NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

MA3005 – CONTROL THEORY
MA3705 – AEROSPACE CONTROL THEORY
MP3001 – DYNAMICS & CONTROL

November/December 2017 — Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FIVE (5)** questions and comprises **SEVEN (7)** pages including **TWO (2)** pages of Appendix.
2. Answer **ALL** questions.
3. Marks for each question are as indicated.
4. This is a **RESTRICTED-OPEN BOOK** examination. One double sided A4 reference sheet is allowed.

1(a) What is the major effect of the left-half pole location on the step response of a system?

(3 marks)

(b) Without evaluating $f(t)$, find the final value of the time function, $f(t)$, with its Laplace function given as:

$$F(s) = \frac{5s^2 + 19s + 20}{s(s^2 + 4s + 5)}$$

(4 marks)

(c) Verify your answer of question (1b), by evaluating the time function, $f(t)$, from its Laplace form, $F(s)$.

(6 marks)

(d) Check the stability of a unity feedback system with a forward-loop transfer function (FLTF) of $G(s) = \dfrac{1}{s^3 + s^2 + 2s + 23}$
Specify how many poles in the right-hand plane (or on imaginary axis) if the system is not stable.

(6 marks)

Note: Question 1 continues on page 2.

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(e) The two feedback control systems in Figure 1 have two different pairs of complex conjugate poles coming from their characteristic equations. System#1 has a pair of poles at $p_{1,2} = -A \pm jB$ and System#2 has a pair of poles at $p_{3,4} = -C \pm jD$.

The locations of the pairs of the complex conjugate poles are represented in Figure 2, where the numbers (1 to 4) in the figure represent the location of the respective pole number.

Express the following parameters:

(i) the damping factors of the System#1 (ζ_1) and System#2 (ζ_2) in terms of system parameters (A, B, C, D, θ_1 and θ_2)

(ii) the damped frequency of the System#1 (ω_{d1}) and System#2 (ω_{d2}) in terms of system parameters (A, B, C, D, θ_1 and θ_2)

(5 marks)

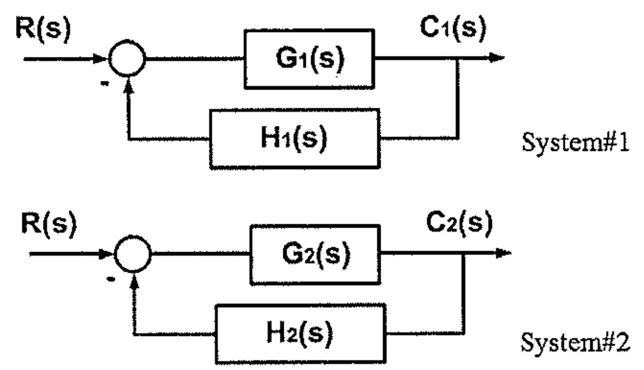


Figure 1: A block diagram.

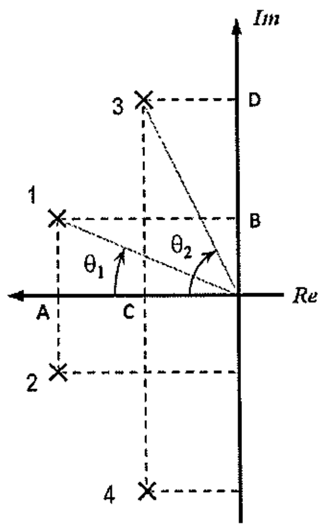


Figure 2: Poles locations.

Note: Question 1 continues on page 3.

(f) Given a unity feedback system as shown in Figure 3, where $G(s) = \dfrac{1}{s(s+2)}$

(i) determine the type number of the system in Figure 3.

(2 marks)

(ii) determine the steady state error, e_{ss}, for a given input, $r(t)$, when $R(s) = \dfrac{1}{s^2}$.

(2 marks)

(iii) calculate the corresponding e_{ss} for $K = 20$ and discuss the role of K.

(2 marks)

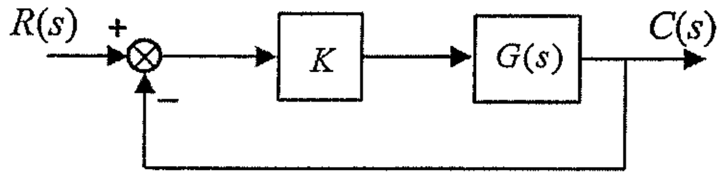


Figure 3: A unity feedback system.

2(a) Find the equivalent transfer function that correlate input, r, and output, c, in Figure 4.

(6 marks)

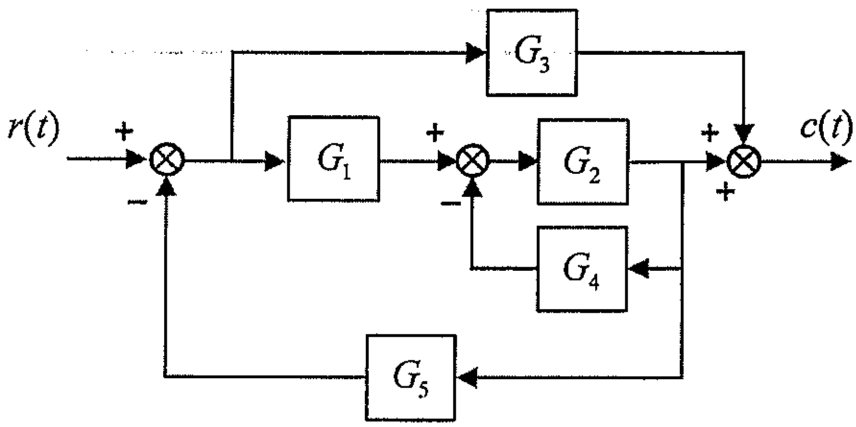


Figure 4: A block diagram.

Note: Question 2 continues on page 4.

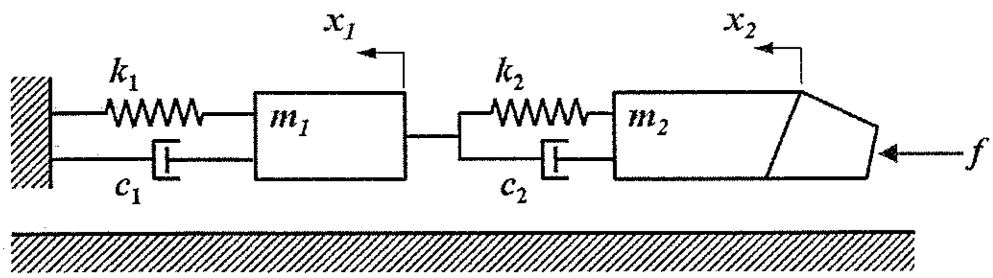
(b) The dynamic structure of a turning tool can be represented as a two-degree-of-freedom system as shown in Figure 5, where m_1 is the equivalent mass of the tool post and m_2 is that of the cutting tool. The turning machine body is considered as a rigid structure.

(i) derive the equation of motion that correlate the cutting force input, f, and the machine body output, x_1, in <u>Laplace domain</u>.

(5 marks)

(ii) find the steady state displacement output, x_1, when a <u>unit</u> step is applied as an input.

(4 marks)



<u>Figure 5</u>: A turning tool dynamic model.

3. A unity feedback system has the following forward-loop transfer function (FLTF):

$$G(s) = \frac{1}{s^2 + 8s + 15}$$

(a) Derive the unit step response of the system (in time domain).

(3 marks)

(b) Derive the response of the system (in time domain) to an impulse input.

(3 marks)

(c) Derive the response of the system (in time domain) to the input, $r(t) = 3\frac{du(t)}{dt} + 2u(t)$.

(4 marks)

(d) Sketch the unit step response in question (3a) of the system and by using the definition of settling time, show that the 2% settling time of the response is 1.45 (sec.).

(5 marks)

4. Given a feedback system shown in Figure 6, where $KG(s) = \dfrac{K}{s(s+1)(s+3)(s+4)}$.

(a) Find the initial poles location ($K = 0$) and the final poles location ($K = \infty$).

(6 marks)

(b) Determine the break in/out points.

(6 marks)

(c) Determine the intersection of the root locus with the imaginary axis

(6 marks)

(d) Sketch the root locus of the system in Figure 6 for $K > 0$.

(7 marks)

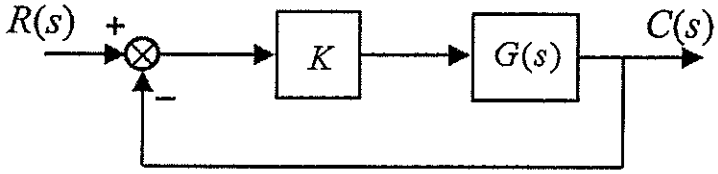


Figure 6: A block diagram.

5. Given a system with a transfer function of $G(s) = \dfrac{(s+10)}{s(s^2+10s+100)}$, sketch the Bode Diagram for magnitude and phase indicating the corner frequency and the asymptotic plot. The real plot can be sketched qualitatively.

(15 marks)

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APPENDIX: Tables of formulas

1. Time Domain specifications:

Transient (2nd Order Systems)

$\sigma = \zeta\omega_n$	$\beta = \cos^{-1}(\zeta)$	$\omega = \omega_n\sqrt{1-\zeta^2}$	$\%M_p = e^{-\sigma\pi/\omega}100$	(1)
2% $t_s = \dfrac{4}{\zeta\omega_n}$	$t_r = \dfrac{\pi-\beta}{\omega}$	$t_p = \dfrac{\pi}{\omega}$	$\zeta = \dfrac{-ln(\%M_p/100)}{\sqrt{\pi^2 + ln^2(\%M_p/100)}}$	(2)
	Static Error Constants (Unity feedback only)			
	$K_p = \lim_{s\to 0} G(s)$;	$K_v = \lim_{s\to 0} sG(s)$;	$K_a = \lim_{s\to 0} s^2G(s)$	(3)

2. Frequency Response

$$\gamma = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}; \text{ or } \zeta = \frac{\tan(\gamma)}{2\left(\sqrt[4]{1+\tan^2(\gamma)}\right)} \qquad (4)$$

$$\omega_b = \omega_n\sqrt{\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}} \qquad (5)$$

APPENDIX: Laplace Transform Pairs

$f(t)$	$F(s)$
(Impulse) $\delta(t)$	1
(Step) $1(t)$	$\frac{1}{s}$
(Ramp) t	$\frac{1}{s^2}$
(Parabolic) $\frac{t^2}{2}$	$\frac{1}{s^3}$
e^{-at}	$\frac{1}{s+a}$
$\frac{1}{(b-a)}\left(e^{-at}-e^{-bt}\right)$	$\frac{1}{(s+a)(s+b)}$
$\frac{1}{ab}\left[1+\frac{1}{(a-b)}\left(be^{-at}-ae^{-bt}\right)\right]$	$\frac{1}{s(s+a)(s+b)}$
$\frac{c-s_1}{s_2-s_1}\cdot e^{-s_1 t}-\frac{c-s_2}{s_2-s_1}\cdot e^{-s_2 t}$	$\frac{s+c}{(s+s_1)(s+s_2)}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}}\cdot e^{-\zeta\omega_n t}\cdot\sin\left(\omega_n\sqrt{1-\zeta^2}\,t\right), \zeta<1$	$\frac{\omega_n^{\,2}}{s^2+2\zeta\omega_n s+\omega_n^{\,2}}$
$\frac{-1}{\sqrt{1-\zeta^2}}\cdot e^{-\zeta\omega_n t}\cdot\sin\left(\omega_n\sqrt{1-\zeta^2}\,t-\phi\right)$ $\tan(\phi)=\frac{\sqrt{1-\zeta^2}}{\zeta},\ \zeta<1$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^{\,2}}$
$1-\frac{1}{\sqrt{1-\zeta^2}}\cdot e^{-\zeta\omega_n t}\cdot\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\phi\right)$ $\tan(\phi)=\frac{\sqrt{1-\zeta^2}}{\zeta},\ \zeta<1$	$\frac{\omega_n^{\,2}}{s\left(s^2+2\zeta\omega_n s+\omega_n^{\,2}\right)}$

END OF PAPER

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1a Damping.

1b. $F(s) = \frac{5s^2+19s+20}{s(s^2+4s+5)}$

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \cancel{s}\frac{5s^2+19s+20}{\cancel{s}(s^2+4s+5)}$

$= \frac{20}{5}$

$= 4$ #.

1c. Let $F(s) = \frac{5s^2+19s+20}{s(s^2+4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+5}$

$5s^2+19s+20 = As^2+4As+5A+Bs^2+Cs$

Compare constants: $20 = 5A \Rightarrow A = 4$

Compare s : $19 = 4A + C \Rightarrow C = 3$

Compare s^2 : $5 = A+B \Rightarrow B = 1$

$\therefore F(s) = \frac{4}{s} + \frac{s+3}{s^2+4s+5}$

$= \frac{4}{s} + \frac{s+3}{(s+2)^2+1}$

$= \frac{4}{s} + \frac{s+2}{(s+2)^2+1} + \frac{1}{(s+2)^2+1}$

$\therefore f(t) = 4 + e^{-2t}\cos t + e^{-2t}\sin t$

$\lim_{t\to\infty} f(t) = 4$ #.

1d. $OLTF = \frac{1}{s^3+s^2+2s+23}$

$CLTF = \frac{1}{1+s^3+s^2+2s+23} = \frac{1}{s^3+s^2+2s+24}$

s^3	1	2
s^2	1	24
s^1	$\frac{2-24}{1} = -22$	
s^0	24	

2 sign changes $\therefore$ 2 right-hand poles

1ei. $\cos\theta_1$, $\cos\theta_2$

1eii. B, D

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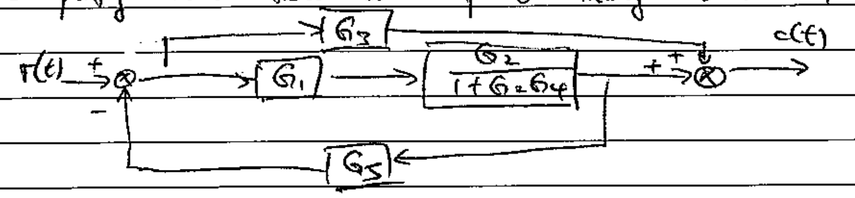
1f i. Type 1

1f ii. $e_{ss} = \frac{1}{K_v} = \frac{1}{\lim_{s\to0} sG(s)} = \frac{1}{\lim_{s\to0} \frac{\cancel{2}}{\cancel{s}(s+4)}} = \frac{1}{\frac{1}{2}} = 2$ #

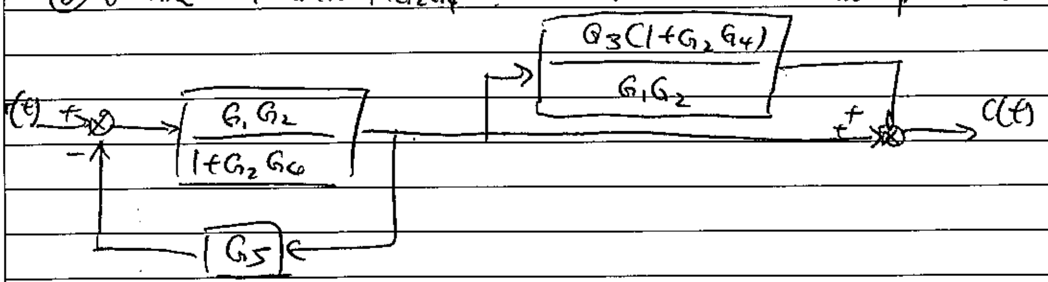
1f iii. $e_{ss} = \frac{1}{\lim_{s\to0} \cancel{s}\frac{20}{\cancel{s}(s+2)}} = \frac{1}{\frac{20}{2}} = 0.1$ #

K reduce steady state error.

