$ $|\vec{V_D}| = \sqrt{(18-9)^2 + (18-9)^2}$

$\vec{g} = -g\,\hat{j}$ (in the y direction) $= \sqrt{162}$

$$\frac{P_C}{9810} + \frac{2}{19.62} + 1 = \frac{200\,000}{9810} + \frac{162}{19.62} + 3$$

$P_C = 299620$ Pa

① $\psi_e = 32 - 16 = 16$ $\psi_e \neq \psi_c \neq \psi_D$ ② Irrotational

Cant Apply B.E.

3b) $\rho\left(\overset{①}{\cancel{\frac{dv_\theta}{dt}}} + \overset{④}{\cancel{v_r\frac{dv_\theta}{dr}}} + \frac{v_\theta}{r}\overset{③}{\cancel{\left(\frac{dv_\theta}{d\theta}\right)}} + \overset{④}{\cancel{\frac{v_r v_\theta}{r}}} + \overset{②}{\cancel{v_z\frac{dv_\theta}{dz}}}\right) = \overset{③}{\cancel{-\frac{1}{r}\frac{dp}{d\theta}}} + \overset{⑥}{\cancel{\rho g_\theta}} + \mu\left\{\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rv_\theta)\right)\right.$

$\left. + \overset{⑤}{\cancel{\frac{1}{r^2}\frac{d^2v_\theta}{d\theta^2}}} + \overset{④}{\cancel{\frac{2}{r^2}\frac{dv_r}{d\theta}}} + \overset{⑤}{\cancel{\frac{d^2v_\theta}{dz^2}}}\right\}$

① steady laminar

② 2D flow ($V_z=0$)

③ $d/d\theta = 0$

④ $V_r = V_z = 0$

⑤ $V_\theta = V_\theta(r)$

⑥ $\vec{g} = -g\hat{k}$ ($g_\theta = 0$)

$$0 = \mu\left\{\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rv_\theta)\right)\right\}$$

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rv_\theta)\right) = 0$$

$$\frac{1}{r}\frac{d}{dr}(rv_\theta) = C_1$$

$$\frac{d}{dr}(rv_\theta) = C_1 r$$

$$rV_\theta = \frac{C_1 r^2}{2} + C_2$$

$$V_\theta = \frac{C_1 r}{2} + \frac{C_2}{r}$$

Boundary Condition ①, $V_\theta = 0$ $r = R_o$

$0 = \frac{C_1 R_o}{2} + \frac{C_2}{R_o}$ — ①

$0 = \frac{C_1 R_o^2}{2} + C_2$ — ①

Boundary Condition ②, $V_\theta = wR_i$, $r = R_i$

$wR_i = \frac{C_1 R_i}{2} + \frac{C_2}{R_i}$ — ②

$wR_i^2 = \frac{C_1 R_i^2}{2} + C_2$ — ②

let ② − ① $\rightarrow wR_i^2 = \left(\frac{C_1 R_i^2}{2} + \cancel{C_2}\right) - \left(\frac{C_1 R_o^2}{2} + \cancel{C_2}\right)$

$$wR_i^2 = \frac{C_1 R_i^2}{2} - \frac{C_1 R_o^2}{2}$$

$$2wR_i^2 = C_1 R_i^2 - C_1 R_o^2$$

$$2wR_i^2 = C_1(R_i^2 - R_o^2) \rightarrow C_1 = \frac{2wR_i^2}{(R_i^2 - R_o^2)}$$

$0 = \left(\frac{2wR_i^2}{(R_i^2 - R_o^2)}\right)\left(\frac{R_o^2}{2}\right) + C_2$

