NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2008-2009

MP4001/MP4F03 – QUALITY ASSURANCE AND MANAGEMENT

April 2009 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 3 questions and comprises 10 pages which include 4 pages of Appendices.
2. Answer all questions.
3. All marks are as indicated.
4. This is a closed book examination.

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1 (a) Briefly describe the advantages and disadvantages of acceptance sampling compared to 100% inspection.

(6 marks)

(b) For a single sampling plan under the standard SS242:1989 (ISO2859-1974), code letter D, normal inspection and AQL=15%, the inspection results for the last 10 lots are as follows:

Lot number	1	2	3	4	5	6	7	8	9	10
No.of defects	1	0	2	1	1	0	0	1	0	1

If the production is at a steady rate and reduced inspection is authorised, can a change from normal to reduced inspection be initiated?

(4 marks)

/2

Note : Question No. 1 continues on page 2.

(c) Samples of n=5 items are taken from a manufacturing process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and R values are calculated for each sample. After 50 subgroups have been analysed, we have

$$\sum \bar{x}_i = 1000 \text{ and } \sum R_i = 180$$

(i) Compute the control limit for the $\bar{x}$ and R control charts.

(6 marks)

(ii) Assume that all points on both charts fall within the control limits. What is the process standard deviation?

(2 marks)

(iii) If the specification limits are 19 ± 4.0, determine c_{pk}. What are your conclusions regarding the ability of the process to produce items conforming to specifications?

(3 marks)

(iv) Assuming that if an item exceeds the upper specification limit it can be reworked, while if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process now producing?

(2 marks)

(v) If the process were centred at $\mu = 19.0$, what would be the effect on percent scrap and rework? What conclusions would you draw from the new arrangement?

(4 marks)

(d) What does ISO 9001 mean by "Involvement of people" of the Eight Principles of Quality Management?

(3 marks)

2 (a) A process engineer wants to examine the effect of amount of catalyst on the yield of chemical process. Three different levels of amount of catalyst can be used economically. She has run a single-factor experiment, and obtained the following experimental data:

Table 1: Yield of chemical process (%)

Amount of Catalysts	Yield for 5 Observations 1	2	3	4	5
20%	2	4	3	5	3
30%	3	4	5	7	9
40%	6	9	10	7	8

Note : Question No. 2 continues on page 3.

(i) State the assumptions of individual data in Analysis of Variance (ANOVA) for this experiment.

(2 marks)

(ii) Let $y_{i.}$ be the total of the observations at the i^{th} level (i =1, 2, 3) and $y_{..}$ be the grand total of all observations. Calculate $y_{i.}$ and $y_{..}$, respectively.

(2 marks)

(iii) Estimate the i^{th} level effect τ_i (i=1,2,3).

(3 marks)

(iv) Calculate the sum of squares using the following formulae:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n} y_{ij}^2 - \frac{y_{..}^2}{an}$$

$$SS_F = \sum_{i=1}^{a} \frac{y_{i.}^2}{n} - \frac{y_{..}^2}{an}$$

(3 marks)

(v) Carry out the Analysis of Variance (ANOVA) for single-factor experiment to determine the effect of amount of catalyst on the response. The type I error probability α is specified as 0.01 and the control limits $F_{0.01,2,12}$ is 6.93.

(4 marks)

(vi) Discuss the meaning of type I error probability in the conclusion of ANOVA for this single-factor experiment briefly.

(2 marks)

(b) A process engineer is trying to improve the life of a cutting tool. She has run a 2^3 factorial experiment using cutting speed (A), metal hardness (B), and cutting angle (C) as the factors. The data from one replicate are summarized as follow.

Table 2

Run	A	B	C	Response
1	-	-	-	72
2	-	-	+	65
3	-	+	-	85
4	-	+	+	83
5	+	-	-	58
6	+	-	+	53
7	+	+	-	68
8	+	+	+	63

Note : Question No. 2 continues on page 4.

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(i) Estimate the main effects A, B, C, and interactions AB, AC, BC, and ABC. Which effects appear to be large?

(7 marks)

(ii) Calculate the sum of squares for using the following formulae:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}, \text{ Sum of square for any effect} = \frac{(\text{contrast})^2}{N}, \text{ where } N$$

denotes the total number of observations.

(8 marks)

(iii) Propose an approach of Analysis of Variance (ANOVA) for this 2^3 factorial experiment with single replicate to confirm your conclusion of part (i). The type I error probability α is specified as 0.1 and the control limit $F_{0.1,1,1}$=39.9, $F_{0.1,1,3}$=5.54.

(4 marks)

3 (a) A disk drive has a constant failure rate and mean time to failure (MTTF) of 5,000 hour.

(i) What will be the probability of failure be for one year of operation?

(3 marks)

(ii) What will the probability of failure before one year of operation if two drives are placed in parallel, and the failures are independent? We assume that both drives are identical.

(3 marks)

(b) An engineer would like to design a system consisting of two subsystems in series. The reliabilities of the subsystems are: R_1 = 0.98, R_2 = 0.94. The cost of the two subsystems is the same. The engineer decides to add two redundant components. Which of the following options would be better to do? Justify your conclusion.

Option 1: Duplicate subsystems 1 and 2 in high-level configuration
Option 2: Duplicate subsystems 1 and 2 in low-level configuration

(3 marks)

(c) A computer in the computing lab in Nanyang Technolgical University has a mean time to failure (MTTF) = 34 hour and a mean time to repair (MTTR) = 2.5 hour.

(i) What is the (time-independent) availability of computer?

(2 marks)

Note : Question 3 continues on page 5.

(ii) If the MTTR is reduced to 1.5 hour, what MTTF can be tolerated without decreasing (time-independent) availability of the computer?

(2 marks)

(d) All six components in a linked system (in Figure 1) have the same reliability equal to r. Express the system reliability as a function of r.

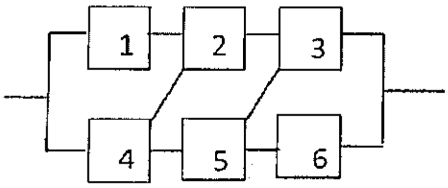

Figure 1

(5 marks)

(e) A laptop in lecture theatre 2 (LT2) in Nanyang Technological University may potentially fair from three failure modes: hardware problem, software problem, and computer virus. Potential time to failure of the laptop for the i^{th} failure mode follows an exponential distribution with mean failure rate λ_i (i=1,2,3). Assume that the laptop is a series system, and the potential times to failure of the laptop for different failure mode are statistically independent. Find the (time-dependent) reliability of the laptop.

(3 marks)

(f) A nonreplacement reliability test is carried out on twenty high speed pumps to estimate the value of the mean failure rate (λ). In order to eliminate wear failures, it is decided to terminate the test after half of the pumps have failed. The times of the first ten failures (in hours) are as follows: 33.7, 36.9, 46.8, 56.6, 62.1, 63.6, 78.4, 79.0, 101.5, 110.2.

(i) Find the total observed operational times of all components.

(2 marks)

(ii) Estimate the mean time to failure (MTTF) and mean failure rate (λ).

(3 marks)

(g) Briefly discuss the following problems:

(i) Describe the purpose, implementation and the recommended action in failure mode and effect analysis (FMEA).

(3 marks)

Note : Question No. 3 continues on page 6.

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(ii) Describe the basic concept, three paradigm, and origin of total quality management (TQM).

(3 marks)

(iii) Describe relative advantage and disadvantage of the use of Weibull distribution in describing mean failure rate in reliability engineering.

(3 marks)

End of Paper

1(a): Advantages:

① More economical as fewer inspection and inspectors are involved.
② Less handling damage during inspection.
③ Applicable to destructive testing.
④ Provides stronger motivation for improvement.

Disadvantages:

① Risk of accepting bad lots and rejecting good lots
② More time and effort are needed for planning and documentation.
③ Less information about the product
④ No insurance that the entire lot confirms to specifications.

1(b) From the table, for code letter D, normal inspection and 15% AQL, the acceptance number $Ac = 3$
The number of the defects of the 10 lots are all less than 3, 10 consecutive batches have been accepted on normal inspection.
Thus a change from normal to reduced inspection can be initiated.

1(c) (i) $\bar{\bar{x}}_t = \frac{1000}{50} = 20$, $\bar{R}_t = \frac{180}{50} = 3.6$

According to shewchart, for $n=5$

$A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.114$

$UCL_{\bar{x}_t} = 20 + 0.577 \times 3.6 = 22.077$

$LCL_{\bar{x}_t} = 20 - 0.577 \times 3.6 = 17.923$

~~$UCL_{R_t} = 2.114 \times 3.6 = 7.6104$~~

$LCL_{R_t} = 0$

(ii) $\sigma = \frac{\bar{R}_t}{d_2} = \frac{3.6}{2.236} = 1.61$

(iii) $USL = 23$, $LSL = 15$

$$C_{pk} = \frac{\min(USL - \bar{\bar{x}}_t, \bar{\bar{x}}_t - LSL)}{3\sigma} = \frac{3}{3 \times 1.61} = 0.621$$

$C_{pk} < 1$, the process is producing product that does not conforms to specifications

(iv) Rework: $z = \frac{USL - \bar{\bar{x}}_t}{\sigma} = \frac{23-20}{1.61} = 1.86$

Percent rework $= 3.14\%$

Scrap:

$z = \frac{\bar{\bar{x}}_t - LSL}{\sigma} = 3.11$

~~Percent scrap $= 0.097\%$~~

(V) when $\mu = 19.0$, Percent rework will decrease and percent scrap will increase.