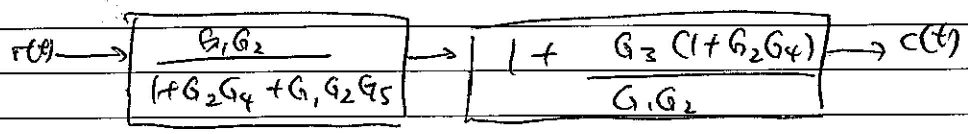
2a. ① Simplify inner feedback loop containing G_2 & G_4



② Combine G_1 and $\frac{G_2}{1+G_2G_4}$ then move first branch point across.



③ Simplify loops

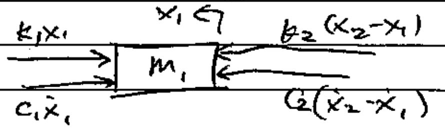


④ Combine

$$\frac{c(t)}{r(t)} = \left(\frac{\cancel{G_1G_2}}{1+G_2G_4+G_1G_2G_5}\right)\left(\frac{G_1G_2+G_3(1+G_2G_4)}{\cancel{G_1G_2}}\right)$$

$$= \frac{G_1G_2+G_3(1+G_2G_4)}{1+G_2G_4+G_1G_2G_5}$$ #

2b i.

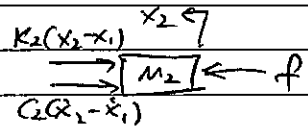


Assume $x_1 < x_2$

$\dot{x}_1 < \dot{x}_2$

$$-k_1x_1 - c_1\dot{x}_1 + k_2(x_2-x_1) + c_2(\dot{x}_2-\dot{x}_1) = m_1\ddot{x}_1$$

$$m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 + (k_1+k_2)x_1 = c_2\dot{x}_2 + k_2x_2 \quad \text{— ①}$$



$$-k_2(x_2-x_1) - c_2(\dot{x}_2-\dot{x}_1) + f = m_2\ddot{x}_2$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 = c_2\dot{x}_1 + k_2x_1 + f \quad \text{— ②}$$

Laplace Transform eq ①, assuming zero initial conditions,

$$X_1(m_1s^2 + (c_1+c_2)s + k_1+k_2) = X_2(c_2s+k_2)$$

$$X_2 = \frac{X_1(m_1s^2+(c_1+c_2)s+k_1+k_2)}{c_2s+k_2}$$

Laplace Transform eq ②, assuming zero initial conditions,

$$X_2(m_2s^2+c_2s+k_2) = X_1(c_2s+k_2)+F$$

Sub X_2 into this equation,

$$\frac{X_1(m_1s^2+(c_1+c_2)s+k_1+k_2)}{c_2s+k_2}(m_2s^2+c_2s+k_2) = X_1(c_2s+k_2)+F$$

$$\frac{X_1}{F} = \frac{c_2s+k_2}{(m_1s^2+(c_1+c_2)s+k_1+k_2)(m_2s^2+c_2s+k_2)-(c_2s+k_2)^2}$$

2b ii When $F = \frac{1}{s}$, $\lim_{t\to\infty} x_1(t) = \lim_{s\to 0} sX_1(s)$

$$= \lim_{s\to 0} s\left(\frac{1}{s}\right)\left(\frac{c_2s+k_2}{(m_1s^2+(c_1+c_2)s+k_1+k_2)(m_2s^2+c_2s+k_2)-(c_2s+k_2)^2}\right)$$

$$= \frac{k_2}{(k_1+k_2)(k_2)-k_2^2}$$

$$= \frac{1}{k_1}$$

3a. FLTF $= G(s) = \frac{1}{s^2+8s+15}$

CLTF $= \frac{C(s)}{R(s)} = \frac{1}{s^2+8s+16}$

For $R = \frac{1}{s}$, $C = \frac{1}{s(s^2+8s+16)}$

Let $C(s) = \frac{1}{s(s^2+8s+16)} = \frac{A}{s} + \frac{B}{(s+4)^2} + \frac{C}{s+4}$

$1 = As^2 + 8As + 16A + Bs + Cs(s+4)$

$= (A+C)s^2 + (8A+B+4C)s + 16A$

Compare constants : $1 = 16A \Rightarrow A = \frac{1}{16}$

Compare s : $0 = 8A + B + 4C \Rightarrow B = -\frac{1}{4}$

Compare s^2 : $0 = A + C \Rightarrow C = -\frac{1}{16}$

$C(s) = \frac{1}{16}\left(\frac{1}{s}\right) - \frac{1}{4}\frac{1}{(s+4)^2} - \frac{1}{16}\frac{1}{s+4}$

$c(t) = \frac{1}{16} - \frac{1}{4}te^{-4t} - \frac{1}{16}e^{-4t}$ #

3b. If response to step input is $c(t)$,

then response to impulse input is $\frac{d}{dt}c(t)$

$\frac{d}{dt}c(t) = te^{-4t} + \frac{1}{4}e^{-4t}$ #.

3c. For $r(t) = 3\frac{du(t)}{dt} + 2u(t)$

$c(t) = 3\left(te^{-4t} + \frac{1}{4}e^{-4t}\right) + 2\left(\frac{1}{16} - \frac{1}{4}te^{-4t} - \frac{1}{16}e^{-4t}\right)$

$= \frac{5}{2}te^{-4t} + \frac{5}{8}e^{-4t} + \frac{1}{8}$ #.

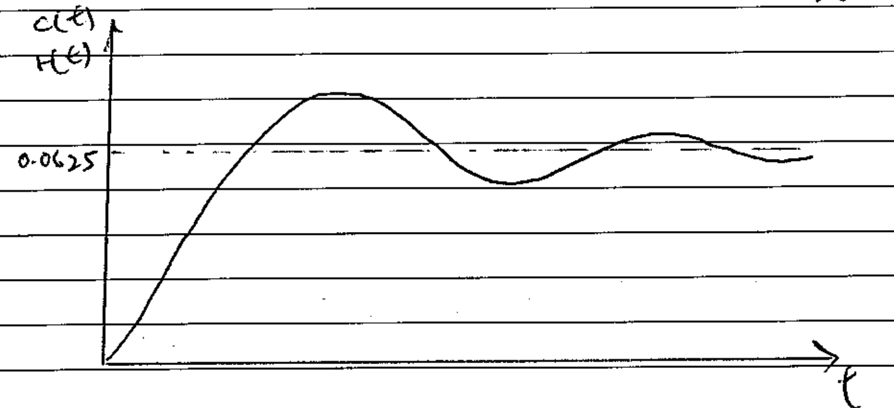
3d. Sub $t = 1.45$ into $c(t) = \frac{1}{16} - \frac{1}{4}te^{-4t} - \frac{1}{16}e^{-4t}$,

$c(1.45) \approx 0.0612$

For $t \to \infty$ $c(\infty) = \frac{1}{16} = 0.0625$

$\therefore$ At $t = 1.45s$, the error is $\frac{0.0625 - 0.0612}{0.0625} = 0.0208 \approx 2\%$ #

$\therefore$ shown.



4a. C.E : $1 + KG(s) = 0$

$1 + \frac{K}{s(s+1)(s+3)(s+4)} = 0$

$s(s+1)(s+3)(s+4) + K = 0.$

Initial pole locations: $p_1 = 0,\ p_2 = -1,\ p_3 = -3,\ p_4 = -4$

Final pole locations: no zeros.

4b. $-\frac{dK}{ds} = \frac{d}{ds}\left(s(s+1)(s+3)(s+4)\right)$

$= \frac{d}{ds}\left((s^2+s)(s^2+7s+12)\right)$

$= \frac{d}{ds}\left(s^4 + 7s^3 + 12s^2 + s^3 + 7s^2 + 12s\right)$

$= \frac{d}{ds}\left(s^4 + 8s^3 + 19s^2 + 12s\right)$

$= 4s^3 + 24s^2 + 38s + 12$

$= 0$

$s = -3.58$ #, -2 (rejected by odd number rule), -0.419 #

4c. At intersection, $s = j\omega$

$(j\omega)^4 + 8(j\omega)^3 + 19(j\omega)^2 + 12(j\omega) + K = 0.$

$\omega^4 - 8j\omega^3 - 19\omega^2 + 12j\omega + K = 0.$

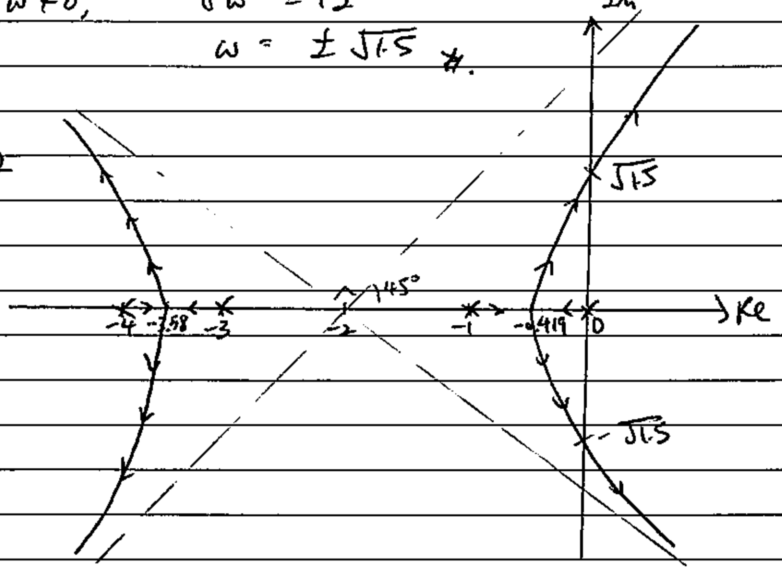
Compare imaginary part,

$-8\omega^3 + 12\omega = 0$

$\omega \neq 0,\quad 8\omega^2 = 12$

$\omega = \pm\sqrt{1.5}$ #.

4d. Asymptote Center $= \frac{0-1-3-4}{4} = -2$



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P(3)

5. $G(s) = \frac{(S+10)}{S(S^2+10S+100)}$ $G(j\omega) = \frac{j\omega+10}{(j\omega)(10j\omega+100-\omega^2)}$

Corner Frequencies: $\omega_1 = 10$

Magnitude

$|G(j\omega)|_{dB} = 20\log\sqrt{100+\omega^2} - 20\log\omega - 20\log\sqrt{(100-\omega^2)^2+100\omega^2}$

Starting point, $\omega = 1$,

$|G(j\omega)|_{dB} = 20\log\sqrt{101} - 20\log 1 - 20\log\sqrt{99^2+100}$

≈ -20

For $\omega < \omega_1$,

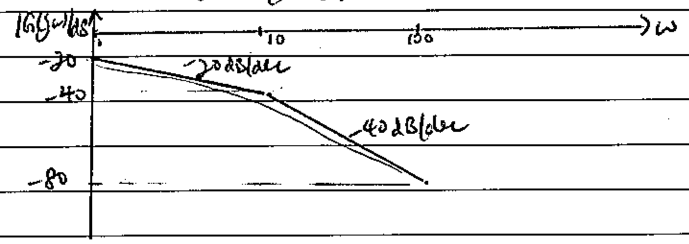
$|G(j\omega)|_{dB} \approx 20\log\sqrt{100} - 20\log\omega - 20\log\sqrt{100^2}$

$= -20\,dB/dec.$

For $\omega > \omega_1$

$|G(j\omega)|_{dB} \approx 20\log\omega - 20\log\omega - 40\log\omega$

$= -40\,dB/dec.$



Phase

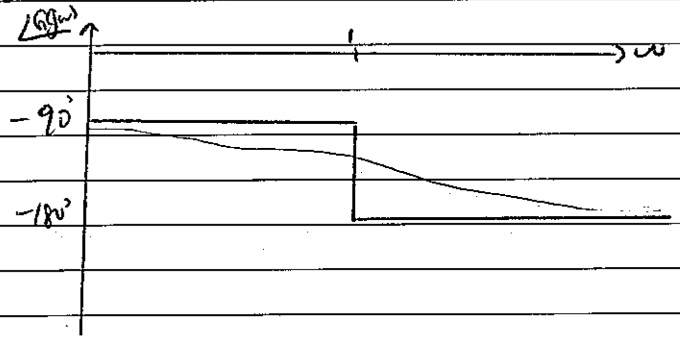
$\angle G(j\omega) = \tan^{-1}\left|\frac{\omega}{10}\right| - 90^\circ - \tan^{-1}\left|\frac{10\omega}{100-\omega^2}\right|$

For $\omega < \omega_1$,

$\angle G(j\omega) = 0^\circ - 90^\circ - 0^\circ = -90^\circ.$

For $\omega > \omega_1$,

$\angle G(j\omega) = 90^\circ - 90^\circ - 180^\circ = -180^\circ$



NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

MA3005 – CONTROL THEORY
MA3705 – AEROSPACE CONTROL THEORY

April/May 2018 Time Allowed: 2½ hours

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1(a) The relationship between the input, $y(t)$, and the output, $x(t)$, of a first order system is represented by the following differential equation:

$$T\dot{x}(t) + x(t) = y(t)$$

Show that the response (in time domain) of the system to $\beta \cdot u_s(t)$, when the initial condition of the system $x(0) = \alpha$ is

$$\beta + (\alpha - \beta)e^{-t/T}$$

[Note: $u_s(t)$ is a unit step function and $\beta \cdot u_s(t)$ is a step function with magnitude of β]

(6 marks)

(b) Find the final value of the response in Question 1(a), if it exists.

(2 marks)

(c) Find the value of the response in Question 1(a) when $t = T$.

(2 marks)

(d) Estimate the time constant of a first order system from its response to a non-unity step input with non-zero initial condition as shown in Figure 1 on page 2.