$C_2 = \frac{-wR_i^2 R_o^2}{(R_i^2 - R_o^2)}$

7

3B)

$$V_\theta = \left(\frac{2\omega R_i^2}{(R_i^2 - R_o^2)} \left(\frac{r}{2}\right) - \frac{\omega R_i^2 R_o^2}{(R_i^2 - R_o^2) r} \right)$$

$$= \frac{\frac{\omega R_i^2 r}{R_i^2}}{\frac{R_i^2}{R_i^2} - \frac{R_o^2}{R_i^2}} - \frac{\frac{\omega R_i^2 R_o^2}{r R_i^2}}{\frac{(R_i^2 - R_o^2)}{R_i^2}}$$

$$= \frac{\omega r - \omega R_o^2 / r}{1 - \frac{R_o^2}{R_i^2}}$$

$$= \frac{\omega \frac{R_o^2}{r} - \omega r}{\frac{R_o^2}{R_i^2} - 1}$$

$$\boxed{= \frac{\omega R_o}{\left(\frac{R_o}{R_i}\right)^2 - 1} \left(\frac{R_o}{r} - \frac{r}{R_o} \right)} \; *$$

3ci) $\psi_{stag} = \frac{m}{2}$

Width $= 2\pi b = 0.09m$

ii) $b = 0.0143m$

At the stagnation point → $U = V_r = \frac{m}{2\pi b} \rightarrow m = 2\pi b U$
$= 11.31 m^2/s$

Distance, $b = 0.0143m$

iii) $\frac{P_s}{\rho g} + \cancel{\frac{V_s^2}{2g}} + \cancel{z_s} = \frac{P_2}{\rho g} + V_2^2 + \cancel{z_2}$

$\psi = Ur\sin\theta + \frac{m}{2\pi}\theta = \frac{m}{2}$

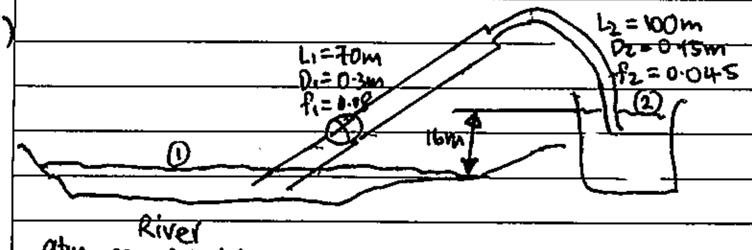
$\frac{P_s}{12.07} = \frac{P_2}{12.07} + \frac{V_2^2}{2g}$

$V_2^2 = U^2\left(1 - 2\frac{b}{r}\cos\theta + \frac{b^2}{r^2}\right)$
$= 400$

$\frac{P_s - P_2}{12.07} = \frac{V_B^2}{19.62}$ ~~$45 = 2\pi b$~~
~~$b = \frac{45}{2\pi} = 0.00716$~~

$\Delta P = \frac{V_B^2}{19.62}(12.07)$ ~~$= 246Pa$~~ (Im not sure, pls check question with Prof)

4ai)



atm reservoir datum

$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + z_2 - H_p + \cancel{\sum\frac{L}{g}\frac{du}{dt}} + f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} + f_2\frac{L_2}{D_2}\frac{V_2^2}{2g}$

steady

atm reservoir

$0 = z_2 - H_p + f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} + f_2\frac{L_2}{D_2}\frac{V_2^2}{2g}$ $Q = 0.15 m^3/s = A_1V_1 = A_2V_2$

$V_1 = 14.08Q$ $V_2 = 56.5Q$. $\left(A_1 = \frac{\pi}{4}D_1^2 = 0.071m^2\right)$

$V_1^2 = 198.2Q^2$ $V_2^2 = 3192.25Q^2$ $\left(A_2 = \frac{\pi}{4}D_2^2 = 0.0177m^2\right)$

4ai) $f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = 0.08\left(\frac{70}{0.3}\right)\left(\frac{2.11^2}{2g}\right)$ $V_1 = 2.11 m/s$ $V_2 = 8.47 m/s$ (when $Q = 0.15 m^3/s$)

$= 4.24 m$

$f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} = 0.045\left(\frac{100}{0.15}\right)\frac{8.47^2}{19.62}$

$= 109.7m$

$0 = 16 - hp + 4.24 + 109.7$

$hp = 130m$

ii) $0 = 16 - h_p + \sum \frac{L}{g}\frac{dV}{dt} + f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$ $V_1^2 = 198.2Q^2$ $V_2^2 = 3192.25Q^2$