~~New $C_{pk} = \frac{4}{3 \times 1.61} = 0.828$~~

More percentage of products produced is conforming to specifications.

(d) Involvement of people:
People at all levels are the essense of an organization and their full involvement enables their ability to be used for the organization's benefit.

2(a)(i) Assumptions:
All the other factors which have effect on the response remain constant for the three levels of experiments or the effect of other factors are negligible.

(a)(ii)

$y_{1.} = 2+4+3+5+3 = 17$

$y_{2.} = 3+4+5+7+9 = 28$

$y_{3.} = 6+9+10+7+8 = 40$

$y_{..} = y_{1.} + y_{2.} + y_{3.} = 85$

(a)(iii)

$\tau_1 = \mu_1 - \mu \approx \bar{y}_{1.} - \bar{y}_{..} = \frac{17}{5} - \frac{85}{15} = -2.267$

$\tau_2 = \mu_2 - \mu \approx \bar{y}_{2.} - \bar{y}_{..} = \frac{28}{5} - \frac{85}{15} = -0.067$

$\tau_3 = \mu_3 - \mu \approx \bar{y}_{3.} - \bar{y}_{..} = \frac{40}{5} - \frac{85}{15} = 2.333$

(a)(iv) $a = 3$, $n = 5$

~~$SS_T = \sum_{i=1}^{3}\sum_{j=1}^{5} y_{ij}^2 - \frac{y_{..}^2}{an} = (2^2+4^2+\dots+8^2) - \frac{85^2}{3\times5}$~~

$= 573 - 481.67 = 91.33$

$SS_F = \sum_{i=1}^{3} \frac{y_{i.}^2}{5} - \frac{y_{..}^2}{an} = \frac{1}{5}(17^2+28^2+40^2) - \frac{85^2}{3\times5} = 52.93$

$SS_E = SS_T - SS_F = 38.4$

(a)(V) Degrees of freedom for factor is 2, for error is 12.

Mean square of factor $= \frac{52.93}{2} = 26.465$

~~MS of error $= \frac{38.4}{12} = 3.2$~~

$F_0 = \frac{MS_F}{MS_E} = 8.27 > 6.93$. Since $F_0 > F_{2, v_1, v_2}$, amount of catalyst has significant effect on the process.

2(a)(vi) Type I error probability is the probability of falsely accepting H1 when in factor H0 is true.

2(b)(i)

Run	A	B	C	AB	AC	BC	ABC	Response
1	−	−	−	+	+	+	−	72
2	−	−	+	+	−	−	+	65
3	−	+	−	−	+	−	+	85
4	−	+	+	−	−	+	−	83
5	+	−	−	−	−	+	+	58
6	+	−	+	−	+	−	−	53
7	+	+	−	+	−	−	−	68
8	+	+	+	+	+	+	+	63

$C_A = -72 - 65 - 85 - \cdots + 63 = -63$
$C_B = -72 - 65 + 85 - \cdots + 63 = 51$
$C_C = -72 + 65 - 85 + \cdots + 63 = -19$
$C_{AB} = 72 + 65 - 85 - \cdots + 63 = -11$
$C_{AC} = 72 - 65 + 85 - \cdots + 63 = -1$
$C_{BC} = 72 - 65 - 85 + \cdots + 63 = 5$
$C_{ABC} = -72 + 65 + 85 - \cdots + 63 = -5$

Main effects:

$A = \frac{-63}{4} = -15.75$; $B = \frac{51}{4} = 12.75$

$C = \frac{-19}{4} = -4.75$; $AB = \frac{-11}{4} = -2.75$

$AC = \frac{-1}{4} = -0.25$; $BC = \frac{5}{4} = 1.25$

$ABC = \frac{-5}{4} = -1.25$;

Effects A and B appear to be large.

b(ii)

$SS_A = \frac{C_A^2}{N} = \frac{(-63)^2}{8} = 496.125$;

$SS_B = \frac{51^2}{8} = 325.125$;

$SS_C = \frac{(-19)^2}{8} = 45.125$;

$SS_{AB} = \frac{(-11)^2}{8} = 15.125$;

$SS_{AC} = \frac{(-1)^2}{8} = 0.125$; $SS_{BC} = \frac{5^2}{8} = 3.125$;

~~$SS_{ABC} = \frac{(-5)^2}{8} = 3.125$~~

$SS_T = (72^2 + 65^2 + \cdots + 63^2) - \frac{547^2}{8} = 887.875$

(iii) Now we assume effects A and B ~~are~~ the main ~~effects~~.

$SS_A = 496.125$; $SS_B = 325.125$; $SS_{AB} = 15.125$;

$SS_E = 887.875 - 496.125 - 325.125 - 15.125 = 51.5$

Source of variation	SS	D.O.F	MS	F_0
A	496.125	1	496.125	28.90
B	325.125	1	325.125	18.94
AB	15.125	1	15.125	
Error	51.5	3	17.167	

A and B have values of $F_0 > F_{0.1,1,3} = 5.54$

Thus effects A and B appear to be large.

3(a)(i) $\lambda = \frac{1}{MTTF} = \frac{1}{5000} = 2 \times 10^{-4}$

Probability of failure $F(t) = 1 - e^{-\lambda t}$

$= 1 - \exp(-2 \times 10^{-4} \times 365 \times 24)$

$= 0.8266$

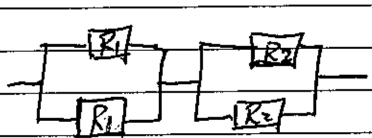
(ii) When two drives are placed in parallel.

$F(t) = 0.8266^2 = 0.6832$

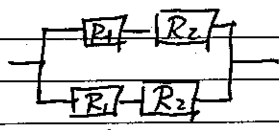
(b) Option 2 will be better to do.

Low-level: $R_S = (2R_1 - R_1^2) \cdot (2R_2 - R_2^2) = 0.9960$

High-level: $R_S = 1 - (1 - 0.98 \times 0.94)^2 = 0.9938$



Low-level



High-level

3(c)(i)

$$\text{Availability} = \frac{34}{34+2.5} = 0.932$$

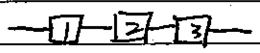
(ii)

$$\frac{MTTF}{MTTF+1.5} \geq 0.932$$

$\therefore$ MTTF $\geq$ 20.4 hours

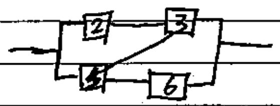
3(d) Use block 4 as the key.

If 4 is failed. The system is:



$R_s^- = r^3$

If 4 is functional, 1 is bypassed. The system is:



Set block 3 as the second key.

when 3 is failed, $R_{ss}^- = r^2$

when 3 is functional, $R_{ss}^+ = 2r - r^2$

Thus $R_s^+ = r^2(1-r) + (2r-r^2)\cdot r$

$= 3r^2 - 2r^3$

The reliability of the original system is:

$$R_s = (1-r)\cdot r^3 + r\cdot(3r^2 - 2r^3)$$
$$= 4r^3 - 3r^4$$

3(e)

$R_i = e^{-\lambda_i t}$

$R_{system} = R_1 \cdot R_2 \cdot R_3 = e^{-\lambda_1 t}\cdot e^{-\lambda_2 t}\cdot e^{-\lambda_3 t}$

$= e^{-(\lambda_1+\lambda_2+\lambda_3)t}$

3(f) (i)

Total times $= 33.7 + 36.9 + 46.8 + 56.6 + 62.1 + 63.6 + 78.4 + 79.0 + 101.5 + 110.2 + 110.2 \times 10$

$= 1770.8$ hours

(ii) $MTTF = \frac{T}{n} = \frac{1770.8}{10} = 177.08$ hours

$\lambda = \frac{1}{MTTF} = 0.00565$

3(g) (i)

FMEA guides how a system should be designed and operated in order to eliminate human error;

It enumerates the potential failure modes of components and traces through the characteristics and consequences of each failure mode on the system as a whole;

It classifies the failures of most products into three main categories: Severity, Occurrence, and Detection.

(ii) TQM

Basic concept: A management approach to long-term success through customer satisfaction.

Three paradigm:

① Increasing employee authority through empowerment

② Forcing on continuous improvement.

③ Everyone must provide data to understand the sources) of variance

Origin: TQM owes origin to Dr. Armand Feigenbaum and his book 'Total Quality Control'.

(iii)

Disadvantage: More complicated:

Advantage: Three types of fails can be represented.

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2009-2010

MP4001/MP4F03 – QUALITY ASSURANCE AND MANAGEMENT

November/December 2009 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 3 questions and comprises 8 pages which include 4 pages of Appendices.
2. Answer all questions.
3. Marks for each question are as indicated.
4. This is a restricted-open book examination which allows one double-sided A4 size cheat sheet.