(Note: In Figure 1, please note that the temperature in the vertical axis does not start from zero)

(5 marks)

Note: Question 1 continues on page 2.
Figure 1 appears on page 2.

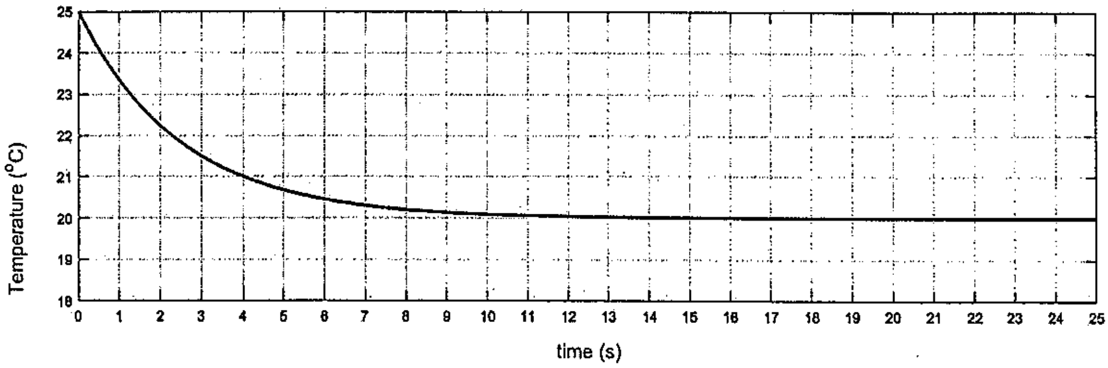

Figure 1: A non-unity step response of a 1st order system.

(e) This is a multiple choice question. No justification is required.

Which of these transfer functions is that of a band-pass filter?

(i) $s^2 + 2s + 100$

(ii) $\frac{s^2+2s+100}{2s}$

(iii) $\frac{1}{s^2+2s+100}$

(iv) $\frac{2s}{s^2+2s+100}$

(v) None of the above

(5 marks)

(f) This is a multiple choice question. No justification is required.

A unity feedback system is shown in Figure 2. Determine the stability of the system, when $K = 3$.

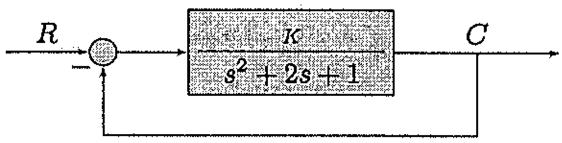

Figure 2: A unity feedback system.

(i) Stable, overdamped

(ii) Stable, critically damped

(iii) Stable, underdamped

(iv) Marginally stable

(v) Unstable

(5 marks)

2. Consider a non-unity feedback system shown in Figure 3.

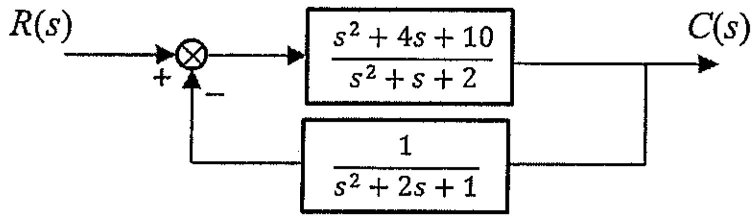

Figure 3: A non-unity feedback system.

(a) Derive the closed-loop transfer function of the system.

(3 marks)

(b) Using Routh-Hurwitz method, define the stability of the system.

(7 marks)

(c) Find the final value of the unit step response of the system, if it exists.

(5 marks)

3. An auxiliary system consisting of a block of mass m_2 is attached to a primary system through a spring k_2. The primary system has a mass of m_1 that is supported to the ground through two parallel springs with identical stiffness of k_1 (see Figure 4).

(a) Define the two differential equations that correlate the input force $f(t)$ to each outputs $x(t)$ and $y(t)$, where $x(t)$ is the displacement of m_1 and $y(t)$is the displacement of m_2 as depicted in Figure 4.

(8 marks)

(b) Rewrite the two differential equations obtained from question 3(a) in Laplace domain, assuming all initial conditions equal to zero.

(6 marks)

(c) Derive the transfer function that correlates the input $F(s)$ to the output $X(s)$, where $F(s)$ is the Laplace of $f(t)$ and $X(s)$ is the Laplace of $x(t)$.

(6 marks)

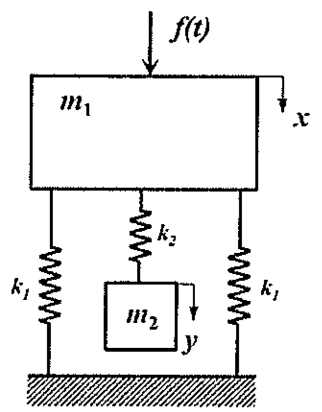

Figure 4: A two-degree-of-freedom system.

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4. Consider a unity feedback system as shown in Figure 5.

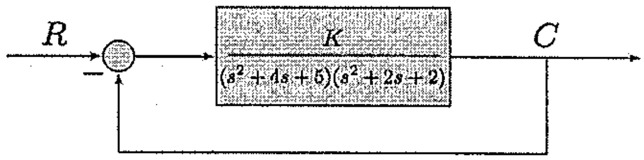

Figure 5: A unity feedback system.

(a) Determine the poles and zeros of the system.

(2 marks)

(b) Determine the number of the asymptotes of the root locus, their configuration and their intersection.

(2 marks)

(c) Find the angles of departure from the start points.

(5 marks)

(d) Find the location(s) of the crossing point(s) at the imaginary axis and the corresponding gain(s) K. For which values of K is the system stable?

(5 marks)

(e) Sketch the root locus of the system.

(5 marks)

(f) What is the steady-state error of the unit step response of the system when $K = 2$?

(2 marks)

(g) Design a lag compensator to decrease the steady-state error of the system to 10% of the error found in Question 4(f), when $K = 2$.

(4 marks)

5. Consider a system with the transfer function $G(s) = \frac{a}{s^2+s+b}$.

(a) For $a = 10$ and $b = 100$, sketch the Bode gain (or magnitude) plot of the system. Detail the steps.

(7 marks)

(b) For $a = 10$ and $b = 100$, sketch the Bode phase shift (or angle) plot of the system. Detail the steps.

(3 marks)

(c) Determine a and b so that the system is a low-pass filter with the following properties:

- The gain at low frequencies is 0 dB;
- The corner frequency is at 0.1 Hz.

(5 marks)

APPENDIX: Tables of formulas

1. Time Domain specifications:

Transient (2nd Order Systems)

$\sigma = \zeta\omega_n$	$\beta = \cos^{-1}(\zeta)$	$\omega = \omega_n\sqrt{1-\zeta^2}$	$\%M_p = e^{-\sigma\pi/\omega}100$	(1)
2% $t_s = \dfrac{4}{\zeta\omega_n}$	$t_r = \dfrac{\pi-\beta}{\omega}$	$t_p = \dfrac{\pi}{\omega}$	$\zeta = \dfrac{-ln(\%M_p/100)}{\sqrt{\pi^2 + ln^2(\%M_p/100)}}$	(2)

Static Error Constants (Unity feedback only)

	$K_p = \lim_{s\to 0} G(s)$;	$K_v = \lim_{s\to 0} sG(s)$;	$K_a = \lim_{s\to 0} s^2G(s)$	(3)

2. Frequency Response

$$\gamma = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}; \text{ or } \zeta = \frac{\tan(\gamma)}{2\left(\sqrt[4]{1+\tan^2(\gamma)}\right)} \qquad (4)$$

$$\omega_b = \omega_n\sqrt{\left(1-2\zeta^2\right) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \qquad (5)$$

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APPENDIX: Laplace Transform Pairs

$f(t)$	$F(s)$
(Impulse) $\delta(t)$	1
(Step) $1(t) = u_s(t)$	$\frac{1}{s}$
(Ramp) t	$\frac{1}{s^2}$
(Parabolic) $\frac{t^2}{2}$	$\frac{1}{s^3}$
e^{-at}	$\frac{1}{s+a}$
$\frac{1}{(b-a)}\left(e^{-at}-e^{-bt}\right)$	$\frac{1}{(s+a)(s+b)}$
$\frac{1}{ab}\left[1+\frac{1}{(a-b)}\left(be^{-at}-ae^{-bt}\right)\right]$	$\frac{1}{s(s+a)(s+b)}$
$\frac{c-s_1}{s_2-s_1}\cdot e^{-s_1 t}-\frac{c-s_2}{s_2-s_1}\cdot e^{-s_2 t}$	$\frac{s+c}{(s+s_1)(s+s_2)}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}}\cdot e^{-\zeta\omega_n t}\cdot\sin\left(\omega_n\sqrt{1-\zeta^2}\,t\right), \zeta<1$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
$\frac{-1}{\sqrt{1-\zeta^2}}\cdot e^{-\zeta\omega_n t}\cdot\sin\left(\omega_n\sqrt{1-\zeta^2}\,t-\phi\right)$ $\tan(\phi)=\frac{\sqrt{1-\zeta^2}}{\zeta},\ \zeta<1$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
$1-\frac{1}{\sqrt{1-\zeta^2}}\cdot e^{-\zeta\omega_n t}\cdot\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\phi\right)$ $\tan(\phi)=\frac{\sqrt{1-\zeta^2}}{\zeta},\ \zeta<1$	$\frac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$

END OF PAPER

MA3005 2017/18 Sem 2 (April / May 2018)

1) a) Laplace transform the equation

$T(sX - x(0)) + X = L(\beta u_s(t))$

$T(sX - \alpha) + X = \frac{\beta}{s}$

$X(Ts+1) = \frac{\beta}{s} + \alpha T$

$X = \frac{\beta}{Ts(s+\frac{1}{T})} + \frac{\alpha T}{T(s+\frac{1}{T})}$

$= \beta \frac{\frac{1}{T}}{s(s+\frac{1}{T})} + \frac{\alpha}{s+\frac{1}{T}}$

$x(t) = L^{-1}(X) = \beta(1 - e^{-\frac{t}{T}}) + \alpha e^{-\frac{t}{T}}$

$= \beta + (\alpha - \beta) e^{-\frac{t}{T}}$

b) $\lim_{t\to\infty} x(t) = \lim_{t\to\infty} (\beta + (\alpha-\beta)e^{-\frac{t}{T}}) = \beta$ $\quad (e^{-\infty} \approx 0)$

c) $x(T) = \beta + (\alpha - \beta) e^{-1}$

d) Using solution from (a) $\rightarrow x(t) = \beta + (\alpha - \beta) e^{-\frac{t}{T}}$

$x(0) = 25 = \beta + (\alpha - \beta) e^0 \Rightarrow \alpha$

$x(\infty) = 20 = \beta$

$x(t) = 20 + (25 - 20) e^{-\frac{t}{T}} = 20 + 5e^{-\frac{t}{T}}$

$x(4) = 21 = 20 + 5e^{-\frac{4}{T}}$

$\frac{1}{5} = e^{-\frac{4}{T}} \rightarrow \ln(0.2) = -\frac{4}{T} \rightarrow T = 2.485\ s$

e) (iv) Band pass filter will filter out low and high frequency

To justify : $F(0) = \frac{2(0)}{0+0+100} = 0$

$\lim_{s\to\infty} F(s) = \lim_{s\to\infty} \frac{2s}{s^2+2s+10} = \lim_{s\to\infty} \frac{\frac{2}{s}}{1+\frac{2}{s}+\frac{10}{s^2}} = 0$

f) $\text{CLTF} = \frac{\frac{3}{s^2+2s+1}}{1+\frac{3}{s^2+2s+1}} = \frac{3}{s^2+2s+4} \rightarrow$ C.E

s^2	1	4
s^1	2	0
s^0	4	

First column all positive $\rightarrow$ stable

Compare CLTF with 2nd order system response : $\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\omega_n = \sqrt{4} = 2$

$2\zeta\omega_n = 2$

$(=) 2\zeta(2) = 2 \rightarrow \zeta = \frac{1}{2} \rightarrow$ underdamped $\rightarrow$ ANS : (iii)

(2) a)

$$G(s) = \frac{\frac{s^2+4s+10}{s^2+s+2}}{1+\left(\frac{1}{s^2+2s+1}\right)\left(\frac{s^2+4s+10}{s^2+s+2}\right)} = \frac{(s^2+4s+10)}{(s^2+s+2)+\frac{s^2+4s+10}{s^2+2s+1}}$$

$$= \frac{(s^2+4s+10)(s^2+2s+1)}{(s^2+s+2)(s^2+2s+1)+(s^2+4s+10)} = \frac{s^4+6s^3+19s^2+24s+10}{s^4+3s^3+6s^2+9s+11}$$

b) C.E : $s^4+3s^3+6s^2+9s+11$

s^4	1	6	11
s^3	3	9	
s^2	3	11	
s^1	-2	0	
s^0	11		

→ 2 sign changes in first column
→ UNSTABLE

c) No final value since the system is not stable

(3) a) m_1 : $m_1\ddot{x} = f - k_1x - k_1x - k_2(x-y)$

$(\Leftrightarrow)\ m_1\ddot{x} + (2k_1+k_2)x - k_2y = f \quad \text{--- (1)}$

m_2 : $m_2\ddot{y} = -k_2(y-x)$

$(\Leftrightarrow)\ m_2\ddot{y} + k_2y = k_2x \quad \text{---- (2)}$

b) (1) : $m_1(s^2X) + (2k_1+k_2)X - k_2Y = F$ $\quad (x(0), \dot{x}(0), y(0), \dot{y}(0) = 0)$

(2) : $m_2(s^2Y) + k_2Y = k_2X$

c) (2) : $Y(m_2s^2+k_2) = k_2X$

$$Y = \frac{k_2X}{k_2+m_2s^2}$$

(1) : $m_1s^2X + (2k_1+k_2)X - k_2\left(\frac{k_2X}{k_2+m_2s^2}\right) = F$

$$\left[m_1s^2 + 2k_1 + k_2 - \frac{k_2^2}{k_2+m_2s^2}\right]X = F$$

$$\frac{X(s)}{F(s)} = \left[m_1s^2 + 2k_1 + k_2 - \frac{k_2^2}{k_2+m_2s^2}\right]^{-1}$$

(4) a)

$$CLTF = \frac{\frac{K}{(s^2+4s+5)(s^2+2s+2)}}{1+\frac{K}{(s^2+4s+5)(s^2+2s+2)}} = \frac{K}{(s^2+4s+5)(s^2+2s+2)+K}$$