$= 16 + \frac{L_1}{g}\frac{dV_1}{dt} + \frac{L_2}{g}\frac{dV_2}{dt} + 0.951V_1^2 + 1.53V_2^2$

$= 16 + 7.14(14.08)\frac{dQ}{dt} + 10.19(56.5)\frac{dQ}{dt} + 0.951(198.2)Q^2 + 1.53(3192.25)Q^2$

$0 = 16 + 676.3\frac{dQ}{dt} + 5072.6Q^2$

$676.3\frac{dQ}{dt} = -16 - 5072.6Q^2$

$\frac{dQ}{dt} = -0.024 - 7.5Q^2$

$\frac{dt}{dQ} = \frac{1}{-0.024 - 7.5Q^2}$

$\frac{dQ}{0.024 + 7.5Q^2} = -dt$

Let $a^2 = 0.024$

$a = 0.155$

$\frac{dQ}{0.155^2 + 7.5Q^2} = -dt$

$\int_{0.15}^{Q_2} \frac{dQ}{0.155^2 + 7.5Q^2} = -\int_0^{2.5} dt$ $\left(\frac{1}{a}\left(tg^{-1}\frac{Q}{a}\right)_{0.15}^{Q_2}\right) = -(2.5 - 0)$

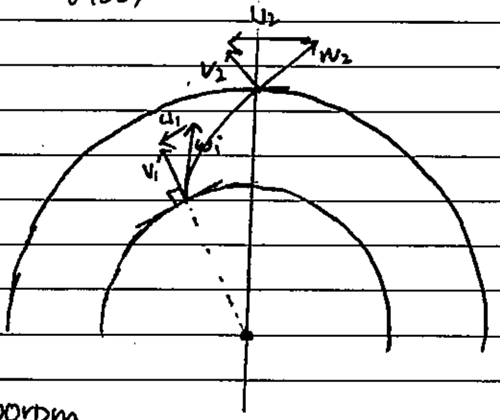
$\frac{1}{0.155}\left[tg^{-1}\left(\frac{Q_2}{0.155}\right)\right]_{0.15}^{Q_2} = -2.5$

4a(ii) $\frac{1}{0.155}\left[tg^{-1}\left(\frac{Q_2}{0.155}\right) - tg^{-1}\left(\frac{0.15}{0.155}\right)\right] = -2.5$

$\left[tg^{-1}\left(\frac{Q_2}{0.155}\right) - 0.769\right] = -0.3875$

$tg^{-1}\left(\frac{Q_2}{0.155}\right) = 0.3815 \rightarrow Q_2 = 0.0622 m^3/s$

4b)

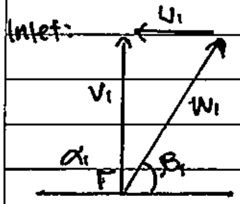


$n = 1200 rpm$

$= 125.7 rad/s \rightarrow U_1 = r_1 \omega_1 = 15.1 m/s \quad U_2 = r_2 \omega = 30.17 m/s$

$Q = 2\pi r_1 b_1 V_{r1} = 2\pi r_2 b_2 V_{r2} = 0.8 m^3/s$

Inlet:

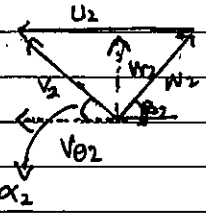


$\alpha_1 = 90°, \alpha_2 = 50°$ $\quad r_1 = 0.12m, b_1 = 0.09m$

$r_2 = 0.24m, b_2 = 0.07m$

(i) $\beta_1 = tg^{-1}\frac{V_1}{U_1} = 38°$ $\quad V_{r1} = 11.79 m/s \quad V_{r2} = 7.58 m/s$

Outlet:



(i) $\beta_2 = tg^{-1}\left(\frac{V_{r2}}{U_2 - V_{\theta 2}}\right) = 17.7°$

$\alpha_2 = 50°$

$V_{r2} = 7.58 m/s$

$\tan 50° = \frac{V_{r2}}{V_{\theta 2}}$

$V_{\theta 2} = 6.36 m/s$

(ii) $T = \rho Q(r_2 V_{\theta 2} - \overset{0}{\cancel{r_1 V_{\theta 1}}})$

$= 1221.12 Nm$

$h_i = \frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g} = 19.6m$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

MA4002 – FLUID DYNAMICS

November/December 2019 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **TWELVE (12)** pages including **EIGHT (8)** pages of Appendices.
2. Answer **ALL** questions.
3. Marks for each question are as indicated.
4. This is a **CLOSED-BOOK** examination.