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1 (a) Control charts for $\bar{x}$ and R are maintained on a certain dimension of a manufactured part, measured in mm. The subgroup size is 4. The values of $\bar{x}$ and R are computed for each subgroup. After 20 subgroups, $\sum \bar{x} = 41.34$ and $\sum R = 0.32$.

(i) Compute the values of the 3σ limits for the $\bar{x}$ and R charts, and estimate the value of σ on the assumption that the process is in statistical control.

(8 marks)

(ii) The specification of the dimension is given as 2.05 ± 0.02 mm. Compute the potential capability index and comment on the ability of this process to meet specifications.

(2 marks)

(iii) If the dimension falls above USL, rework is required; if below LSL, the part must be scrapped. If the process is in statistical control and normally distributed, what can you conclude regarding its ability to meet specifications? Can you make any suggestions for improvement?

(8 marks)

(b) The producer's risk is defined by $\alpha = 0.05$ for 1.5% defective product and the consumer's risk is defined by $\beta = 0.10$ for 4.5% defective product. Derive a single sampling plan that exactly meets the producer's stipulation and comes as close as possible to the consumer's stipulation.

(8 marks)

Note : Question No. 1 continues on page 2.

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(c) What does ISO 1901 mean by "Continual improvement" of the Eight Principles of Quality Management?

(4 marks)

2 (a) An ANOVA study has concluded that the strength (y) of a new synthetic fiber (in kpsi) is significantly affected by the percentage (x) of cotton used in the blend of materials for the fiber. The data of the ANOVA are shown in Table 1.

Table 1

No.	Cotton percentage (x)	Fiber strength (y)	
1	10	9	12
2	15	15	14
3	20	22	22

(i) Calculate the error for each observation of y.

(4 marks)

(ii) Check if the mean (μ_1) of the fiber strength at ($x = 10$) differs from the mean (μ_3) of the fiber strength at ($x = 20$) using the following information:

$SS_T = 141.33$ and $SS_F = 136.33$, obtained during ANOVA.
$\pm t_{0.05}(6) = 2.447$, $\pm t_{0.05}(3) = 3.182$ (α is specified as 0.05).

(6 marks)

(b) A 2^2 experiment with a replicate of ($n = 3$) was run to study two variables A and B that probably affect the quality of an electronic component. It is also suspected that there is an interaction between A and B. Various sum of squares have been computed and displayed in the ANOVA table (Table 2). Complete this ANOVA table and find which effects are significant for $\alpha = 0.05$.

Note: $F_{0.05,1,4} = 7.71$, $F_{0.05,1,8} = 5.32$, $F_{0.05,4,8} = 3.84$.

Table 2

Source of variation	Sum of squares	Degrees of freedom	Mean square	F_0
A	4.08			
B	168.75			
AB	90.75			
Error	135.33			
Total				

(7 marks)

Note : Question No. 2 continues on page 3.

(c) A 2^3 design includes three factors A, B and C. The four runs — (1), *a*, *bc* and *abc* — from this 2^3 design are used to constitute a 2^{3-1} one-half fractional design.

(i) What is the generator in this fractional design and what are the aliases of the main effect B and interaction AC?

(6 marks)

(ii) Comment on the goodness of this one-half fractional design.

(3 marks)

(d) A 2^{5-2} fractional design is carried out to investigate the effects of five factors A, B, C, D and E on yield (A = condensation temperature, B = amount of water, C = solvent volume, D = condensation time, E = amount of raw material). The basic generators are ACE and BDE. Work out the plus-minus-sign table for this fractional design using factors A, B and C as the basic factors. Also estimate the main effect A based on the following observations from the 2^{5-2} design.

$e = 23$	$ad = 29$	$cd = 24$	$bde = 14$
$ab = 25$	$bc = 16$	$ace = 35$	$abcde = 28$

(9 marks)

3 (a) Briefly answer the following questions:

(i) What are the main methods to improve the reliability of a system?

(3 marks)

(ii) Interpret the following equations:

$$R(0) = 1, \quad R(\infty) = 0, \quad R(t) + F(t) = 1$$

(3 marks)

(b) An aircon system is started once at 8:30 am every day with a demand failure probability of 0.00008. It will then work 10 hours continuously with a failure rate of 0.00002/hr. Calculate the reliability of the aircon system for a time period of one year (365 days).

(7 marks)

(c) A system consists of m parallel branches. Each branch has two components in series connection. Figure 1 shows the block diagram when $m = 2$. All of the components have the same constant failure rate $\lambda = 0.0002$/hr. If the reliability of the system must be no less than 0.95 for a time period of 1000 hours, determine the number m of the branches.

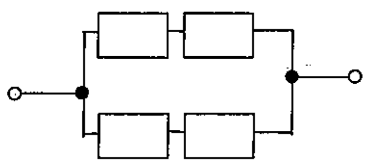

Figure1

(7 marks)

Note : Question No. 3 continues on page 4.

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(d) Figure 2 displays the reliability block diagram of a system. Each block represents a component. The reliability of block i is R_i. Express the system reliability R_s in terms of the component reliabilities R_i (i = 1, 2, ..., 7).

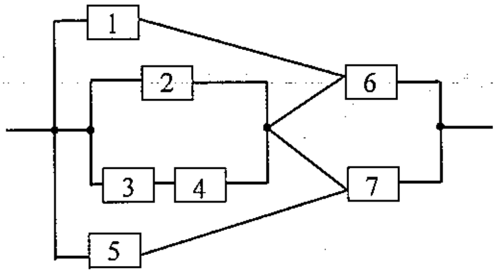

Figure 2

(9 marks)

(e) A nonreplacement test is carried out on 10 specimens of a product. The first five failure times (in hours) are recorded as 10.5, 13.5, 20.0, 23.6 and 27.4. The test is terminated immediately after the fifth failure is observed. Examination of the failed specimens reveals that the second failure is caused by a failure mode that is not under investigation. Estimate the failure rate of the product.

(6 marks)

End of Paper

Appendices

DERIVATION OF SINGLE SAMPLING PLANS

Values of np_1 and c for constructing single sampling plans whose OC curve is required to pass through the two points $(p_1, 1 - \alpha)$ and (p_2, β)*

(Here p_1 is the fraction defective for which the risk of rejection is to be α, and p_2 is the

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1.(a) (i) $n=4$ $m=20$ $A_2=0.729$ $D_3=0$ $D_4=2.282$ $d_2=2.059$ $d_3=0.8798$

$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{m} = \frac{41.34}{20} = 2.067$ $\bar{R} = \frac{\Sigma R}{m} = \frac{0.32}{20} = 0.016$

$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$
$= 2.067 + (0.729)(0.016)$
$= 2.079$

$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$
$= 2.067 - (0.729)(0.016)$
$= 2.055$

$UCL_R = D_4\bar{R}$
$= 2.282(0.016)$
$= 0.0365$

$LCL_R = 0$

$\sigma = \frac{\bar{R}}{d_2} = \frac{0.016}{2.059} = 0.0078$

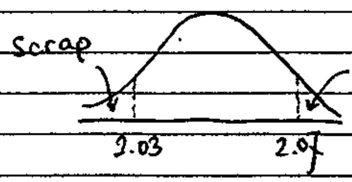
(ii)

$C_p = \frac{2.07 - 2.03}{6 \times 0.0078} = 0.8547 < 1$

$\Rightarrow$ This process is NOT capable.

(iii)

$\bar{x} = 2.067$ $\sigma = 0.0078$



① $\frac{2.03 - 2.067}{0.0078} = -4.74$

$P(Z < -4.0) = 0.00003 = 0.003\%$
$\Rightarrow P(Z < -4.74) < 0.003\%$
$\Rightarrow$ Less than 0.003% will be scrapped

② $\frac{2.07 - 2.067}{0.0078} = 0.38$

$P(Z > 0.38) = 0.352 = 35.2\%$
$\Rightarrow$ 35.2% will be reworked.

Suggestion: The center & spread of the process could be improved to reduce rework and scrap.
(e.g. the center 2.067 is too close to the upper limit 2.07; it results in great amount of rework (35.2%).)

1(b) $\alpha = 0.05 \Rightarrow P_a = 0.95$ $\beta = 0.10 \Rightarrow P_a = 0.10$

$AQL = 1.5\% = 0.015$ $LQL = 4.5\% = 0.045$

$$\frac{P_2}{P_1} = \frac{0.045}{0.015} = 3 = \frac{P_{0.1}}{P_{0.95}}$$

$\Rightarrow c=6$ or $c=7$ from Appendix 1. (Page 5)

Producer: $c=6$ $AQL = 1.5\% = 0.015$

$nP_{0.95} = 3.286$ $n = \frac{3.286}{0.015} = 219$

$c=7$ $nP_{0.95} = 3.981$ $n = \frac{3.981}{0.015} = 265$

Consumer: $c=6$ $LQL = 4.5\% = 0.045$ $P_a = 0.10$ $n = 219$

$nP_{0.1} = 10.532$ $P_{0.10} = \frac{10.532}{219} = 0.048 = 4.8\%$

$c=7$ $n=265$

$nP_{0.10} = 11.71$ $P_{0.10} = \frac{11.71}{265} = 0.044 = 4.4\%$

$\Rightarrow$ 4.4% is closer to consumer's stipulation (4.5%)

$\Rightarrow$ Sampling Plan. $c=7$ $n=265$

* $nP_{0.95}$ & $nP_{0.1}$ values found from Appendix 2 (Page 6).