C.E : $K + (s^2+4s+5)(s^2+2s+2) = 0$

$N(s) = 1 \rightarrow$ no zeros

$D(s) = (s^2+4s+5)(s^2+2s+2)$

4

a) Continued

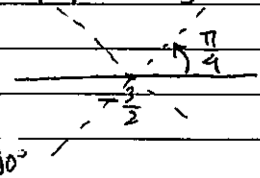
Roots of $s^2+4s+5=0 \rightarrow s = \dfrac{-4 \pm \sqrt{16-20}}{2} = -2 \pm j$

Roots of $s^2+2s+2=0 \rightarrow s = \dfrac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$

Poles : $p_1 = -2+j$; $p_2 = -2-j$; $p_3 = -1+j$; $p_4 = -1-j$

b) No of asymptotes $= 4-0 = 4$

$\sigma_a = \dfrac{-2+j-2-j-1+j-1-j}{4-0} = -\dfrac{3}{2}$



c) $\angle(p_1 \rightarrow p_2) = \angle(-2-j+2-j) = \angle(-2j) = -90°$

$\angle(p_2 \rightarrow p_1) = \angle(2j) = 90°$

$\angle(p_1 \rightarrow p_3) = \angle(-1+j+2-j) = \angle(1) = 0°$

$\angle(p_3 \rightarrow p_1) = \angle(-1) = 180°$

$\angle(p_1 \rightarrow p_4) = \angle(-1-j+2-j) = \angle(1-2j) = -63.43°$

$\angle(p_4 \rightarrow p_1) = \angle(-1+2j) = 116.57°$

$\angle(p_2 \rightarrow p_3) = \angle(-1+j+2+j) = \angle(1+2j) = 63.43°$

$\angle(p_3 \rightarrow p_2) = \angle(-1-2j) = -116.57°$

$\angle(p_2 \rightarrow p_4) = \angle(-1-j+2+j) = \angle(1) = 0°$

$\angle(p_4 \rightarrow p_2) = \angle(-1) = 180°$

$\angle(p_3 \rightarrow p_4) = \angle(-1-j+1-j) = \angle(-2j) = -90°$

$\angle(p_4 \rightarrow p_3) = \angle(2j) = 90°$

$\rightarrow \alpha_{dep}(p_1) = 180° - 90° - 180° - 116.57° = -206.57° = 153.43°$

$\rightarrow \alpha_{dep}(p_2) = 180° + 90° + 116.57° - 180° = 206.57°$

$\rightarrow \alpha_{dep}(p_3) = 180° - 0° - 63.43° - 90° = 26.57°$

$\rightarrow \alpha_{dep}(p_4) = 180° + 63.43° - 0° + 90° = 333.43° = -26.57°$

d) $K + (-\omega^2 + 4j\omega + 5)(-\omega^2 + 2j\omega + 2) = 0$

$(K + \omega^4 - 15\omega^2 + 10) + j(-6\omega^3 + 18\omega) = 0$

$\rightarrow -6\omega^3 + 18\omega = 0 \rightarrow \omega = 0, \pm\sqrt{3}$

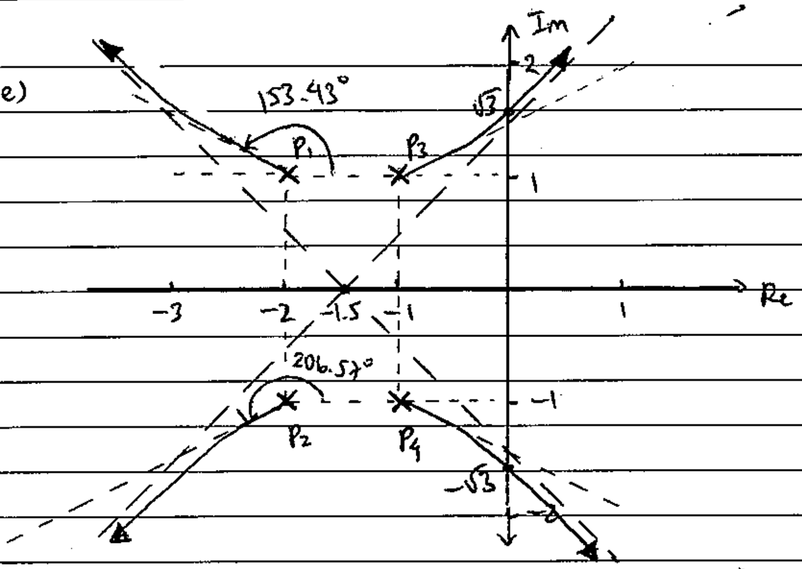
At $\omega = 0 \rightarrow K + 0 - 0 + 10 = 0 \rightarrow K = -10 < 0$

At $\omega = \pm\sqrt{3} \rightarrow K + 9 - 45 + 10 = 0 \rightarrow K = 26$

$\rightarrow$ Crossing points at $(0, \sqrt{3})$ and $(0, -\sqrt{3})$

System is stable at $0 < K < 26$

4 e)



f) Type 0 system

$K_p = \dfrac{2}{(0+0+5)(0+0+2)} = \dfrac{2}{10} = 0.2$

$e_{ss} = \dfrac{1}{1+K_p} = \dfrac{1}{1.2} = \dfrac{5}{6}$

g) Open loop TF : $G(s) = \left(\dfrac{s+z_c}{s+p_c}\right)\dfrac{2}{(s^2+4s+5)(s^2+2s+2)}$

$K_p = \dfrac{2(0+z_c)}{(0+p_c)(0+0+5)(0+0+2)} = \dfrac{2z_c}{10p_c} = \dfrac{z_c}{5p_c}$

$e'_{ss} = 10\% \times \dfrac{5}{6} = \dfrac{1}{1+\frac{z_c}{5p_c}}$

$\dfrac{1}{12} = \dfrac{1}{1+\frac{z_c}{5p_c}} \rightarrow \dfrac{z_c}{5p_c} = 11$

Choose $z_c = 0.11$, $p_c = 0.002$

5 $G(s) = \dfrac{10}{s^2+s+100} = \dfrac{N(s)}{D(s)} \rightarrow$ system type 0

Second order term : $s^2+s+100 \rightarrow \omega = \sqrt{100} = 10 = \omega_{max} = \omega_{min}$

→ Left asymptote : $0 \rightarrow 0.1\,\omega_{min}$: $0 \rightarrow 1$ $(x : -\infty \rightarrow 0)$ $x = \log \omega$

Gain : $y = 20\log\dfrac{|10|}{|0+0+100|} - 20(0)x = -20$

Phase : $\dfrac{N(0)}{D(0)} = \dfrac{10}{100} > 0 \rightarrow y = -(0)\dfrac{\pi}{2} = 0$

→ Right asymptote : $10\,\omega_{max} \rightarrow \infty$: $100 \rightarrow \infty$ $(x : 2 \rightarrow \infty)$

$n = \deg(D) - \deg(N) = 2 - 0 = 2$

(5) Continued

Gain = $y = 20\log\left(\frac{10}{1}\right) - 20(2)x = 20 - 40x$

Phase : $A = \frac{10}{1} > 0 \rightarrow y = -(2)\frac{\pi}{2} = -\pi$

⇒ Sampling in middle

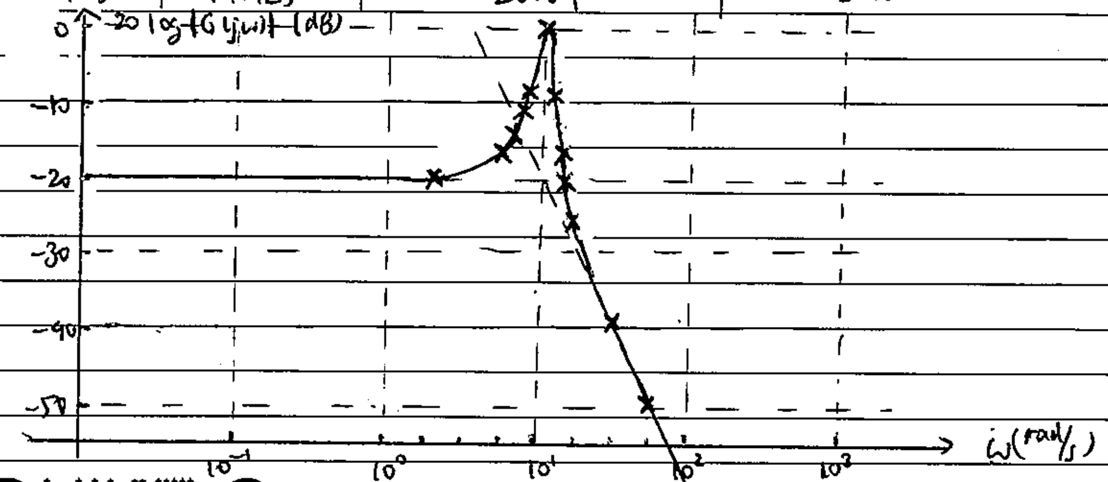
$$G(j\omega) = \frac{10}{-\omega^2 + j\omega + 100} = \underbrace{\frac{10}{\sqrt{(100-\omega^2)^2+\omega^2}}}_{|G(j\omega)|} e^{j\underbrace{\left(-\tan^{-1}\left(\frac{\omega}{100-\omega^2}\right)\right)}_{\angle G(j\omega)}}$$

x	ω	$20\log\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0.25	1.78	−19.72	−0.018
0.5	3.16	−19.09	−0.035
0.75	5.62	−16.73	−0.081
1	10	0	−1.570
1.25	17.8	−26.75	−3.059
1.5	31.6	−39.08	−3.106
1.75	56.2	−49.71	−3.123

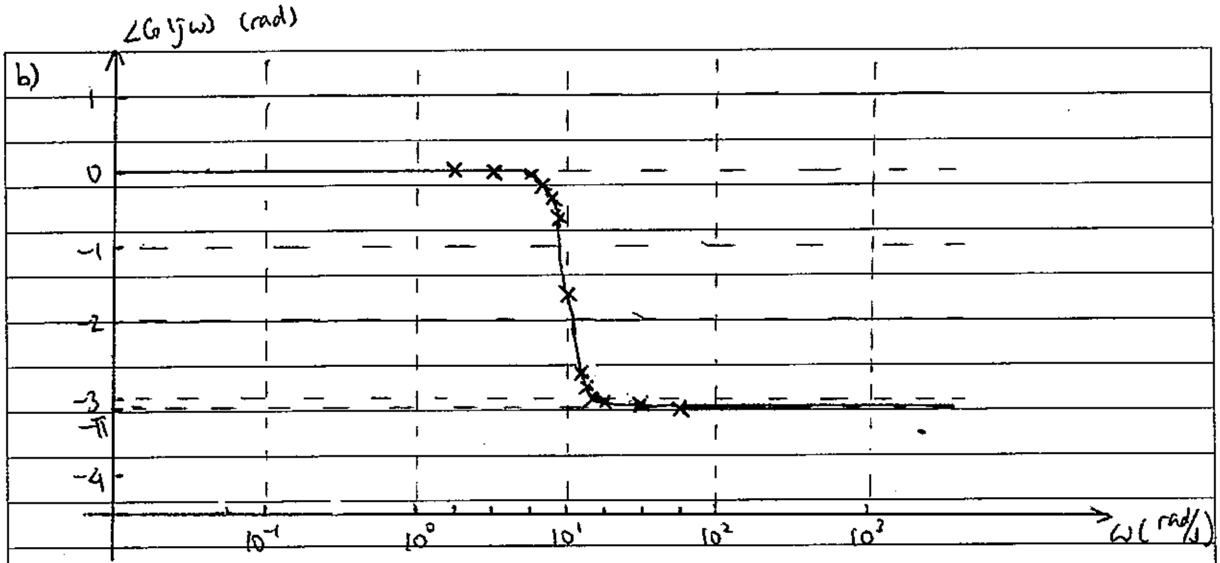
⇒ Sampling near corner frequency

x	ω	$20\log\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0.85	7.079	−14.05	−0.141
0.9	7.943	−11.54	−0.212
0.95	8.912	−7.01	−0.409
1.05	11.220	−9.01	−2.733
1.1	12.589	−15.54	−2.930
1.15	14.125	−20.04	−3.001

a)



(5) b)



c) Gain on left : $0 = 20 \log \left(\frac{a}{0+0+b} \right)$

$20 \log (1) = 20 \log \left(\frac{a}{b} \right) \rightarrow a = b$

Corner freq : $(0.1 \times 2\pi) = \sqrt{b}$

$b = 0.395 = a$

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: http://bit.ly/2lW2C32

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

MA3005 – CONTROL THEORY
MA3705 – AEROSPACE CONTROL THEORY

April/May 2019

Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FIVE (5)** questions and comprises **NINE (9)** pages including **TWO (2)** pages of Appendix.
2. Answer **ALL** questions.
3. Marks for each question are as indicated.
4. This is a **RESTRICTED-OPEN BOOK** examination. One double sided A4 reference sheet is allowed.

1(a) For a function $f(t)$, where the Laplace transformation is

$$F(s)=\frac{3s+8}{s^2+2s+10}$$

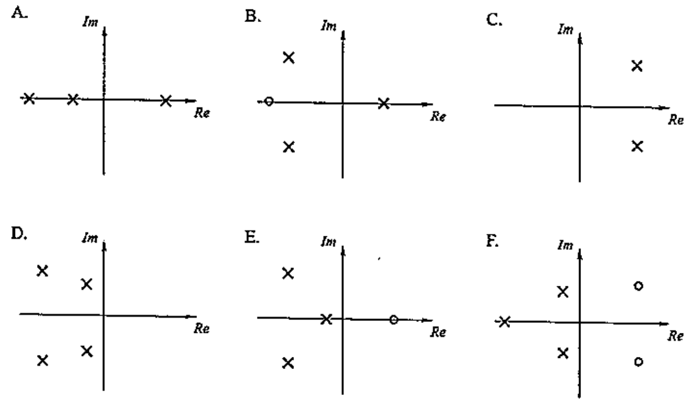
Without evaluating $f(t)$, find the initial value of $f(t)$, the final value of $f(t)$ and the initial value of $\frac{d}{dt}f(t)$

(4 marks)

(b) Which system among those whose pole-zero plots (from A to F) shown in Figure 1(a), might result in a step response as shown in Figure 1(b)? Explain your answer (your rationale is compulsory).

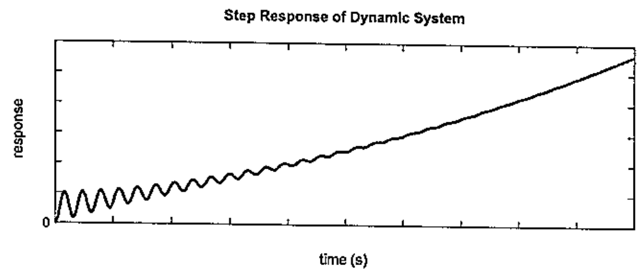
(6 marks)

Note: Question 1 continues on page 2.
Figure 1 appears on page 2.