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1 (a) Air flows through a converging-diverging nozzle (Figure 1) with an area ratio $A_2/A_t = 3$. Stagnation pressure and stagnation temperature at the inlet of the nozzle are $P_0 = 2$ MPa and $T_0 = 370$ K, respectively. A constant-area duct is attached to the nozzle outlet.

(i) If the Mach number at the duct exit (point 3) is $M_3 = 1.5$, compute the Mach number at point 2, the stagnation temperature at point 3 and the heat exchange through the duct. Determine the heat transfer direction and sketch the process from point 1 to point 3 on a T-S diagram.

(ii) If the cross-sectional area of the duct is $A = 0.2\ \text{m}^2$, calculate the mass flow rate.

For air, the specific heat ratio $k = 1.4$, the specific heat at constant pressure $c_p = 1004\ \text{J kg}^{-1}\ \text{K}^{-1}$, and the gas constant $R = 287\ \text{J kg}^{-1}\ \text{K}^{-1}$.

(15 marks)

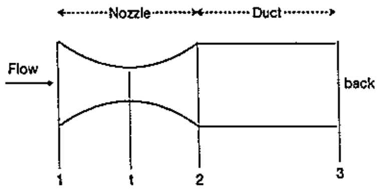

Figure 1

(b) Air flows in a converging-diverging channel. A normal shock occurs at one point of the converging section and the Mach number right in front of the shock is $M_1 = 2$. Another normal shock occurs in the diverging section.
If the areas where the two shocks occur are the same, find the entropy generation in the channel.
For air, the specific heat ratio $k = 1.4$, the specific heat at constant pressure $c_p = 1004\ \text{J kg}^{-1}\ \text{K}^{-1}$, and the gas constant $R = 287\ \text{J kg}^{-1}\ \text{K}^{-1}$.

(10 marks)

---

2 (a) A fluid with density $\rho = 800\ \text{kg m}^{-3}$ and viscosity $\mu = 0.02\ \text{kg m}^{-1}\text{s}^{-1}$ flows at velocity $U = 3\ \text{m s}^{-1}$ over a flat plate of length $L = 3$ m and width $b = 1$ m. The flow direction is parallel to the longer side of the plate. Compute the drag force and drag coefficient for the top surface of the plate given that the dependence of skin friction c_f on the Reynolds number Re_x is

$$c_f = \frac{0.73}{\sqrt{Re_x}}$$

Note that the Reynolds number Re_x is defined as

$$Re_x = \frac{\rho U x}{\mu}$$

Here, x is the streamwise distance from the leading edge of the plate, and τ_w is the shear stress at the surface of the plate.

(15 marks)

(b) Consider two spheres having the same diameter: sphere A is made of steel with density $\rho_A = 7700\ \text{kg m}^{-3}$ and sphere B is made of aluminium with density $\rho_B = 2700\ \text{kg m}^{-3}$. The spheres fall down vertically in a water pool. Due to the difference in surface roughness of the two spheres, they have different drag coefficients. If the two spheres have the same terminal velocity, calculate the drag coefficient ratio C_A/C_B, where C_A and C_B are the drag coefficients of sphere A and sphere B, respectively. Note that water has density $\rho_w = 1000\ \text{kg m}^{-3}$.

(10 marks)

3 (a) Gasoline with a SG=0.72 is pumped by a centrifugal pump. When the flow rate is 0.025 m^3/s, the pump requires 15 kW input, and its efficiency is 85%. Calculate the pressure rise produced by the pump. Express this result as the height of gasoline. The pump efficiency is given by $\eta = \frac{\gamma Q h_a}{Input\ power}$.