1. (c) Continual Improvement:-

Continual Improvement of the organization's overall performance should be a permanent objective of the organization.

* Performance advantage through improved organizational capabilities, alignment of improvement activities at all levels to an organization's strategic intent, flexibility to react quickly to opportunities

2(a) (i) error $\varepsilon_{ij} = y_{ij} - \bar{y}_{i.}$

$\bar{y}_{1.} = \frac{1}{2}(9+12) = 10.5$ $9-10.5 = -1.5$ $12-10.5 = 1.5$
$\bar{y}_{2.} = \frac{1}{2}(15+14) = 14.5$ $15-14.5 = 0.5$ $14-14.5 = -0.5$
$\bar{y}_{3.} = \frac{1}{2}(22+22) = 22$ $22-22 = 0$ $22-22 = 0$

(ii) $\mu_1 = 10.5$ $\mu_3 = 22$
$DOF_E = 3\times(2-1) = 3$ $SS_E = SS_T - SS_F = 141.33 - 136.33 = 5$
$MS_E = \frac{5}{3}$ $n = 2$

$$t = \frac{\bar{y}_{i.} - \bar{y}_{j.}}{\sqrt{2MS_E/n}} = \frac{10.5 - 22}{\sqrt{2\times(5/3)/2}} = -8.9$$

v.s. $\pm t_\alpha(dof) = + t_{0.05}(3) = 3.182$

$t >$ $\pm t_\alpha(dof) \Rightarrow$ there is difference between 2 levels ($x=10$ and $x=20$)

2(b) $a=2$ $b=2$ $n=3$ $MS = \frac{SS}{DOF}$ e.g. $\frac{135.33}{8} = 16.92$

	SS	DOF	MS	$F_0 = \frac{MS_F}{MS_E}$	$F_{0.05,1,8}$
A	4.08	2-1=1 [* a-1]	4.08	$\frac{4.08}{16.92} = 0.24$	< 5.32
B	168.75	2-1=1 [* b-1]	168.75	168.75/16.92 = 9.97	> 5.32
AB	90.75	(2-1)(2-1)=1 [*(a-1)(b-1)]	90.75	90.75/16.92 = 5.36	> 5.32
Error	135.33	2x2x(3-1)=8 [*ab(n-1)]	16.92		
Total	398.91	8+1+1+1=11 [*abn-1] or. 2x2x3-1=11			

* choose $F_{0.05,1,8} = F_{\alpha,\nu_1,\nu_2}$ for A, B & AB.
∵ A: $\alpha = 0.05$ $\nu_1 = a-1 = 1$ $\nu_2 = ab(n-1) = 8$
B: $\alpha = 0.05$ $\nu_1 = b-1 = 1$ $\nu_2 = ab(n-1) = 8$ → $F_{0.05,1,8}$
AB: $\alpha = 0.05$ $\nu_1 = (a-1)(b-1) = 1$ $\nu_2 = ab(n-1) = 8$

F_0 v.s $F_{0.05,1,8}$
⇒ A: Not significant ($F_0 < F_{0.05,1,8}$)
B & AB: Significant ($F_0 > F_{0.05,1,8}$)

2(c) (i)

	A	B	C	AC	AB
(1)	-	-	-	+	+
a	+	-	-	-	-
bc	-	+	+	-	-
abc	+	+	+	+	+

*① Sign table of A . B . C from (1) . a . bc . abc

e.g. (1) ⇒ - - -
a ⇒ + - -
bc ⇒ - + +
abc ⇒ + + +

② Sign table of AC . AB from signs of A . B . C

e.g. - - - ⇒ + for AC & AB
+ - - ⇒ - for AC & AB

It can be observed that:

$B \hat{=} C$ $AC \hat{=} AB$ (equal by sign)

we can know that:

$B(\underline{BC}) \hat{=} C$ & $AC(\underline{BC}) \hat{=} AB$

Thus. generator: BC
alias of B : C
alias of AC : AB

(ii)
Efficient ① reduce experiment cost ;
② Accuracy is remained although there are less runs.

2 (d) ACE & BDE $(ACE)(BDE) \hat{=} ABCD$ (3 generators)

$D(ABCD) \hat{=} ABC$ $E(ACE) \hat{=} AC$

	A	B	C	$D \hat{=} ABC$	$E \hat{=} AC$	
e	-	-	-	-	+	23
ad	+	-	-	+	-	29
bde	-	+	-	+	+	14
cd	-	-	+	+	-	24
ab	+	+	-	-	-	25
ace	+	-	+	-	+	35
bc	-	+	+	-	-	16
abcde	+	+	+	+	+	28

* e.g. "-" for A . B . C ⇒ "-" for $ABC \hat{=} D$ & "+" for $AC \hat{=} E$

~~⇒ "-" for A . B . C . D & "+" for E~~

From "+" or "-" of A . B . C . D . E ⇒ run notations e ad...

$C_A = -e + ad - bde - cd + ab + ace - bc + abcde$
$= -23 + 29 - 14 - 24 + 25 + 35 - 16 + 28 = 40$

$A \approx l_A = \frac{C_A}{N/2} = \frac{40}{8/2} = 10$

3. (a) (i) ① Improve reliability of the components;
② Adopt redundancy;
③ Improve maintenance.

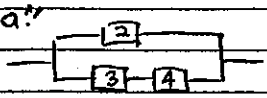
(ii) $R(0)=1$: $t=0$, the system must be working.
$R(\infty)=0$: $t=\infty$, the system will definitely fail.
$R(t)+F(t)=1$: at any time, the sum of reliability and unreliability is 1.

(b) demand failure. $R_d = e^{-np}$
continuous failure. $R_c = e^{-\lambda t}$

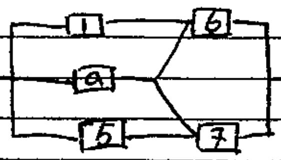
$R_s = R_d \times R_c = e^{-365\times 0.00008} \times e^{-10\times 365 \times 0.00002} \approx 90\%$

(c) $R_1 = R\times R = e^{-\lambda t} \times e^{-\lambda t} = e^{-2\lambda t} = e^{-2\times 0.0002 \times 1000} = e^{-0.4} = 0.67$

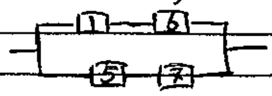
Parallel: $R_s = 1-(1-R_1)^m \geq 0.95$ (R_s → System Reliability)
$(1-0.67)^m \leq 0.05$
$0.33^m \leq 0.05$
$0.33^2 = 0.1089 > 0.05$ & $0.33^3 = 0.036 < 0.05$
$\Rightarrow m \geq 3 \Rightarrow (1-0.67)^m \leq 0.05 \Rightarrow R_s = 1-(1-0.67)^m \geq 0.95$

(d) "a":


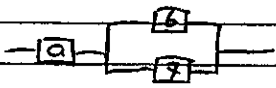
$R_a = R_2 + R_3R_4 - R_2R_3R_4$

System:


*a is the key

① when "a" fails. $F_K = 1 - R_a$


$R_s^- = R_1R_6 + R_5R_7 - R_1R_6R_5R_7$

② when "a" functions. $R_K = R_a$
① & ⑤ are bypassed.


$R_s^+ = R_a(R_6 + R_7 - R_6R_7)$

System reliability: $R_s = F_K R_s^- + R_K R_s^+$
$= (1-R_a)R_s^- + R_a R_s^+$

3 (e). $t_1 \quad t_3 \quad t_4 \quad t_5 \quad t_5$ → the last 5 specimens

$$T^* = 10.5 + 20 + 23.6 + 27.4 + 27.4 \times 5$$
$$= 218.5 \text{ hrs}$$

$$MTTF = \frac{T^*}{5-1} = \frac{218.5 \text{ hrs}}{4} = 54.625 \text{ hrs}$$

$$\lambda = \frac{1}{MTTF} = \frac{1}{54.625 \text{ hrs}} = 0.0183/hr$$

* exclude $t_2 = 13.5$, because the 2^{nd} failure is caused by a failure mode that's not under investigation.

T^*: total observed operational time of all components.

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2010-2011

MP4001/MP4F03 – QUALITY ASSURANCE AND MANAGEMENT

May 2011 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 3 questions and comprises 5 pages which include 1 page of Appendix.
2. Answer all 3 questions.
3. Marks for each question are as indicated.
4. This is a restricted open-book examination which allows one double-sided A4 size cheat sheet.

---

1 (a) Table 1 displays the observations from a single factor experiment (replicate $n = 2$).