(× indicates pole and ○ indicates zero)

(a) Pole-zero plots.



(b) Step response of a dynamic system.

Figure 1

(c) Determine (graphically) the system type and system order of a unity feedback system whose open-loop transfer function, $G(s)$, has a Bode plot (magnitude) as shown in Figure 2. No justification is required.

(5 marks)

Note: Question 1 continues on page 3.
Figure 2 appears on page 3.

33

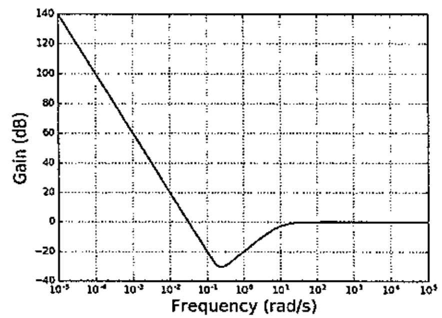


Figure 2: Bode plot (magnitude).

(d) Considering the root-locus plot shown in Figure 3, determine (graphically) the departure angle(s) (in degrees) from the pole at -3+2*j* and the number of asymptotes. No justification is required.

(5 marks)

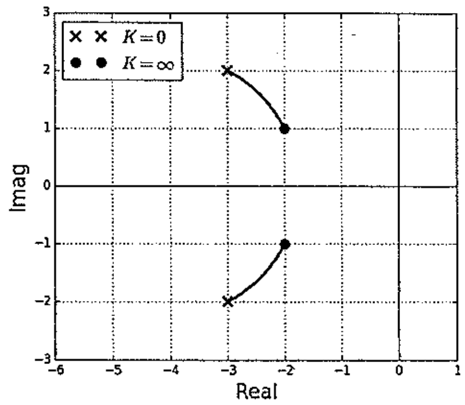


Figure 3: Root-locus plot.

2(a) Figure 4 shows a complex block diagram of a system. Determine the equivalent single block diagram correlating the response C and the input R. Show the working steps in detail.

(6 marks)

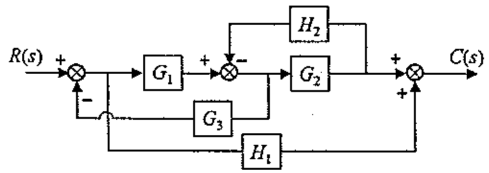


Figure 4: A block diagram.

(b) In the case where $G_3(s) = H_1(s) = 0$, the block diagram in Figure 4 can be redrawn as shown in Figure 5.

If it is known that $G_1(s) = \dfrac{1}{s+2}$, $G_2(s) = \dfrac{1}{s^4 + 2s^3 + 5s^2 + 4s + 39}$ and $H_2(s) = 1$, check the stability of the system using Routh-Hurwitz method (if the system is unstable, specify how many poles in RHP).

(6 marks)

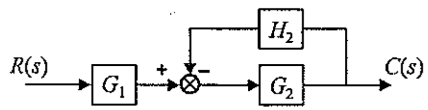


Figure 5: Simplified block diagram.

(c) Determine the steady-state unit-step response of the system in 2(b), if it exists.

(3 marks)

3(a) Figure 6 shows a rotational system comprising two disks with moment of inertia J_1 and J_2, respectively. The two disks are interconnected to each other through a flexible shaft with stiffness of k_2. Disk J_1 is supported to the ground by a bearing with damping value of c_1 and disk J_2 by bearing with damping of c_2. The system is driven from the first disk through a flexible shaft with the stiffness k_1 by an angular displacement input $\varphi(x)$.

Define the transfer function, $\frac{\Theta_2(s)}{\Phi(s)}$, that correlates the displacement output on the second disk, θ_2, to the displacement input, $\varphi(x)$. Show the working steps.

(8 marks)

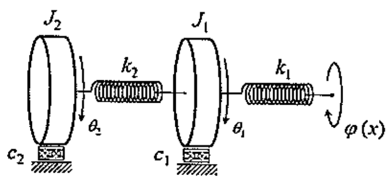


Figure 6: Rotational system.

(b) Figure 7 illustrates a single degree-of-freedom rotational system, where J is the moment of inertia of the disk, k is the stiffness of the shaft and c is the damping value from the bearing. The system is driven by an angular displacement input $\varphi(x)$ through the shaft, while the displacement output from the disk is $\theta(x)$.

(i) Derive the transfer function that correlates the input and output of the system.

(3 marks)

(ii) If a unit-step (radian) displacement input $\varphi(x)$ is given to the system, the displacement response is shown in Figure 8.

From the response, estimate the stiffness values, k_0 and k_1 and the damping value of the systems, c, if the moment of inertia $J = 5$ kg·m^2.

(7 marks)

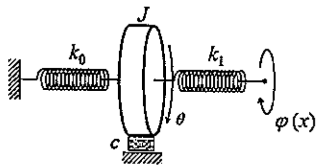


Figure 7: Single degree-of-freedom rotational system.

Note: Question 3 continues on page 6.
Figure 8 appears on page 6.

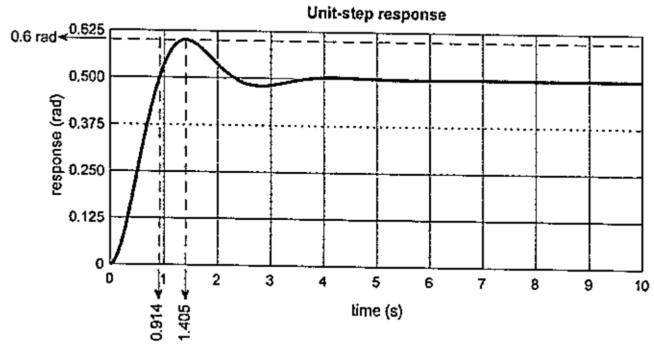


Figure 8: Unit-step response of the system.

(iii) Represent again the transfer function by correlating the displacement input to the velocity output of the system.
(You can use ω as the notation of the velocity output).

(3 marks)

(iv) If a unit-ramp displacement input is given to the system, $\varphi(t) = t$, find the steady-state velocity output, $\omega(t)$, of the system.

(4 marks)

34

4. Consider the unity-feedback system as shown in Figure 9.

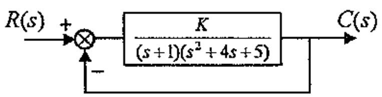


Figure 9: Unity-feedback system.

(a) Determine the poles and zeros of the system. (2 marks)

(b) Determine the number of asymptotes, their configuration and their intersection. (2 marks)

(c) Determine the location(s) of the break-in/break-out point(s), if any. (4 marks)

(d) Determine the location(s) of the crossing(s) of the imaginary axis, if any, and the corresponding gain(s), K. For which values of K is the system stable? (4 marks)

(e) Sketch the root locus of the system. (5 marks)

(f) Determine graphically the root s such that the system has a damping value of 0.707. (Hint: $\cos(\pi/4) \approx 0.707$) (3 marks)

(g) Determine numerically the root, s, and the corresponding gain, K, so that the system has a damping value of 0.707. Comment on the validity of the second-order approximation at that gain K. (5 marks)

5. Consider a system with transfer function $G(s)=\dfrac{1}{s^2+5s+100}$.

(a) Sketch the Bode plot (magnitude) of $G(s)$. Detail the steps. (6 marks)

(b) Sketch the Bode plot (phase shift) of $G(s)$. Detail the steps. (4 marks)

(c) Determine numerically, with 3 significant digits, the frequency, ω_{peak}, for which the amplitude $|G(j\omega)|$ is maximum.
(Hint: $|G(j\omega)|$ is maximum if $1/|G(j\omega)|$ is minimum)

Determine the amplitude at ω_{peak} and the amplitude at the corner frequency. Give comments on the results. (5 marks)

APPENDIX: Tables of formulas

1. Time Domain specifications:

Transient (2nd Order Systems)

$\sigma = \zeta\omega_n$	$\beta = \cos^{-1}(\zeta)$	$\omega = \omega_n\sqrt{1-\zeta^2}$	$\%M_p = e^{-\sigma\pi/\omega}100$	(1)
2% $t_s = \dfrac{4}{\zeta\omega_n}$	$t_r = \dfrac{\pi-\beta}{\omega}$	$t_p = \dfrac{\pi}{\omega}$	$\zeta = \dfrac{-ln(\%M_p/100)}{\sqrt{\pi^2 + ln^2(\%M_p/100)}}$	(2)
	Static Error Constants (Unity feedback only)			
	$K_p = \lim_{s\to 0} G(s)$;	$K_v = \lim_{s\to 0} sG(s)$;	$K_a = \lim_{s\to 0} s^2G(s)$	(3)

2. Frequency Response

$$\gamma = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}};\ \text{or}\ \zeta = \frac{\tan(\gamma)}{2\left(\sqrt[4]{1+\tan^2(\gamma)}\right)} \quad (4)$$

$$\omega_b = \omega_n\sqrt{\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}} \quad (5)$$

1. (a) by initial value theorem,

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} s\cdot F(s)$$

$$= \lim_{s\to\infty} \frac{s(3s+8)}{s^2+2s+10}$$

$$= \lim_{s\to\infty} \frac{3s^2+8s}{s^2+2s+10}$$

$$= \lim_{s\to\infty} \frac{3+8/s}{1+2/s+10/s^2}$$

$$= 3 \ \#$$

$$D(s) = s^2+2s+10$$

$$= (s+1)^2+9$$

$$D(s) = 0$$

$$(s+1)^2+9 = 0$$

$$s = -1 \pm 3j$$

real part of all poles are negative and non-zero

$\therefore$ final value exists

$\therefore$ by final value theorem,

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\cdot F(s)$$

$$= \lim_{s\to 0} \frac{3s^2+8s}{s^2+2s+10}$$

$$= 0 \ \#$$

for $L[f(t)] = F(s)$,

$$L\left[\frac{d}{dt}f(t)\right] = L[\dot{f}(t)]$$

$$= sF(s) - f(0)$$

$$= \frac{3s^2+8s}{s^2+2s+10} - 3$$

$$= \frac{3s^2+8s-3s^2-6s-30}{s^2+2s+10}$$

$$= \frac{2s-30}{s^2+2s+10}$$

$\therefore$ by initial value theorem,

$$\lim_{t\to 0}\left[\frac{d}{dt}f(t)\right] = \lim_{s\to\infty} s\cdot L\left[\frac{d}{dt}f(t)\right]$$

$$= \lim_{s\to\infty} \frac{s(2s-30)}{s^2+2s+10}$$

$$= \lim_{s\to\infty} \frac{2s^2-30s}{s^2+2s+10}$$

$$= \lim_{s\to\infty} \frac{2-30/s}{1+2/s+10/s^2}$$

$$= 2 \ \#$$

1. (b) from Fig. 1(b),
the response is divergent
and does not converge to
a steady-state value.
∴ system is unstable.
∴ there are poles whose
real parts are positive,
non-zero i.e. poles in
the Right Hand Plane.
∴ eliminate C, D, E, F.

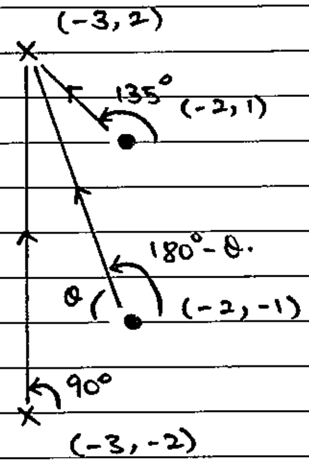
from response, there is
clear oscillatory behaviour
∴ there must also be
complex conjugate poles
∴ eliminate A.

∴ system is B #

1. (c) left asymptote:
slope = $-20k$ / decade
$-20k = 100 - 140$
$= -40$
∴ system type, $k = 2$ #

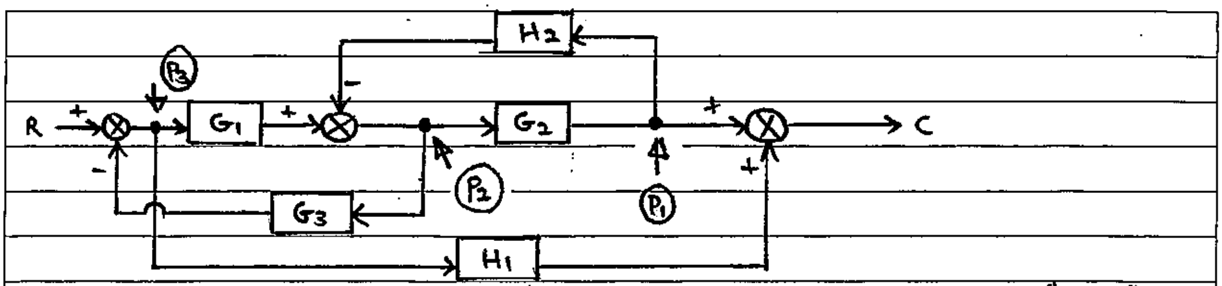
right asymptote:
slope = $-20n$ / decade
$-20n = 0$
∴ system order, $n = 0$ #

1. (d) α dep (p)
$= \pi - \sum \alpha_{poles \Rightarrow p} + \sum \alpha_{zeros \Rightarrow p}$
$= 180° - 90° + 135°$
$+ (180° - 71.6°)$
$= 333.4°$
$= -26.6°$ # (1 dp).

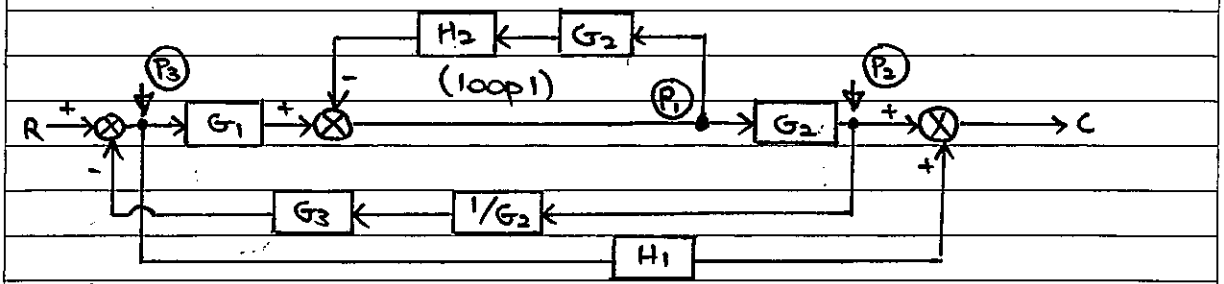


$\tan\theta = \left(\frac{3}{1}\right)$
∴ $\theta = \tan^{-1}(3) = 71.6°$ (1dp)

2.(a)



bring point P_1 before, infront of G_2, compensate with "G_2".
bring point P_2 after, behind of G_2, compensate with "$1/G_2$".