(5 marks)

(b) A small centrifugal pump, when tested at 2875 rpm with water, delivered Q = 0.016 m^3/s and h_a = 40 m at its best efficiency of 70%. Determine the specific speed of the pump at this test condition. The specific speed of the pump is given by $N_S = \frac{\omega Q^{1/2}}{(gh_a)^{3/4}}$.

And circle the correct pump type in Figure 2.

(5 marks)

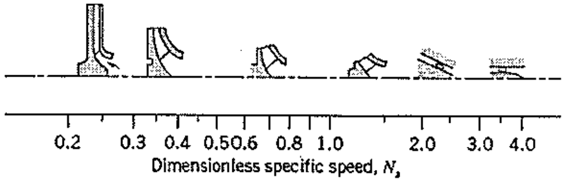

Figure 2

(c) A pump with impeller diameter D = 500 mm delivers Q = 0.725 m^3/s of water at h_a = 10 m at its best efficiency. The specific speed of the pump is 1.74. (The formula for specific speed can be found in previous question) If the pump is now run at 900 rpm, using scaling laws, determine the new flow rate and required input power.

Power coefficient is: $C_P = input\ power/\rho\omega^3 D^5$

Flow coefficient is: $C_Q = Q/\omega D^3$

(5 marks)

(d) A centrifugal pump is used to pump water from a reservoir whose surface is 6.1 m above the centerline of the pump inlet. The diameter of the pipe is 3.05 cm, and the length is 3.66 m. The total minor loss coefficient is 6.8, the friction coefficient f = 0.0174. The pump's required net positive suction head is provided by the manufacturer as a curve fit: $NPSH_{required} = 0.3m + 1.36\,Q^2$ (Q is in L/s). If the pump is operated at Q=2.52 L/s, determine if cavitation will occur by comparing available NPSH and required NPSH. Atmospheric pressure is 100 kPa, and vapor pressure is 3169 Pa.

(15 marks)

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4 (a) A rigid pipe is 200 m long, and the water velocity inside is 3 m/s. If a valve is shut off in 0.2 seconds. Can it induce Joukowski pressure? Assume wave celeric is 1500 m/s. The maximum closure time to give Joukowski head is t = 2L/c. Show your working.

(6 marks)

(b) Consider a two-dimensional flow whose velocity field is given by $\vec{V} = (0.2x^3 + 3xy)\vec{\imath} + (y^3 - 2x^2y^2)\vec{\jmath}$. This velocity fields means the horizontal velocity component is $u(x,y) = 0.2x^3 + 3xy$, and the vertical velocity component is $v(x,y) = y^3 - 2x^2y^2$. Determine if this flow satisfies the continuity condition of an incompressible flow. Show your working.

(8 marks)

Choose the correct answers for the following questions.

(c) How much is the Betz limit (maximum power coefficient) of any wind turbine?

(A) 0.99
(B) 0.59
(C) 0.50

(2 marks)

(d) In order to include the unsteady effects in an incompressible flow in a pipe, the extra term in the energy equation is:

(A) $\frac{L}{g}\frac{dV}{dt}$
(B) $Lg\frac{dV}{dt}$
(C) dV^2/dt

(2 marks)

(e) If one potential flow is expressed as a function of f, and the other potential flow is expressed as a function of g, then the superposition of these two potential flows can be expressed as:

(A) f/g
(B) $f \times g$
(C) $f+g$

(2 marks)

1a) $\frac{A_2}{A_t} = 3$ From Table, $M_2 = 2.64$

$T_0 = T_{02} = 370$ K

$\frac{T_2}{T_{02}} = 0.4177$ (From table)

$T_2 = 154.55$

$\frac{T_{02}}{T_0^*} = 0.6921$ (From raylegh table)

$T_0^* = 534.6$K

$\frac{T_{03}}{T_0^*} = 0.9093$ (From Ray table)