Table 1

Factor Temperature (T) ($^{\circ}$C)	Observations Number of products per hour (m)
10	2, 3
20	3, 5
30	7, 7

Use Analysis of Variance (ANOVA) to determine whether the temperature (T) has significant effect on the number of products per hour (m) (Note: use α = 0.05, $F_{0.05,2,6} = 5.14$, $F_{0.05,2,3} = 9.55$, $F_{0.05,3,3} = 9.28$)

(7 marks)

Note : Question No. 1 continues on page 2.

(b) Table 2 displays the observations from a 2^3 design.

Table 2

Run	A	B	C	Observations
1	-	-	-	1
2	+	-	-	3
3	-	+	-	1
4	+	+	-	2
5	-	-	+	-1
6	+	-	+	4
7	-	+	+	-2
8	+	+	+	3

Determine the test ratio F_0 of interaction BC for the purpose of ANOVA.

(7 marks)

(c) For the 2^{5-2} design in Table 3,

Table 3

Run	A	B	C	D	E	result
1	-	-	-	+	+	1
2	+	-	-	-	-	3
3	-	+	-	-	+	0
4	+	+	-	+	-	2
5	-	-	+	+	-	-1
6	+	-	+	-	+	4
7	-	+	+	-	-	-2
8	+	+	+	+	+	3

(i) Which of ABD, ACE and ABC are the generators for this 2^{5-2} design?

(3 marks)

(ii) Write down all the generators for this 2^{5-2} design and all the aliases of interaction BC.

(3 marks)

(iii) Project the 2^{5-2} design onto a 2^2 design in terms of the two most effective factors. Present this 2^2 design, together with the results (observations), by means of a Plus and Minus Signs Table.

(7 marks)

(iv) For the resultant 2^2 design in (iii), calculate the sum of squares, mean square and degrees of freedom of the interaction between these two most effective factors, and also determine the degrees of freedom of the error.

(6 marks)

2 (a) An individual thermocouple of a particular design has a constant failure rate of $\lambda = 0.008$ hr^{-1}. How many thermocouples must be placed in parallel if the system reliability for 100 hours operation must be no smaller than 0.95?

(7 marks)

(b) All of the five components in Figure 1 have the same reliability R. Calculate the system reliability in terms of R.

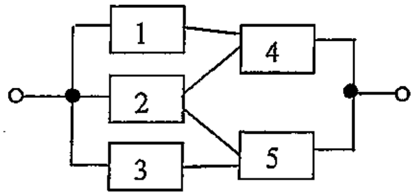


Figure 1

(8 marks)

(c) Four time intervals (in hours) of operation between successive failures of an air-conditioning equipment are listed below:

413, 14, 58, 37

The times appear in the order of occurrence (i.e., the first failure occurred at 413 hr, the second at 413 + 14 = 427 hr, and so on). Calculate $F(t_2)$, $R(t_2)$, $f(t_2 \le t \le t_3)$, $\lambda(t_2 \le t \le t_3)$ and MTTF (note, ($t_2 = 427$) is the time when the second failure occurs).

(10 marks)

(d) A bearing in a compressor is designed for 5 years of operation. The failure of the bearing is mainly due to wearout and is described by a Weibull distribution with the parameters of $\theta = 8$ and $m = 3$.

(i) What is the reliability of the bearing if no preventive maintenance is performed over the 5-year design life?

(3 marks)

(ii) What is the reliability of the bearing over the 5-year life if it is regularly maintained with a time interval $T = 1$ year and the repair error p (i.e. the probability that a faulty maintenance causes the system to fail immediately) is negligible?

(3 marks)

(iii) What is the reliability of the bearing over the 5-year life if it is regularly maintained with a time interval $T = 1$ year and the repair error p is equal to 0.03?

(3 marks)

24

3 (a) A np control chart is to be designed to monitor the fraction nonconforming p of canned food from a production line. The in-control fraction nonconforming p_0 is estimated to be 0.02 from the historic data. The sample size n is selected as 30.

(i) Determine the central line and the integral control limits of the np chart.

(6 marks)

(ii) If it is required that the np chart be able to detect decreasing p shift (i.e., $p < p_0$), what is the minimum value of the sample size n?

(7 marks)

(b) An $\bar{x}$ &R chart is monitoring the mean μ and variance σ^2 of the diameter x of a shaft.

(i) In the 5th sample, the sample mean $\bar{x}$ falls below the lower control limit $LCL_{\bar{x}}$ of the $\bar{x}$ chart and the sample range R falls between the lower control limit LCL_R and upper control limit UCL_R of the R chart. What is the process status in terms of the mean and variance of x?

(3 marks)

(ii) In the 15th sample, $\bar{x}$ falls above the upper control limit $UCL_{\bar{x}}$ of the $\bar{x}$ chart and R also falls above the upper control limit UCL_R of the R chart. What is the process status in terms of the mean and variance of x?

(3 marks)

(c) One of the following two single sampling plans is to be selected to inspect the lots of a machine component in which AQL = 0.01 and RQL = 0.05 (AQL and RQL indicate the Acceptable Quality Level and Rejectable Quality Level, respectively).

Plan 1: sample size ($n = 50$) and acceptance number ($c = 1$).
Plan 2: sample size ($n = 50$) and acceptance number ($c = 2$).

The quality cost U is calculated by

$$U = 70\alpha + 20\beta,$$

where α is the probability of the producer's risk and β is the probability of the customer's risk. Which is the better plan in terms of the quality cost? Or in other words, which plan results in lower U value?

(9 marks)

(d) The length x of a component follows a normal distribution with $\mu = 10$ cm and $\sigma = 0.2$ cm. It is required that x be maintained between 9.5 cm and 10.4 cm, i.e., $LSL = 9.5$ cm and $USL = 10.4$ cm. Calculate the number of defective Parts Per Million (PPM).

(5 marks)

End of Paper

Semester 2 Examination 2010-2011

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MP4001/MP4F03

1)(a) $n=2$, $a=3$. $y_{..}=27$

$$SS_T = \sum_{i=1}^{3}\sum_{j=1}^{2} y_{ij}^2 - \frac{y_{..}^2}{an}$$

$$= (2^2+3^2+3^2+5^2+7^2+7^2) - 27^2/6$$

$$= 145 - 121.5$$

$$= 23.5$$

$$SS_F = \sum_{i=1}^{3} y_{i.}^2/n - y_{..}^2/an$$

$$= \frac{(5^2+8^2+14^2)}{2} - \frac{27^2}{6}$$

$$= \frac{285}{2} - \frac{729}{6}$$

$$= 21$$

$$SS_E = SS_T - SS_F$$

$$= 2.5$$

DOF for $SS_T = an-1 = 5$

DOF for $SS_F = a-1 = 2 = V_1$

DOF for $SS_E = a(n-1) = 3 = V_2$

$MS_F = \frac{21}{2} = 10.5$

$MS_E = \frac{2.5}{3} = 0.833$

$F_0 = MS_F / MS_E = 12.6$

using $F_{0.05,2,3} = 9.55$, $F_0 > F_{0.05,2,3}$.

Hence, temperature (T) has significant effect on the number of products per hour (m).

(1) (b) This is a single replicate design.

Run	A	B	C	BC	ABC	Observations
1	−	−	−	+	−	1
2	+	−	−	+	+	3
3	−	+	−	−	+	1
4	+	+	−	−	−	2
5	−	−	+	−	+	−1
6	+	−	+	−	−	4
7	−	+	+	+	−	−2
8	+	+	+	+	+	3

$C_{BC} = +(1) + (3) - (1) - (2) - (-1) - (4) + (-2) + 3$
$= -1$

$N = 2^k n = 2^3 \times 1 = 8$

$SS_{BC} = \frac{C_{BC}^2}{N} = \frac{(-1)^2}{8} = \frac{1}{8}$ //

$SS_E = SS_{ABC}$ due to single replicate.

$C_{ABC} = -1 + 3 + 1 - 2 - 1 - 4 + 2 + 3$
$= 1$

$SS_{ABC} = \frac{1}{8}$
$= SS_E$

$DOF_{BC} = DOF_E = 1$ //

$MS_{BC} = MS_E = \frac{1}{8}$

Hence, $F_0 = MS_{BC} / MS_E$
$= 1$ //

(I) (c) Plus-minus sign Table for ABD and ACE and ABC.

(i)

Run	ABD	ACE	ABC
1	+	+	−
2	+	+	+
3	+	+	+
4	+	+	−
5	+	+	+
6	+	+	−
7	+	+	−
8	+	+	+

Since ABD and ACE constantly have "+" sign, they are the generators for this 2^{5-2} design

(ii) Third generator (induced):

$(ABD)(ACE) \hat{=} A^2BCDE \hat{=} BCDE$

∴ All generators are ABD, ACE and BCDE.

Alias for interaction BC

from ABD : $BC \hat{=} BC(ABD) \hat{=} ACD$

from ACE : $BC \hat{=} BC(ACE) \hat{=} ABE$

from BCDE : $BC \hat{=} BC(BCDE) \hat{=} DE$

(iii) $A \hat{=} BD \hat{=} CE \hat{=} ABCDE$

$B \hat{=} AD \hat{=} ABCE \hat{=} ACDE$

$C \hat{=} ABCD \hat{=} AE \hat{=} BDE$

$D \hat{=} AB \hat{=} ~~B~~ ACDE \hat{=} BCE$

$E \hat{=} ABDE \hat{=} AC \hat{=} BCD$.