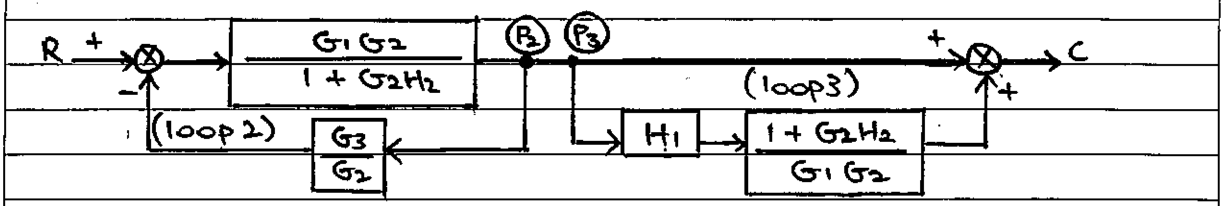


loop 1 : FLTF = 1

OLTF = G_2H_2 , bring point P_3 after behind G_2,

negative feedback   compensate with "$\frac{1+G_2H_2}{G_1G_2}$".

$\therefore$ CLTF = $\frac{1}{1+G_2H_2}$



loop 2 : FLTF = $\frac{G_1G_2}{1+G_2H_2}$

OLTF = $\frac{G_1G_3}{1+G_2H_2}$

CLTF = $\frac{G_1G_2/1+G_2H_2}{1+G_1G_3/1+G_2H_2}$

$= \frac{G_1G_2}{1+G_1G_3+G_2H_2}$

loop 3 : (positive feed forward)

equivalent transfer fuction

$= 1 + \frac{H_1+G_2H_1H_2}{G_1G_2}$

$= \frac{G_1G_2+G_2H_1H_2+H_1}{G_1G_2}$

2.(a) $\therefore \dfrac{C}{R} = \dfrac{G_1G_2}{1+G_1G_3+G_2H_2} \times \dfrac{G_1G_2+G_2H_1H_2+H_1}{G_1G_2} = \dfrac{G_1G_2+G_2H_1H_2+H_1}{1+G_1G_3+G_2H_2}$

$$R(s) \longrightarrow \boxed{\dfrac{G_1G_2+G_2H_1H_2+H_1}{1+G_1G_3+G_2H_2}} \longrightarrow C(s)$$

#

2.(b) $FLTF = G_2$, $OLTF = G_2H_2$

$CLTF = \dfrac{G_2}{1+G_2H_2}$

$\therefore \dfrac{C}{R} = \dfrac{G_1G_2}{1+G_2H_2}$

$$= \frac{\left(\frac{1}{s+2}\right)\left(\frac{1}{s^4+2s^3+5s^2+4s+39}\right)}{1+\frac{1}{s^4+2s^3+5s^2+4s+39}}$$

$$= \frac{\frac{1}{(s+2)(s^4+2s^3+5s^2+4s+39)}}{\frac{(s^4+2s^3+5s^2+4s+40)}{(s^4+2s^3+5s^2+4s+39)}}$$

$$= \frac{1}{(s+2)(s^4+2s^3+5s^2+4s+40)}$$

$$= \frac{1}{s^5+4s^4+9s^3+14s^2+48s+80}$$

$\therefore$ characteristic equation :

$s^5+4s^4+9s^3+14s^2+48s+80=0$

s^5	1	9	48
s^4	4	14	80
s^3	$11/2$	28	0
s^2	$-70/11$	80	0
s^1	$680/7$	0	
s^0	80	0	

#

$b_1 = -\frac{1}{4}\left[(1)(14)-(4)(9)\right] = \frac{11}{2}$

$b_2 = -\frac{1}{4}\left[(1)(80)-(4)(48)\right] = 28$

$c_1 = -\frac{2}{11}\left[(4)(28)-(\frac{11}{2})(80)\right] = -\frac{70}{11}$

$c_2 = -\frac{2}{11}\left[(4)(0)-(\frac{11}{2})(80)\right] = 80$

$d_1 = \frac{11}{70}\left[(\frac{11}{2})(80)-(-\frac{70}{11})(28)\right] = \frac{680}{7}$

$e_1 = -\frac{7}{680}\left[(-\frac{70}{11})(0)-(\frac{680}{7})(80)\right] = 80$

there are 2 changes in sign of coefficients found in first column of Routh-Hurwitz array.

$\therefore$ system is unstable #

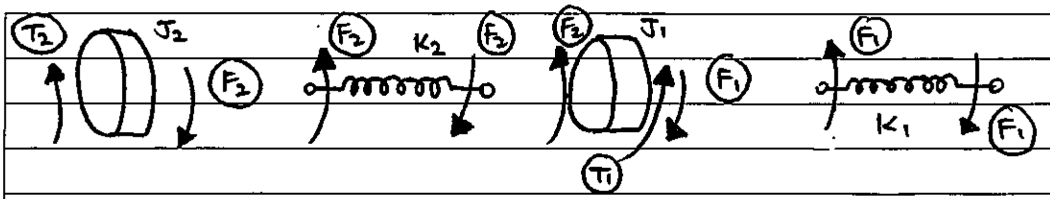
and no. of poles in RHP $= 2$ #

2.(c) since system is unstable, a steady-state value response to a unit-step input does not exist #.

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: http://bit.ly/2lW2C32

3.(a)



$T_2 = C_2\dot{\theta}_2$ $\qquad$ $T_1 = C_1\dot{\theta}_1$

assume $\varphi > \theta_1 > \theta_2$

$\therefore\ F_1 = K_1(\varphi - \theta_1)$

$F_2 = K_2(\theta_1 - \theta_2)$.

for J_1 disk,

$\Sigma T = J\alpha$

$F_1 - F_2 - T_1 = J_1\alpha_1$

$K_1(\varphi - \theta_1) - K_2(\theta_1 - \theta_2) - C_1\dot{\theta}_1 = J_1\ddot{\theta}_1$

$K_1\varphi - K_1\theta_1 - K_2\theta_1 + K_2\theta_2 - C_1\dot{\theta}_1 = J_1\ddot{\theta}_1$

$K_1\varphi + K_2\theta_2 = J_1\ddot{\theta}_1 + C_1\dot{\theta}_1 + (K_1+K_2)\theta_1$

assuming zero initial conditions,

$K_1\Phi + K_2\Theta_2 = J_1 s^2\Theta_1 + C_1 s\Theta + (K_1+K_2)\Theta_1$

$\therefore\ K_1\Phi + K_2\Theta_2 = \Theta_1[J_1s^2 + C_1s + K_1 + K_2]$ —(1)

for J_2 disk,

$\Sigma T = J\alpha$

$F_2 - T_2 = J_2\alpha_2$

$K_2(\theta_1 - \theta_2) - C_2\dot{\theta}_2 = J_2\ddot{\theta}_2$

$K_2\theta_1 - K_2\theta_2 - C_2\dot{\theta}_2 = J_2\ddot{\theta}_2$

$K_2\theta_1 = J_2\ddot{\theta}_2 + C_2\dot{\theta}_2 + K_2\theta_2$

assuming zero initial conditions,

$K_2\Theta_1 = J_2s^2\Theta_2 + C_2s\Theta_2 + K_2\Theta_2$

$K_2\Theta_1 = \Theta_2[J_2s^2 + C_2s + K_2]$

$\therefore\ \Theta_1 = \frac{\Theta_2}{K_2}[J_2s^2 + C_2s + K_2]$ —(2)

substitute (2) into (1),

$K_1\Phi + K_2\Theta_2$

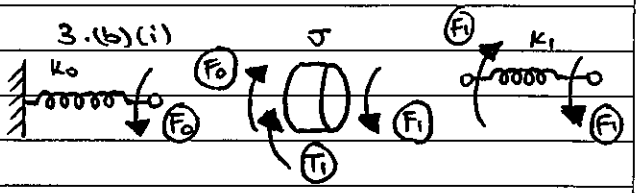
$= \frac{\Theta_2}{K_2}[J_1s^2 + C_1s + K_1 + K_2][J_2s^2 + C_2s + K_2]$

$\therefore\ K_1\Phi$

$= \frac{\Theta_2[J_1s^2 + C_1s + K_1 + K_2][J_2s^2 + C_2s + K_2] - K_2^2\Theta_2}{K_2}$

$\therefore\ \frac{\Theta_2}{\Phi} = \frac{K_1K_2}{[J_1s^2 + C_1s + K_1 + K_2][J_2s^2 + C_2s + K_2] - K_2^2}$ #

3.(b)(i)



for J disk, $\Sigma T = J\alpha$

$F_1 - F_0 - T_1 = J\alpha$

$K_1(\varphi - \theta) - K_0\theta - C\dot{\theta} = J\ddot{\theta}$

$K_1\varphi - K_1\theta - K_0\theta - C\dot{\theta} = J\ddot{\theta}$

$K_1\varphi = J\ddot{\theta} + C\dot{\theta} + (K_0 + K_1)\theta$

assuming zero initial condition,

$K_1\Phi = Js^2\Theta + Cs\Theta + (K_0 + K_1)\Theta$

$K_1\Phi = \Theta[Js^2 + Cs + K_0 + K_1]$

$\therefore\ \frac{\Theta}{\Phi} = \frac{K_1}{Js^2 + Cs + K_0 + K_1}$ #

3.(b)(ii)

$$\frac{\Theta}{\Phi} = \frac{K_1}{Js^2 + cs + (K_0+K_1)}$$

$$\frac{\Theta}{\Phi} = \frac{K_1/J}{s^2 + \frac{c}{J}s + \frac{K_0+K_1}{J}} \quad -(1)$$

2nd order system general form:

$$\frac{C}{R} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad -(2)$$

by comparing (1) and (2) form:

$$K\omega_n^2 = \frac{K_1}{J}$$

$$2\zeta\omega_n = \frac{c}{J}$$

$$\omega_n^2 = \frac{K_0+K_1}{J}$$

since given, unit step input, from Fig 8, steady-state value $= 0.500$

$\therefore$ gain $K = 0.5$

$$\therefore 0.5\,\omega_n^2 = \frac{K_1}{J}$$

$$\omega_n^2 = \frac{2K_1}{J}$$

also, $\omega_n^2 = \frac{K_0+K_1}{J}$

$$\therefore \frac{2K_1}{J} = \frac{K_0+K_1}{J}$$

$$\therefore K_0 = K_1$$

from Fig 8, $M_p = 0.625 - 0.5$
$= 0.125$

$t_p = 1.405$

$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}} = 0.870 \text{ (3sf)}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\therefore \omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}} = 4.545 \text{ (4sf)}$$

$\therefore K = 0.5,\ \omega_n = 4.545,\ \zeta = 0.870.$

$$K\omega_n^2 = \frac{K_1}{J}$$

$$\therefore K_1 = KJ\omega_n^2 = (0.5)(5)(4.545)^2 = 51.6 \text{ (3sf)}.$$

$$2\zeta\omega_n = \frac{c}{J}$$

$$\therefore c = 2\zeta J\omega_n = (2)(0.870)(5)(4.545) = 39.5 \text{ (3sf)}.$$

$\therefore c = 39.5$ # $K_0 = 51.6$ # $K_1 = 51.6$ #

3.(b)(iii)

$L[\theta] = \Theta$

$L[\omega] = L[\dot{\theta}] = s\Theta + \theta(0)$

assuming zero initial condition

$\therefore \theta(0) = 0$

let $L[\omega] = \varpi$

$\therefore \varpi = s\Theta$

$$\frac{\Theta}{\Phi} = \frac{K_1}{Js^2 + cs + (K_0+K_1)}$$

$$\frac{\varpi}{\Phi} = \frac{K_1 s}{Js^2 + cs + (K_0+K_1)} \ \#$$

3.(b)(iv)

$\varphi(t) = t$

$$\Phi(s) = \frac{1}{s^2}$$

$$\frac{\varpi}{\Phi} = \frac{K_1 s}{Js^2 + cs + (K_0+K_1)}$$

by final-value theorem,

$$\lim_{t\to\infty} \omega(t) = \lim_{s\to 0} s\cdot\varpi(s)$$

$$= \lim_{s\to 0} s\cdot\frac{1}{s^2}\cdot\frac{K_1 s}{Js^2 + cs + (K_0+K_1)}$$

$$= \frac{K_1}{K_0+K_1}$$

$= 0.5$ # $(K_0 = K_1)$.

4. FLTF = OLTF = $\dfrac{K}{(s+1)(s^2+4s+5)}$

CLTF = $\dfrac{\frac{K}{(s+1)(s^2+4s+5)}}{1+\frac{K}{(s+1)(s^2+4s+5)}}$

$= \dfrac{K}{K+(s+1)(s^2+4s+5)}$

∴ characteristic equation:

$K + (s+1)(s^2+4s+5) = 0$

$K \cdot N(s) + D(s) = 0$

$N(s) = 1$

$D(s) = (s+1)(s^2+4s+5)$

$D(s) = 0$

$(s+1)(s^2+4s+5) = 0$

$s+1 = 0 \quad s^2+4s+5 = 0$

$(s+2)^2+1 = 0$

$s = -1$ or $s = -2 \pm i$

∴ there are 3 poles:

$s = -1$ #

$s = -2+i$ # $s = -2-i$ #

$N(s) = 0$

no solutions.