$T_{03} = 486.12$

From $M_2 = 2.64$ to $M_3 = 1.5$

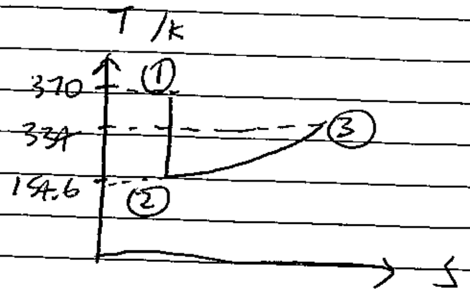
∴ It is heating

$q = C_p(T_{03} - T_{02})$

$= 1004(486.12 - 370) = 116.6$ kJ/kg

$\frac{T_3}{T_{03}} = 0.6869$ (Table)

$T_3 = 334$K



1aii) $M_t = 1$

$\frac{T_t}{T_0} = 0.8333$ (table $\quad$ $\frac{P_t}{P_0} = 0.5283$

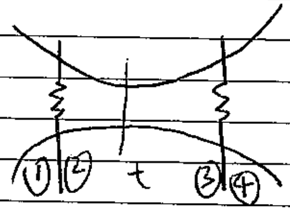
$T_t = 0.8333 \times 370 = 308.3k$ $\quad$ $P_t = 0.5283 \times 2000000$

$V = c = \sqrt{kRT} = 352\ m/s$ $\quad$ $= 1056.6\ kPa$

$\rho = \frac{P}{RT} = 11.94$

$\dot{m} = VA\rho = 8407\ kg/s$

b)



$M_1 = 2$ $\quad$ $M_2 = 0.5774$

$\frac{P_{02}}{P_{01}} = 0.7209$

$\frac{A_2}{A^*} =$ ~~[illegible]~~ 1.2130

$\frac{A_3}{A^*} = 1.2130$ $\quad$ $\therefore$ $M_3 = 1.55$

$M_4 = 0.6841$

$\Delta s = \Delta s_{2-1} + \Delta s_{4-3}$

$= -R \ln \frac{P_{02}}{P_{01}} - R \ln \frac{P_{04}}{P_{03}}$

$= -287 \ln 0.7209 - 287 \ln 0.9132$

$= 119.98\ J/kgk$

2a)

$$C_D = \frac{1}{A}\int_{\text{surface}} \frac{\tau_w}{\frac{1}{2}\rho U^2}\,dA$$

$$= \frac{1}{bL}\int_{\text{surface}} C_f\, b\,dx$$

$$= \frac{1}{3}\int_{\text{surface}} 0.73\left(\frac{\mu}{\rho U x}\right)^{0.5} dx$$

$$= 7.0244\times10^{-4}\int_0^3 x^{-0.5}\,dx$$

$$= 7.0244\times10^{-4}\left[\frac{x^{0.5}}{0.5}\right]_0^3$$

$$= 2.433\times10^{-3}$$

$$F_D = 2.433\times10^{-3}\times\frac{1}{2}(800)(3)^2(3)$$

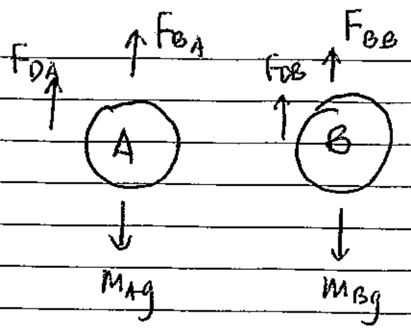
$$= 26.28\ \text{N}$$

2b)

$V \rightarrow$ speed

Diameter same $\therefore \forall$, volume same

A, area same



FBD for A:

$$\rho_A \forall g = C_A \tfrac{1}{2} \rho_w V^2 A + \rho_w \forall g$$

$$(7700 - 1000) \forall g = C_A \tfrac{1}{2} (1000) V^2 A \quad \text{—} ①$$

FBD for B

$$\rho_B \forall g = C_B \tfrac{1}{2} \rho_w V^2 A + \rho_w \forall g$$

$$(2700 - 1000) \forall g = C_B \tfrac{1}{2} (1000) V^2 A \quad \text{—} ②$$

$$\frac{①}{②} \Rightarrow \frac{C_A}{C_B} = \frac{7700-1000}{2700-1000} = 3.94$$

3) a) $\gamma = \rho_g g = (S.G)(\rho_{water})(g)$

$= 0.72(1000)(9.81) = 7063.2\ N/kg$

$\eta = \frac{\gamma Q h_a}{\text{Input Power}} \rightarrow (0.85) = \frac{(7063.2)(0.025)(h_a)}{(15000)}$