$N = 2^{5-2} n$
$= 8$

$l_A = \left(\frac{1}{8}\right)(-1+3+0+2+1+4+2+3)$
$= 1.75$

$l_B = -0.5$

$l_C = -0.25$

$l_D = 0$

$l_E = 0.75$

Assuming higher order effects are negligible, $A \approx l_A$ and $E \approx l_E$, The ~~most~~ two most significant factors are A and E

Please turn over.

(iii) continued.

The 2^2 design table.

Run	A	E	AE	Results (Observations)
(1)	−	−	+	−1, −2
a	+	−	−	3, 2
e	−	+	−	1, 0
ae	+	+	+	4, 3

(iv) C_{AE} * • Calculate the totals of the runs

$(1) = -3$

$a = 5$

$e = 1$

$ae = 7$

• Calculate the contrast

$C_{AE} = (1) - a - e + ae = (-3) - 5 - 1 + 7$

$= -2$

• Calculate Sum of Squares

$SS_{AE} = \frac{C_{AE}^2}{N} = \frac{(-2)^2}{8} = 0.5$ //

DOF for $SS_{AE} = (a-1)(e-1)$ where $a=2$, $e=2$

$= 1$ //

$MS_{AE} = SS_{AE} / \text{DOF for } AE$

$= 0.5$ //

DOF for error $= ab(n-1)$

$= (2)(2)(2-1)$

$= 4$ //

2 (a)

For each thermocouple, $R = e^{-\lambda t}$

$= e^{-(0.008)(100)}$

$= 0.449$

For parallel N thermocouples, $R_s = 1-(1-R)^N$ and $R_s \geq 0.95$.

$1-(1-R)^N \geq 0.95$

$0.05 \geq (1-R)^N$

let $(1-R)^N = 0.05$

$N \ln(1-0.449) = \ln 0.05$

$N \ln(0.551) = \ln 0.05$

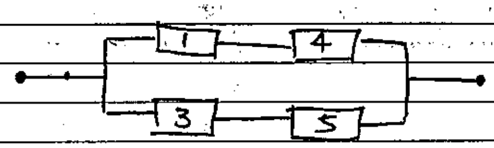
$N(-0.596) = -2.9957$

$N = 5.026$

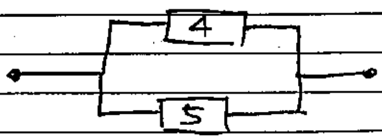
Hence, $N \geq 5.026$ and final $N = 6$ //.

(b) Linked configuration: [2] is key.

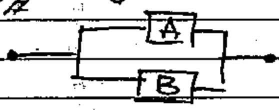
R_s^-



R_s^+



R_s^-:



$R_s^+ = R_4 + R_5 - R_4R_5$

$= R + R - RR$

$= 2R - R^2$

$R_A = R_1R_2 = R^2$

$R_B = R_4R_5 = R^2$

$R_s^- = R_A + R_B - R_AR_B$

$= R^2 + R^2 - R^2R^2$

$= 2R^2 - R^4$

$\therefore R_s = R_s^-(1-R_k) + R_s^+R_k$ whr $R_k = R$

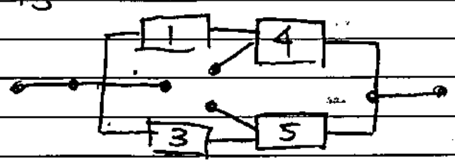
$= (2R^2 - R^4)(1-R) + R(2R-R^2)$

$= 2R^2 - 2R^3 - R^4 + R^5 + 2R^2 - R^3$

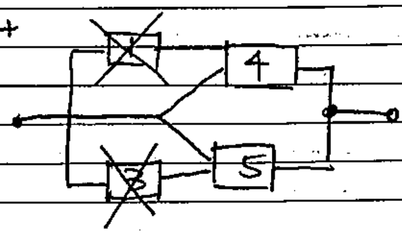
$= R^5 - R^4 - 3R^3 + 4R^2$ //.

* Bonus: for illustration only

R_s^-



R_s^+



* 1 & 3 are bypassed!

2(c)

i	t_i
0	0.00
1	413
2	427
3	485
4	522

$\hat{F}(t_2) = \frac{2}{N+1} = \frac{2}{4+1} = \frac{2}{5}$ //

$\hat{R}(t_2) = 1 - \hat{F}(t_2) = \frac{3}{5}$ //

$f(t_2 \leq t \leq t_3) = \frac{\hat{F}(t_3) - \hat{F}(t_2)}{t_3 - t_2} = \frac{\frac{3}{5} - \frac{2}{5}}{58}$

$= 0.00345$ //

$\lambda(t_2 \leq t \leq t_3) = \frac{0.00345}{3/5} = 0.00575$ //

$MTTF = \frac{1}{N}\sum_{i=1}^{N} t_i = \frac{1}{4}(413 + 427 + 485 + 522)$

$= 461.75$ hours.

2(d) (i) $R_b(T_d) = e^{-(T_d/\theta)^m}$ where T_d = design life span.

$= e^{-(5/8)^3}$

$= 0.7834$ //

(ii) $T = 1$ yr

With maintenance, $R_M(T_d) = R_b(T)^N$ where $N = 5$

and $R(T) = e^{-(T/\theta)^m} = e^{-(1/8)^3} = 0.998$

$\therefore R_M(T_d) = (0.998)^5 = 0.9903$ //

(iii) $T = 1$ yr, $p = 0.03$.

$R_M(T_d) = R(T)^N (1-p)^{N-1}$

$= (0.9903)(1 - 0.03)^4$

$= 0.8767$ //

3(a)

(i) $n = 30$, $p_0 = 0.02$

$UCL_d = CL_d + 3\sigma_d = np_0 + 3\sqrt{np_0(1-p_0)}$
$= 30(0.02) + 3\sqrt{(30)(0.02)(1-0.02)}$
$= 0.6 + 2.3$
$= 2.9$ //

$CL_d = np_0 = 0.6$ //

$LCL_d = CL_d - 3\sigma_d = 0.6 - 2.3$
$= -1.7$

but LCL_d cannot be negative $\therefore LCL_d = 0$ //

(ii) In order to detect decreasing p shift, $LCL_d = 0.6$

$n(0.02) - 3\sqrt{n(0.02)(1-0.02)} = 0.6$
$0.02n - 0.42n^{1/2} - 0.6 = 0.$

let $0.02A^2 - 0.42A - 0.6 = 0$ where $A = n^{1/2}$.

$A = 22.343$ or $A = -1.343$
$n^{1/2} = 22.343$ $n^{1/2} = -1.343$
* $n = 500$ // $n = 1.8$ //
↳ gives $LCL_d = 0.609$ //. ↳ gives $LCL_d = -0.6$ (wrong answer).

$\therefore$ Minimum n is 500 to detect decreasing p shift.

(b)
(i)
It means that the mean value of x has decreased but the variance/spread of x is still within specification. Process is considered to be out of control.

(ii) It means that the mean value of x has increased ~~but~~ and the variance of x has increased. The product quality deteriorates and process is considered out of control.

3(c) AQL = 0.01 , RQL = 0.05

For Plan 1, n = 50, c = 1

$$P_a|_{p=AQL} = \sum_{i=0}^{1} \frac{50!}{i!(50-i)!}(0.01)^i(1-0.01)^{50-i}$$

$$= \frac{50!}{0!\,50!}(0.01)^0(0.99)^{50} + \frac{50!}{1!\,49!}(0.01)(0.99)^{49}$$

$$= 0.605 + 0.30556$$

$$= 0.911$$

$$P_a|_{p=RQL} = \sum_{i=0}^{1} \frac{50!}{i!(50-i)!}(0.05)^i(1-0.05)^{50-i}$$

$$= \frac{50!}{0!\,50!}(0.05)^0(0.95)^{50} + \frac{50!}{1!\,49!}(0.05)^1(0.95)^{49}$$

$$= 0.0769 + 0.2025$$

$$= 0.2794$$

Hence, $\alpha = 1 - P_a|_{p=AQL} = 0.089$ AND $\beta = P_a|_{p=RQL} = 0.2794$

~~Profit~~ Quality Cost, $U = 70(0.089) + 20(0.2794) = \underline{11.818}$ //

For Plan 2, n = 50, c = 2.

$$P_a|_{p=AQL} = 0.911 + \frac{50!}{2!\,48!}(0.01)^2(0.99)^{48}$$

$$= 0.9866$$

$$P_a|_{p=RQL} = 0.2794 + \frac{50!}{2!\,48!}(0.05)^2(0.95)^{48}$$

$$= 0.541$$

Hence, $\alpha = 1 - 0.9866 = 0.0134$ AND $\beta = 0.541$.

Quality cost, $U = 70(0.0134) + 20(0.541) = \underline{11.758}$ //

∴ Hence, <u>Plan 2</u> gives a lower Quality Cost and is a better plan!

3 (d) $\mu = 10 cm$, $\sigma = 0.2 cm$.
LSL = 9.5cm , USL = 10.4cm.