∴ there are no zeros #

no. of asymptotes

$= n_{poles} - n_{zeros}$

$= 3$ #

$\sigma_a = \dfrac{\Sigma poles - \Sigma zeros}{n_{poles} - n_{zeros}}$

$= \dfrac{(-1)+(-2)+(-2)}{3-0}$

$= -\frac{5}{3}$

∴ intersection point of asymptotes: $(-\frac{5}{3}, 0)$ #

for break-in/out points:

$D(s) = (s+1)(s^2+4s+5)$

$= s^3+5s^2+9s+5$

$D'(s) = 3s^2+10s+9$

$N'(s)D(s) - D'(s)N(s) = 0.$

$0 - (3s^2+10s+9)(1) = 0$

$s^2 + \frac{10}{3}s + 3 = 0$

$(s+\frac{5}{3})^2 + \frac{2}{9} = 0$

∴ $s = -\frac{5}{3} \pm \frac{\sqrt{2}}{3} i$

$K < 0$ ∴ no break-in/out points #

for intersection points,

$K + (s+1)(s^2+4s+5) = 0$

$K + s^3+5s^2+9s+5 = 0$

let $s = j\omega$

$K + (j\omega)^3 + 5(j\omega)^2 + 9(j\omega) + 5 = 0$

$K - \omega^3 j - 5\omega^2 + 9\omega j + 5 = 0$

$(K+5-5\omega^2) + (9\omega - \omega^3)j = 0$

$9\omega - \omega^3 = 0 \qquad K+5-5\omega^2 = 0$

$\omega(\omega^2-9) = 0 \qquad K = 5\omega^2 - 5 = 0$

$\omega = 0$ or $\omega = \pm 3$ if $\omega = 0$, $K = -5$ (reject)

if $\omega = \pm 3$, $K = 40$

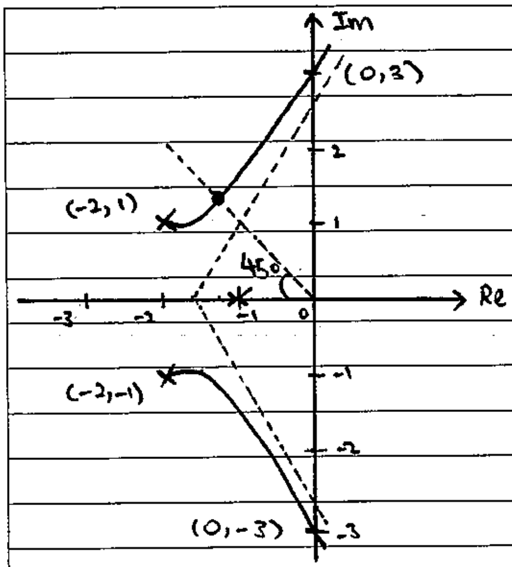
∴ points where locus cross imaginary axis are $(0,3)$#, $(0,-3)$#

the corresponding K value = 40#

∴ range of values of K for which system remains stable:

$0 \le K \le 40$ #

4.



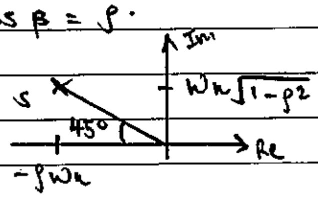
approximation to 2nd order :

$$G(s) = \frac{K\omega_n^2}{s^2+2\rho\omega_n s+\omega_n^2}$$

where for underdamped system of $0<\rho<1$,

$s_{1,2} = -\rho\omega_n \pm j\omega_n\sqrt{1-\rho^2}$

$\cos\beta = \rho$.



from graph,

$s = -1.3 + 1.4i$ #

5. $G(s) = \dfrac{1}{s^2 + 5s + 100}$

system type, $k = 0$

system order, $n = 2 - 0 = 2$

corner frequency : $\omega_{cor} = 10$

$\omega_{cor} = 10 \quad \log(\omega_{cor}) = 1$

$0.1\omega_{cor} = 1 \quad \log(0.1\omega_{cor}) = 0$

$10\omega_{cor} = 100 \quad \log(10\omega_{cor}) = 2$

$\left|\dfrac{N(0)}{D(0)}\right| = \left|\dfrac{1}{0+0+100}\right| = \dfrac{1}{100}$

$|A| = \left|\dfrac{\text{coeff. of } N}{\text{coeff. of } D}\right| = \dfrac{1}{1} = 1$

5. (a) gain plot :

left asymptote :

for $0 < \omega < 0.1\omega_{cor}$

slope $= -20k$/decade

$= 0$/decade

intercept $= 20\log\left|\dfrac{N(0)}{D(0)}\right|$

$= 20\log\left|\dfrac{1}{100}\right|$

$= -40$

$\therefore$ left asymptote:

$20\log|G(j\omega)| = 0 \cdot \log(\omega) + (-40)$

$20\log|G(j\omega)| = -40$.

right asymptote:

for $10\omega_{cor} < \omega < \infty$

slope $= -20n$/decade

$= -40$/decade

intercept $= 20\log|A|$

$= 20\log|1|$

$= 0$

$\therefore$ right asymptote:

$20\log|G(j\omega)| = -40\log(\omega) + 0$

$20\log|G(j\omega)| = -40\log(\omega)$.

(plot on next page).

5. (b) phase plot :

left asymptote : for $0 < \omega < 0.1\omega_{cor}$

$\dfrac{N(0)}{D(0)} = \dfrac{1}{100}, > 0$

$\therefore \; y = -k\dfrac{\pi}{2}$

$y = 0 \quad (k=0)$

$\therefore$ left asymptote: $\angle G(j\omega) = 0$.

right asymptote : for $10\omega_{cor} < \omega < \infty$.

$A = 1, > 0$

$\therefore \; y = -n\dfrac{\pi}{2}$

$y = -\dfrac{\pi}{2}$

$\therefore$ right asymptote: $\angle G(j\omega) = -\pi$. $(n=2)$

(plot on next page).

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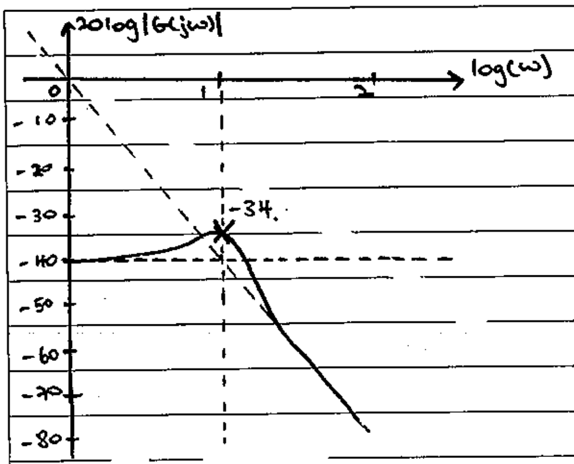
Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: http://bit.ly/2lW2C32

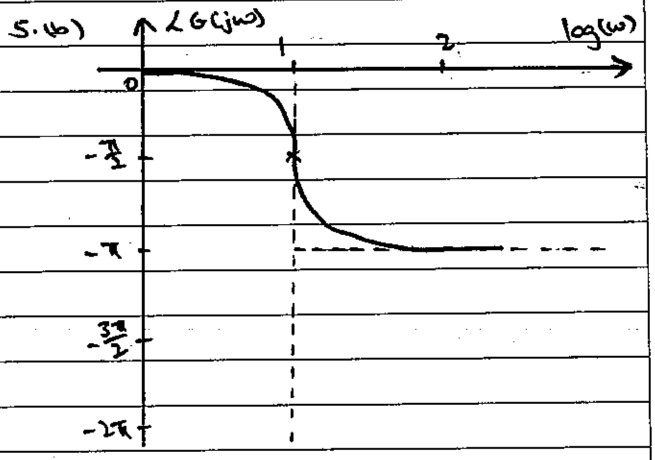
GAIN PLOT :

5.(a)



PHASE PLOT :

5.(b)



5.(a)

$\log(\omega) = 1$

$\omega = 10$

$G(j\omega) = \dfrac{1}{100 + 50j - 100}$

$\therefore |G(j\omega)| = \dfrac{1}{50}$

$\therefore 20\log|G(j\omega)| = -33.98$

$\approx -34.$

5.(b)

$\log(\omega) = 1$

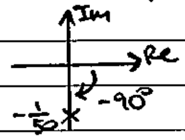
$\omega = 10,$

$G(j\omega) = \dfrac{1}{50j}$ (purely imaginary)

$\therefore \quad = -\dfrac{1}{50} j$

$\therefore \angle G(j\omega) = -90°$

$= -\dfrac{\pi}{2}$



5.(c)

$G(s) = \dfrac{1}{s^2 + 5s + 100}$

for all $s = j\omega$ (general),

$G(j\omega) = \dfrac{1}{(j\omega)^2 + 5(j\omega) + 100}$

$= \dfrac{1}{(100-\omega^2) + (5\omega)j}$

$|G(j\omega)| = \dfrac{1}{\sqrt{(100-\omega^2)^2 + (5\omega)^2}}$

given, $|G(j\omega)|$ is maximum,

when $\dfrac{1}{|G(j\omega)|}$ is minimum.

$\therefore \dfrac{1}{|G(j\omega)|} = \sqrt{(100-\omega^2)^2 + (5\omega)^2}$

let $f(\omega) = \sqrt{(100-\omega^2)^2 + (5\omega)^2}$

$f(\omega)$ is min. when $f'(\omega) = 0$

$f'(\omega) = 2(100-\omega^2)(-2\omega) + 50\omega$

$f'(\omega) = 0$

$50\omega - 4\omega(100-\omega^2) = 0$

$50\omega + 4\omega(\omega^2 - 100) = 0$

$2\omega[25 + 2\omega^2 - 200] = 0$

$\omega(2\omega^2 - 175) = 0$

$\therefore \omega = 0$ (reject) or $\omega = \sqrt{175}$

$\omega = 13.2$ (3sf) #

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

MA3005 – CONTROL THEORY
MA3705 – AEROSPACE CONTROL THEORY

November/December 2019 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FIVE (5)** questions and comprises **SEVEN (7)** pages including **TWO (2)** pages of Appendix.
2. Answer **ALL** questions.
3. Marks for each question are as indicated.
4. This is a **RESTRICTED-OPEN BOOK** examination. One double sided A4 reference sheet is allowed.

1(a) What is the stability of a system that contains some LHP poles, three imaginary poles, one negative-real zero and a pair of RHP zero as illustrated in Figure 1? Will the system oscillate as a response to a step input? Give the rationale of your answer.

(5 marks)

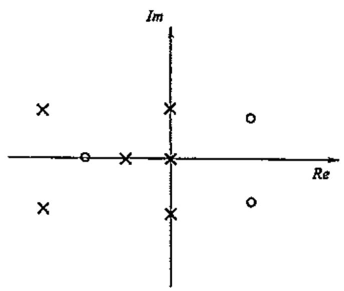


Figure 1: s-plane for the system.

Note: Question 1 continues on page 2.

(b) The transfer function of a second order system is $G(s)=\dfrac{1}{s^2-2s+5}$.

(i) Is the system stable, marginally stable or unstable? Explain your answer.

(2 marks)

(ii) Find, mathematically, the impulse response of the system, $G(s)$.

(5 marks)

(iii) Based on your answer in *(ii)*, what is the frequency of oscillation of the system?

(3 marks)

(c) Consider the unity-feedback system shown in Figure 2.

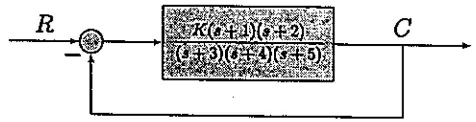


Figure 2: Unity feedback system.

Determine the number of break-in/break-out points on the real axis. No justification is required.

(5 marks)

(d) Consider a system with the Bode plots shown in Figure 3.

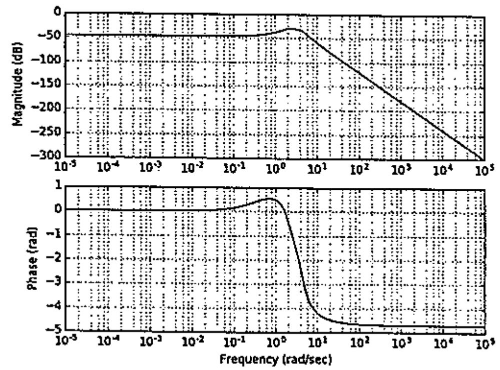


Figure 3: Bode plots.

Suppose that the input to the system is $r(t)=50\cos(0.01t+0.5)$, determine the output of system. No justification is required.

(5 marks)

40

2. An electronic company is planning to carry out a drop test on its product. A drop test is meant to evaluate the durability of the product by releasing the product to free fall (with initial velocity, $v_0 = 0$) from a target altitude, where typically the product is dropped from an elevated hook of a crane (see Figure 4). The manufacturer aims to test the durability of the product under impact at a testing velocity $v_t = 25$m/s, where the mass of the test item is 0.2kg.

It is known that the damping coefficient of air is 0.1Ns/m, and the gravitational acceleration is 10m/s^2.

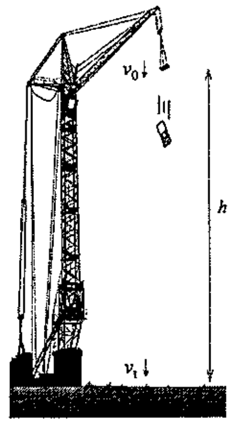


Figure 4: Drop test for the system.

(a) Show that no matter how high the initial position of the product before it is dropped, the specified testing velocity, v_t, will never be reached. This is to say that the drop test cannot achieve the goal.
(Hint: Determine the final velocity of the product under the test).

(8 marks)

(b) In order to achieve the goal, the test engineer considers to give an initial velocity to the test item ($v_0 \neq 0$).

Discuss whether it is a good idea or not to achieve the testing velocity target ($v_t = 25$m/s). If it is, find the ideal initial velocity for the test product to reach the testing velocity.

(7 marks)

3(a) Figure 5 shows the block diagram model of a dynamic system. Determine the equivalent transfer function of the system that correlates the output, C, to the input R.

(5 marks)

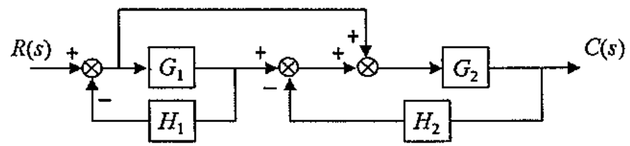


Figure 5: A block diagram.

(b) Referring to the problem in 3(a), if it is known that:

$G_1(s) = 127$

$H_1(s) = 0$

$$G_2(s) = \frac{1}{s^8 + 3s^7 + 10s^6 + 24s^5 + 48s^4 + 96s^3 + 128s^2 + 192s + 127}, \text{ and}$$

$H_2(s) = 1$,

Check the stability of the system using Routh-Hurwitz method. If the system is unstable, specify the number of poles in RHP.

(4 marks)

(c) Figure 6 shows a rotational system comprising two disks with moment of inertia J_1 and J_2, respectively. The two disks are interconnected to each other through a flexible shaft with stiffness of k. The first disk J_1 is supported to the ground by a bearing with damping value of c, while and disk J_2 is freely spinning without bearing support (no damping effect). The system is driven on the second disk (J_2) by a torque input $\tau(t)$.

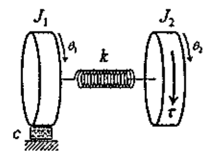


Figure 6: A rotational system.

(i) With the help of a free-body-diagram, define the transfer function, $\frac{\Omega_1(s)}{\mathrm{T}(s)}$, that correlates the angular velocity output on the second disk, ω_1, to the torque input, $\tau(t)$. Show the working step.
(Hint: $\Omega_1 = \mathcal{L}[\omega_1]$, where velocity ω_1 is the first derivative of θ_1.)

(5 marks)

(ii) Find the steady state angular velocity output ω_1, if a step torque input is given to the system, $\tau(t) = 1$.

(6 marks)

4. Consider the PD-controlled system shown in Figure 7.

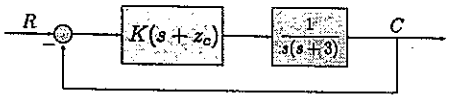


Figure 7: A PD-controlled system.