$h_a = 72.205m$ #

b) $N_s = \frac{w Q^{\frac{1}{2}}}{(gh_a)^{\frac{3}{4}}} \rightarrow N_s = \frac{(2875 \times (\frac{2\pi}{60}))(0.016)^{\frac{1}{2}}}{(9.81 \times 40)^{\frac{3}{4}}}$

$= 0.43195$ #

second pump from the left chousen

c) $N_s = \frac{W_1 (Q_1)^{\frac{1}{2}}}{(gh_a)^{\frac{3}{4}}} \rightarrow W_1 = \frac{N_s (gh_a)^{\frac{3}{4}}}{(Q_1)^{\frac{1}{2}}} = \frac{(1.74)(9.81 \times 10)^{\frac{3}{4}}}{(0.725)^{\frac{1}{2}}} \times \frac{60}{2\pi}$

$= 608.28\ rpm$

[illegible] [illegible]

$C_{Q_1} = C_{Q_2} \rightarrow \frac{Q_1}{W_1 D^3} = \frac{Q_2}{W_2 D^3} \rightarrow \frac{0.725}{(608.28)} = \frac{Q_2}{(900)}$

$Q_2 = 1.0727\ m^3/s$ #

Input power 1 $= \frac{\rho_w g h_a Q_1}{\eta}$ assume $\eta = 0.7$

Input power $= \frac{(1000)(9.81)(10)(0.725)}{0.7} = 101.6036\ kW$

$C_{P_1} = C_{P_2} \rightarrow \frac{\text{input power } 1}{\rho_w W_1^3 D^5} = \frac{\text{input power } 2}{\rho_w W_2^3 D^5}$

$\downarrow$

$\frac{101.6036 \text{ kw}}{(608.28)^3} = \frac{\text{input power } 2}{(900)^3} \rightarrow$ input power 2 = 329.1 kw #

~~d)~~ ~~$Z + \frac{P_0}{\rho g} = NPSH +$~~

d) $NPSH = Z + \frac{P_0}{\rho g} - \Sigma h_L - \frac{P_v}{\rho g}$

$= Z + \frac{P_0}{\rho g} - \frac{P_v}{\rho g} - \left[f \frac{L}{D} \frac{Q^2}{2gA^2} + K \frac{Q^2}{2gA^2} \right]$

$A = \frac{\pi D^2}{4} = \frac{\pi (0.0305)^2}{4} = 7.30617 \times 10^{-4} \text{ m}^2$

$NPSH = 6.1 + \frac{100 \text{kPa} - 3169 \text{Pa}}{(1000)(9.81)} - \left[0.0174 \left(\frac{3.66}{0.0305} \right) + 6.8 \right] \frac{(2.52 \times 10^{-3})^2}{2(9.81)(7.306 \times 10^{-4})^2}$

$= 6.1 + 9.87064 - [8.488] \cdot [0.60635]$

$=$ ~~10.81~~ 10.824 m

NPSH required $= 0.3 + 1.36 (2.52)^2 = 8.936544 < 10.824$

NPSH available > NPSH required → cavitation would not occur #

4) a) maximum closure time $t = \frac{2L}{c} = \frac{2(200)}{(4500)} = 0.2667\,s > 0.28$

~~Thus~~ Joukowski pressure can be induced!

b) for continuity of a 2-D incompressible flow!

~~$\frac{dx}{dx} + \frac{dv}{dy}$~~ $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$\frac{\partial u}{\partial x} = 0.6x^2 + 3y$ $\quad \frac{\partial v}{\partial y} = 3y^2 - 4x^2y$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.6x^2 + 3y + 3y^2 - 4x^2y \neq 0$

~~from~~ · flow <u>does</u> not satisfy continuity #

c) B

d) A

e) C