$PPM_U = 1000000 \times prob(X > USL)$
$= 1000000 \times [1 - prob(X < USL)]$
$= 1000000 \times \left[1 - \Phi\left(\frac{USL - \mu}{\sigma}\right)\right]$
$= 1000000 \times \left[1 - \Phi\left(\frac{10.4 - 10}{0.2}\right)\right]$
$= 1000000 \times [1 - \Phi(2)]$
$= 1000000 \times (1 - 0.9773)$ *based on std stat table in Appendix.
$= 22700$

$PPM_L = 1000000 \times prob(X < LSL)$
$= 1000000 \times \Phi\left(\frac{LSL - \mu}{\sigma}\right)$
$= 1000000 \times \Phi\left(\frac{9.5 - 10}{0.2}\right)$
$= 1000000 \times \Phi(-2.5)$
$= 1000000 \times [1 - \Phi(2.5)]$
$= 1000000 \times [1 - 0.9938]$ *based on std stat table in Appendix
$= 6200$.

∴ Number of defective parts, PPM = 6200 + 22700
= 28900 //

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. The MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2011-2012

MP4001/MP4F03 – QUALITY ASSURANCE AND MANAGEMENT

April/May 2012 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 3 questions and comprises 7 pages which include 2 pages of Appendices.

2. Answer all 3 questions.

3. Marks for each question are as indicated.

4. This is a restricted open-book examination which allows one double-sided A4 size cheat sheet.

---

1 (a) A component has a constant failure rate which is estimated from five failure times (at 40, 50, 90, 120, 150 in hours) observed in a complete reliability test. Evaluate the reliability of this component for a specified time period of 10 hours.

(5 marks)

(b) The linked system in Figure 1 consists of six components, each of them has a reliability of 0.90. Calculate the system reliability.

(8 marks)

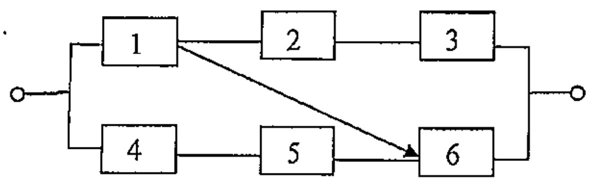


Figure 1

Note: Question No. 1 continues on page 2.

(c) The failure rate of a device, λ, is a function of time t in years as shown:

$$\lambda = 0.015 + 0.02t$$

(i) Calculate the reliability of this device for a design life of 5 years, assuming that no maintenance is performed.

(6 marks)

(ii) Calculate the reliability again for the same design life of 5 years, but assuming that an annual preventive maintenance always restores the device to an as-good-as-new condition.

(4 marks)

(iii) Repeat part (ii), with the assumption that there is a 5% chance that the preventive maintenance will cause immediate failure.

(3 marks)

(d) A standby system has a primary component a with a failure rate of λ_a and a backup component b with a failure rate of λ_b. Establish the state transition diagram and the state transition equations of this standby system, and express the system reliability in terms of the state probabilities such as P_1, P_2 and so on.

(8 marks)

2 (a) A quality improvement team uses a designed experiment (DOE) to study an injection molding process in order to reduce the number of defects. The team decides to investigate four factors, each at two levels (Table 1). Due to resource limitation, only eight tests (observations) can be made.

Table 1

Factor	Low level	High level
A, mold temperature (C°)	30	80
B, screw speed (mm/s)	2	5
C, holding time (minute)	10	14
D, cycle time (minute)	45	60

(i) Build the plus-and-minus-sign table of an appropriate fractional design for this experiment and find the aliases of the main effect A and interaction BC.

(8 marks)

(ii) Project the above fractional design into a full factorial design in terms of the two most effective factors, given the following values of the runs:

(1) = 5, cd = 1, ac = 5, ab = 3, ad = 4, bd = 0, bc = 4, abcd = 6

(7 marks)

Note : Question No 2. continues on page 3.

(iii) Calculate the test ratio F_0 of each of the two remaining factors and their interaction.

(8 marks)

(b) Consider the data in Table 2 obtained from an experiment studying two factors A and B.

Table 2

Factor A	Factor B		
	1	2	3
1	10	15	20
2	1	5	9

Using ANOVA to analyze the data and draw the conclusions regarding the effects of factors A and B based on ($\alpha = 0.05$).

(10 marks)

3 (a) Sketch a Pareto chart using the data in the check sheet in Table 3. What kinds of information can be acquired from the time-oriented summary and the category-oriented summary of a check sheet?

(7 marks)

Table 3

Defect	Months in 1988												Total
	1	2	3	4	5	6	7	8	9	10	11	12	
Part damaged	10	1		3	1	2		1		10	3	12	43
Machining problems	5		3	3				1	8		3	8	31
Supplied parts rusted	1		1	1								2	5
Masking insufficient	4	3	6	4	3	1						3	24
Misaligned weld	2											1	3
Wrong part issued	1	1						2				1	5
Total	23	5	10	11	4	3	0	4	8	10	6	27	111

(b) In a process producing bearings, it is found that ($\bar{\bar{x}} = 34.00\ mm$) and ($\bar{R} = 4.71\ mm$) from the preliminary samples of size $n = 5$. Set up the $\bar{x}$ and R charts for this process.

(6 marks)

Note: Question No. 3 continues on page 4.

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(c) Apply the following three runs rules to the top and bottom control charts in Figure 2.

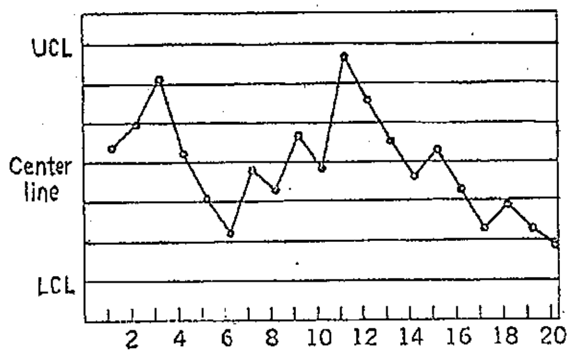
Rule 1: Two out of three consecutive points plot beyond the 2-sigma warning limits on one side.

Rule 2: Four out of five consecutive points plot at a distance of 1-sigma or beyond from the center line on one side.

Rule 3: Eight consecutive points plot on one side of the center line.

Do the three runs rules signal any out-of-control conditions in each of the two control charts? If so, indicate where the out-of-control condition has occurred and with which rule.

(6 marks)



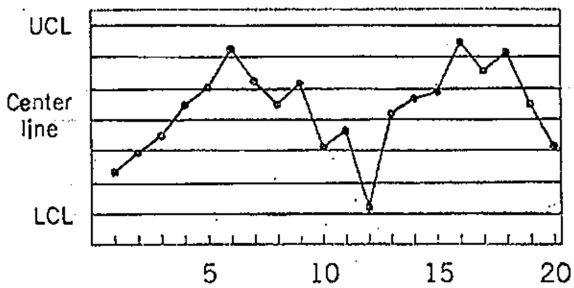


Figure 2

Note: Question No. 3 continues on page 5.

(d) Design an np control chart based on the data in Table 4, which gives the numbers d of nonconforming bearings in 20 samples of size 100.

(6 marks)

Table 4

Sample Number	Number of Nonconforming bearings	Sample Number	Number of Nonconforming bearings
1	0	11	0
2	0	12	1
3	1	13	0
4	0	14	0
5	0	15	0
6	0	16	1
7	0	17	0
8	0	18	0
9	2	19	0
10	0	20	1
			Total = 6

(e) A single sampling plan with ($n = 100$, $c = 2$) is to inspect the lots of a consumer product, for which AQL = 0.01 and RQL = 0.1. Calculate the producer's risk α and the customers' risk β. State whether this sampling plan can meet the requirement of ($\alpha \le 0.01$, $\beta \le 0.05$) ?

(8 marks)

End of Paper

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Appendix

Table: Percentage Points of the F Distribution ($\alpha = 0.05$)

ν_1 degrees of freedom of the numerator
ν_2 degrees of freedom of the denominator

$$F_{0.05,\nu_1,\nu_2}$$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211

1) a) $T = 40 + 50 + 90 + 120 + 150 = 450\,hr$

$n = 5$

$\lambda = \frac{1}{MTTF} = \frac{n}{T} = \frac{5}{450} = \frac{1}{90}/hr$

$t = 10$

$R(t) = e^{-\lambda t} = e^{-\frac{1}{90}\cdot 10} = e^{-\frac{1}{9}} = 0.895$ //

b) Choose component 1 as key component

$R_S^- = R_4 R_5 (R_2 R_3 + R_6 - R_2 R_3 R_6)$

$= 0.9 \cdot 0.9\,(0.9 \cdot 0.9 + 0.9 - 0.9 \cdot 0.9 \cdot 0.9)$

$= 0.795$

$R_S^+ = R_2 R_3 + R_6 - R_2 R_3 R_6$

$= 0.9 \cdot 0.9 + 0.9 - 0.9 \cdot 0.9 \cdot 0.9$

$= 0.981$

$R_S = R_S^- (1 - R_K) + R_S^+ R_K$

$= 0.795(1 - 0.9) + 0.981 \cdot 0.9$

$= 0.9624$ //

c) i) $t = 5$ years $\lambda = 0.015 + 0.02t$

assume $A = \int_0^5 \lambda(t)\,dt = \int_0^5 (0.015 + 0.02t)\,dt = [0.015t + 0.01t^2]_0^5 = 0.325$

$R(t) = e^{-A} = e^{-0.325} = 0.7225$ //

ii) $t = 1$ year (Annual preventive maintenance) for $N = 5$ years

assume $A = \int_0^1 \lambda(t)\,dt = \int_0^1 (0.015 + 0.02t)\,dt = 0.025$

$R(t) = e^{-A} = e^{-0.025} = 0.9753$

$R_M(NT) = R(T)^N = (0.9753)^5 = 0.8824$ //

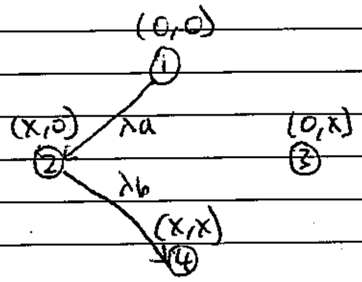
iii) $R_M(NT) = R(T)^N (1-p)^{N-1}$ where $R(T) = 0.9753$, $p = 0.05$, $N = 5$ years

$R_M(5) = (0.9753)^5 (1 - 0.05)^{5-1}$

$= 0.7188$ //

1d)

Component	State 1	2	3	4
a	O	X	O	X
b	O	O	X	X



Equations:

$$\frac{d}{dt}P_1(t) = -\lambda_a P_1(t)$$

$$\frac{d}{dt}P_2(t) = \lambda_a P_1(t) - \lambda_b P_2(t)$$

$$P_3(t) = 0$$

$$\frac{d}{dt}P_4(t) = \lambda_b P_2(t)$$

Or $P_4 = 1 - P_1 - P_2$

2a)i) This is a 2^{4-1} design, Basic generator = ABCD

Run	A	B	C	$D \hat{=} ABC$	value (from 2b)
1	−	−	−	−	5
ad	+	−	−	+	4
bd	−	+	−	+	0
ab	+	+	−	−	3
cd	−	−	+	+	1
ac	+	−	+	−	5
bc	−	+	+	−	4
abcd	+	+	+	+	6

Aliases:
$A \hat{=} BCD$
$BC \hat{=} AD$

ii) $C_A = -5 + 4 - 0 + 3 - 1 + 5 - 4 + 6 = 8$
$C_B = -5 - 4 + 0 + 3 - 1 - 5 + 4 + 6 = -2$
$C_C = -5 - 4 - 0 - 3 + 1 + 5 + 4 + 6 = 4$
$C_D = -5 + 4 + 0 - 3 + 1 - 5 - 4 + 6 = -6$

Most effective factors: A & D (the 2 largest)

Run	A	D	AD	values
1	−	−	+	5, 4
a	+	−	−	3, 5
d	−	+	−	0, 1
ad	+	+	+	4, 6

2a)(iii) $N = 8$

$SS_B = \frac{C_B^2}{N} = \frac{(-2)^2}{8} = \frac{1}{2} = MS_B$

$SS_C = \frac{C_C^2}{N} = \frac{4^2}{8} = 2 = MS_C$

$C_{BC} = +5 + 4 - 0 - 3 - 1 - 5 + 4 + 6 = 10$

$SS_{BC} = \frac{C_{BC}^2}{N} = \frac{10}{8} = 1.25 = MS_{BC}$

→ $SS_B = MS_B$, $SS_C = MS_C$, $SS_{BC} = MS_{BC}$ because $b = 2$ and $C = 2$, hence $b-1 = 1$, $C-1 = 1$. $SS_B = MS_B/(b-1) = MS_B$, the same applies to c & bc.

In this case, single replicate, hence $SS_E = SS_{ABC}$

$C_{ABC} = C_D = -6$

$SS_{ABC} = \frac{C_{ABC}^2}{N} = \frac{(-6)^2}{8} = 4.5 = MS_{ABC}$; since $DOF_{ABC} = 1$

$MS_E = 4.5$

$F_{0B} = \frac{MS_B}{MS_E} = \frac{0.5}{4.5} = \frac{1}{9}$ //

$F_{0C} = \frac{MS_C}{MS_E} = \frac{2}{4.5} = \frac{4}{9}$ //

$F_{0BC} = \frac{MS_{BC}}{MS_E} = \frac{1.25}{4.5} = \frac{5}{18}$ //

b) $a = 2$, $b = 3$, $n = 1$

Factor A	Factor B: 1	2	3	$y_{i.}$
1	10	15	20	45
2	1	5	9	15
$y_{.j}$	11	20	29	$y_{..} = 60$

$SS_A = \frac{1}{3}(45^2 + 15^2) - \frac{1}{6} \cdot 60^2 = 150$

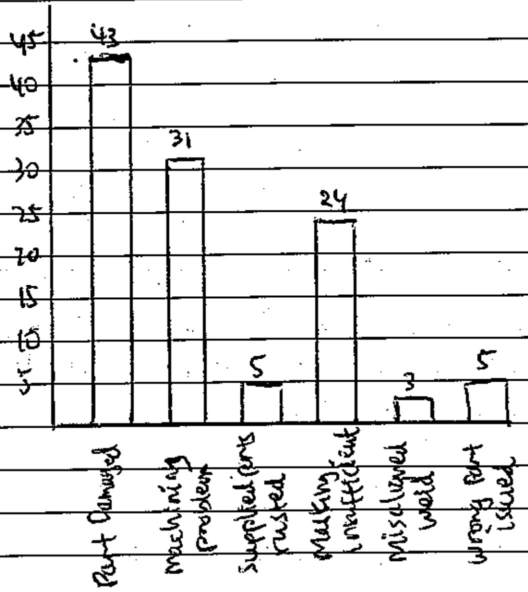
$SS_B = \frac{1}{2}(11^2 + 20^2 + 29^2) - \frac{1}{6} \cdot 60^2 = 81$

$SS_{AB} = 10^2 + 15^2 + 20^2 + 1^2 + 5^2 + 9^2 - \frac{1}{6} \cdot 60^2 - 150 - 81 = 1$

Single replicate: $SS_E = SS_{AB} = 1$

$MS_A = 150$	$v_{1a} = 1$ $v_{2a} = 2$
$MS_B = 40.5$	$v_{1b} = 2$ $v_{2b} = 3$
$MS_{AB} = 0.5$	$F_{\alpha,1,2} = 18.513 > F_{0A}$ } Both A & B has
$MS_E = 0.5$	$F_{\alpha,2,3} = 9.552 > F_{0B}$ } no effect //
$F_{0A} = 300$	where $\alpha = 0.05$
$F_{0B} = 81$	
$F_{0AB} = 1$	

3) a)



→ Time oriented summary:
Valuable in looking for trends or other patterns of defects
→ Category oriented summary:
Tells which type of defects occur more frequently

b) $\bar{\bar{x}} = 34$ mm , $\bar{R} = 4.71$ mm , $n = 5$

$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 34 + 0.577 \cdot 4.71 = 36.72$ mm

$CL_{\bar{x}} = \bar{\bar{x}} = 34$mm

$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 34 - 0.577 \cdot 4.71 = 31.28$ mm

$UCL_R = D_4\bar{R} = 2.114 \cdot 4.71 = 9.96$ mm

$CL_R = \bar{R} = 4.71$ mm

$LCL_R = D_3\bar{R} = 0$ mm

c) - Points 16, 17, 18 at bottom control chart violates rule 1
- Points 16, 17, 18, 19, 20 at top control chart violates rule 2

d) $p = D/N = 6/(20 \cdot 100) = 3 \cdot 10^{-3}$; $n = 100$

$UCL_d = np + 3\sqrt{np(1-p)} = 100 \cdot 3 \cdot 10^{-3} + 3\sqrt{100 \cdot 3 \cdot 10^{-3}(1 - 3 \cdot 10^{-3})} = 1.94 \simeq 2$

$CL_d = np = 0.3 \simeq 1$

$LCL_d = np - 3\sqrt{np(1-p)} = -1.34 = 0$ (can't be negative)

e) $n = 100$, $c = 2$, $AQL = 0.01$, $RQL = 0.1$

~~$P_a|_{p=0.01} = \frac{100!}{0!(100)!} 0.01^0 (1-0.01)^{100} + \frac{100!}{1!(99)!} 0.01^1 (0.99)^{99} + \frac{100!}{2!(98)!} 0.01^2 (0.99)^{98} = 0.921$~~

$\alpha = 1 - 0.921 = 0.079$

~~$\beta = P_a|_{p=0.1} = \frac{100!}{0!100!} 0.1^0 (1-0.1)^{100} + \frac{100!}{1!99!} 0.1^1 (0.9)^{99} + \frac{100!}{2!98!} 0.1^2 (0.9)^{98} = 1.945 \cdot 10^{-3}$~~

$\alpha > 0.01$ can't meet requirement, while $\beta < 0.05$ meets requirement