(a) In this question, we assume that $z_c = 1$. Determine the open-loop pole(s) and open-loop zero(s) and sketch the root locus of the system.

(4 marks)

(b) In this question, we assume that $z_c = 4$.

(i) Determine the open-loop pole(s) and open-loop zero(s). Determine the location(s) of the break-in/break-out point(s), if any.

(4 marks)

(ii) Determine the location(s) of the closed-loop pole(s) when $K = 5$.

(3 marks)

(iii) Sketch the root locus of the system.

(5 marks)

(iv) Show numerically that $s = -5 \pm 2j$ does not belong to the root locus.

(3 marks)

(c) Find the value of z_c such that $s = -5 \pm 2j$ belongs to the root locus.

(6 marks)

5. Consider a lead-lag compensator with the following transfer function:

$$G(s) = K \cdot \frac{(s+\frac{1}{T_1})(s+\frac{1}{T_2})}{(s+\frac{\alpha}{T_1})(s+\frac{1}{\alpha T_2})}$$

(a) For $K = 1$, $T_1 = 10$, $T_2 = 1000$ and $\alpha = 10$, sketch

(i) the Bode plot (magnitude) of $G(s)$.

(6 marks)

(ii) the Bode plot (phase) of $G(s)$.

(4 marks)

(b) In this question, we assume only that $K>0$, $T_1>0$, $T_2>0$ and $\alpha>0$. Let $\omega_1 = \frac{1}{\sqrt{T_1 \cdot T_2}}$. Show that $\angle G(\omega_1) = 0$.

(5 marks)

41

Nov/Dec 2019

(a) $F(s) = \frac{3s+8}{s^2+2s+10}$

$IVT = \lim_{s\to\infty} s \cdot \left(\frac{3s+8}{s^2+2s+10}\right) = \lim_{s\to\infty} \frac{3s+8}{s+2+\frac{10}{s}}$

$= \lim_{s\to\infty} \frac{(s)}{(s)} \frac{3+\frac{8}{s}}{1+\frac{2}{s}+\frac{10}{s^2}}$

$= \frac{3}{1} = 3.$

$FVT = \lim_{s\to 0} s \cdot \frac{3s+8}{s^2+2s+10} = 0$ (case 2 funct^n).

$\frac{d}{dt}f(t) = sF(s) = s \cdot \frac{3s+8}{s^2+2s+10}.$

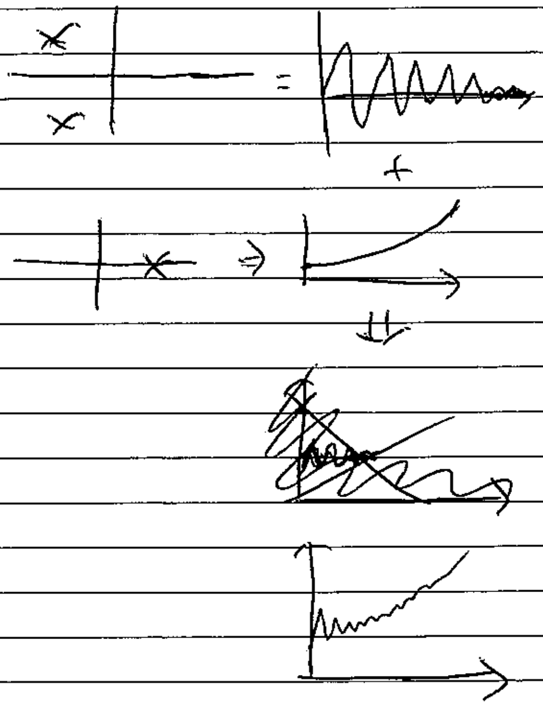
$IVT = \lim_{s\to\infty} s \cdot \frac{d}{dt}f(t) = \lim_{s\to\infty} s^2 \cdot \frac{3s+8}{s^2+2s+10}$

$= \lim_{s\to\infty} \frac{3s+8}{1+\frac{2}{s}+\frac{10}{s^2}} = \infty$

b.) - Step response of dynamic system ~~response~~ shows.
dying oscillat^{n}, unstable.
- zero do not change the look on response graph.
- So there's no need to check zeroes.

① need dying oscillat^{n}

② need unstable



∴ B suits graph.

c. ① draw left asymptote & right asymptote.

left asymptote:

gradient: $\frac{20-60}{-2-(-3)} = -40$

equation: $y = 20\log\left(\frac{N(0)}{D(0)}\right) - 20k\log(\omega)$ → gradient.

$20k = 40$ where k is system type.

$k = 2$ ∴ System type 2.

Right asymptote:

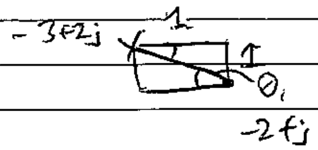
gradient = 0.

equation: $y = 20\log|A| - 20n\log(\omega)$

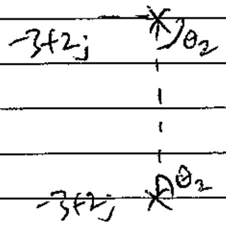
$20n = 0$ where n = S system is order of.

∴ System order 0.

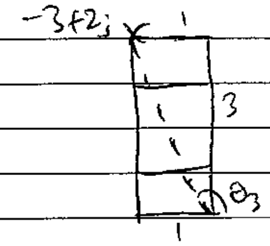
d. use graph and geometrics to find angle.

① 

$\theta_1 = 135°$

② 

$\theta_2 = 90°$

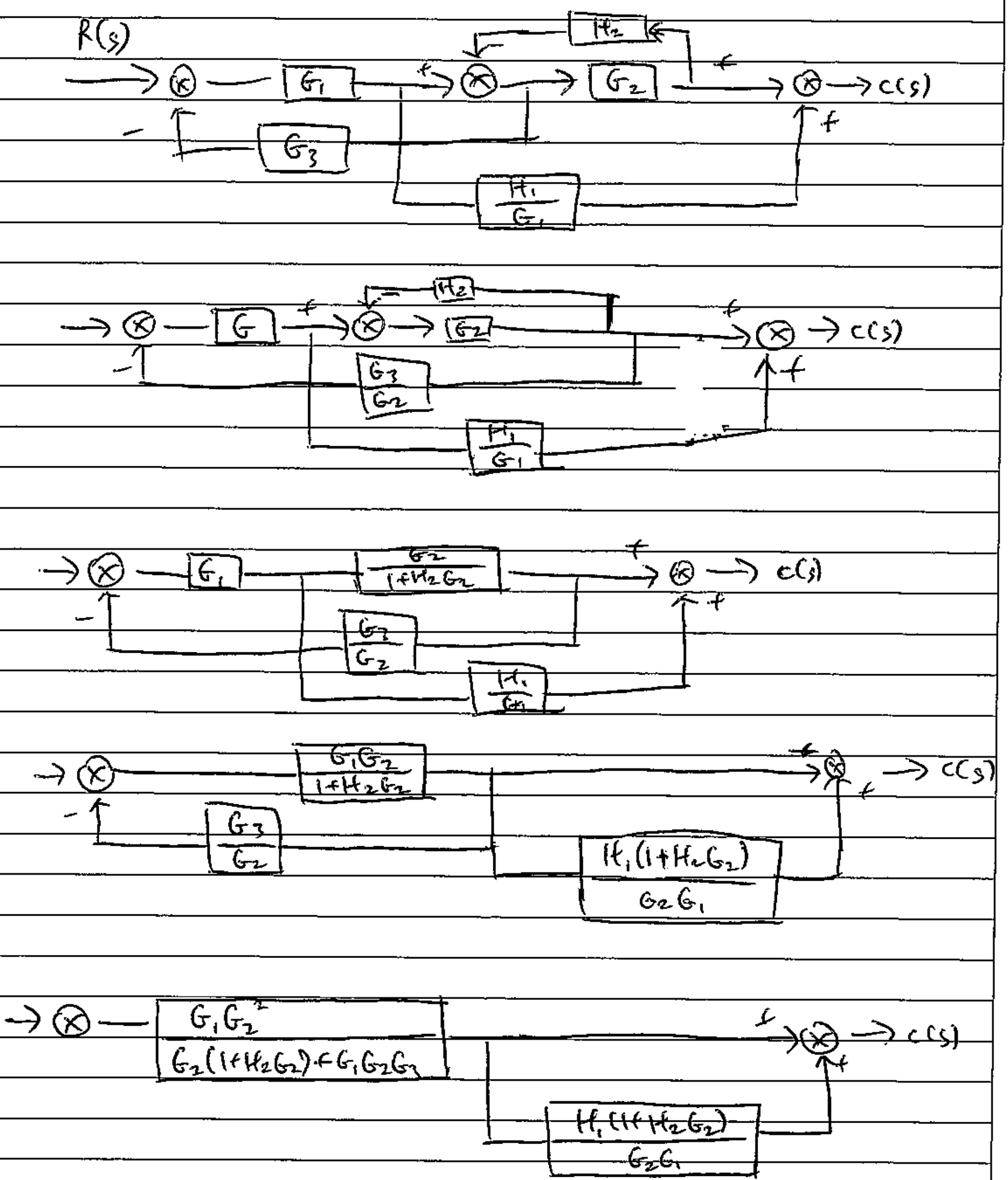
③ 

$\theta_3 = 180° - \tan^{-1}\left(\frac{3}{1}\right) = 108.48°$

$$\alpha_{dep} = 180 - \sum_{poles\,p_i} \alpha_{p_i \to p} + \sum_{zeros\,z_i} \alpha_{z_i \to p}$$

$$= 180° - 90° + 135° + 108.48° = 333.48°$$

2(a).


R(s)
G1
H2
G2
G3
C(s)
H1/G1
G3/G2
G2/(1+H2G2)
G1G2/(1+H2G2)
H1(1+H2G2)/(G2G1)
G1G2^2/(G2(1+H2G2)+G1G2G3)

$$R(s) \rightarrow \boxed{\frac{G_1G_2}{1+H_2G_2+G_1G_3} + \frac{H_1(1+H_2G_2)}{1+H_2G_2+G_1G_3}} \rightarrow C(s)$$

$$R(s) \rightarrow \boxed{\frac{G_1G_2 + H_1 + H_1H_2G_2}{1+H_2G_2+G_1G_3}} \rightarrow C(s)$$

b)

$$R(s) \rightarrow \boxed{G_1} \rightarrow \boxed{\frac{G_2}{1+H_2G_2}} \rightarrow C(s)$$

$$R(s) \rightarrow \boxed{\frac{G_1G_2}{1+H_2G_2}} \rightarrow C(s)$$

$$\frac{\left(\frac{1}{s+2}\right)\left(\frac{1}{s^4+2s^3+5s^2+4s+39}\right)}{1+\frac{1}{s^4+2s^3+5s^2+4s+39}}$$

$$\frac{\frac{1}{s+2}\left(\frac{1}{\cancel{s^4+2s^3+5s^2+4s+39}}\right)}{\frac{s^4+2s^3+5s^2+4s+39+1}{\cancel{s^4+2s^3+5s^2+4s+39}}}$$

$$\frac{1}{(s+2)(s^4+2s^3+5s^2+4s+40)}$$

$$\frac{1}{s^5+2s^4+5s^3+4s^2+40s+2s^4+4s^3+10s^2+8s+80}$$

$$\frac{1}{s^5+4s^4+9s^3+14s^2+48s+80.}$$

s^5	1	9	48
s^4	4	14	80.
s^3	5.5	28	
s^2	-6.36	80.	
s^1	97.18		
s^0	80.		

Since change of pole twice, $\therefore$ unstable and I pole in RHS.

c) There's no steady-state as it is not stable.

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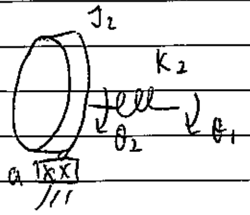
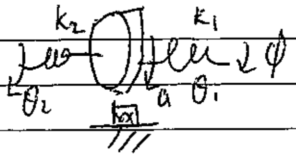
3.(a)



$$J_2\ddot{\theta}_2 + C_2\dot{\theta}_2 + K_2\theta_2 - K_1\theta_1 = 0$$

$$J_2 s^2 \Theta_2 + C_2 s \Theta_2 + K_2 \Theta_2 - K_1 \Theta_1 = 0$$

$$\frac{(J_2 s^2 + C_2 s + K_2)\Theta_2}{K_1} = \Theta_1$$



$$J_1\ddot{\theta}_1 + C_1\dot{\theta}_1 + K_1\theta_1 + K_2(\theta_1 - \theta_2) = K_1\phi$$

$$J_1\ddot{\theta}_1 + C_1\dot{\theta}_1 + (K_1 + K_2)\theta_1 - K_2\theta_2 = K_1\phi$$

$$J_1 s^2 \Theta_1 + C_1 s \Theta_1 + (K_1 + K_2)\Theta_1 - K_2\Theta_2 = K_1\Phi$$

$$(J_1 s^2 + C_1 s + K_1 + K_2)\Theta_1 - K_2\Theta_2 = K_1\Phi$$

Sub both eq:

$$\left[\frac{(J_1 s^2 + C_1 s + K_1 + K_2)(J_2 s^2 + C_2 s + K_2)}{K_1} - K_2\right]\Theta_2 = K_1\Phi$$

$$\left[\frac{(J_1 s^2 + C_1 s + K_1 + K_2)(J_2 s^2 + C_2 s + K_2)}{K_1}\right]\Theta_2 = K_1\Phi$$

$$\frac{\Theta_2}{\Phi} = \frac{K_1^2}{(J_1 s^2 + C_1 s + K_1 + K_2)(J_2 s^2 + C_2 s + K_2) - K_1 K_2}$$

(b) (i)

$$J\ddot{\theta} + C\dot{\theta} + K_1\theta + K_0\theta = K_1\varphi$$

$$Js^2\Theta + Cs\Theta + (K_1 + K_0)\Theta = K\Phi$$

$$(Js^2 + Cs + K_1 + K_0)\Theta = K_1\Phi$$

$$\frac{\Theta}{\Phi} = \frac{K_1}{Js^2 + Cs + K_1 + K_0}$$

(ii) typical 2OPE: $G(s) = \frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$\frac{\Theta}{\Phi} = \frac{\frac{K_1}{J}}{s^2 + \frac{C}{J}s + \frac{K_1 + K_0}{J}}$$

unit response when $t \to \infty$ $\lim_{s\to 0} sC(s) = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{K\omega_n^2}{\cancel{s^2} + \cancel{2\zeta\omega_n s} + \omega_n^2}$

$= K = 0.5$

$\lim_{s\to 0} s\Theta = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} - \frac{\frac{K_1}{J}}{\cancel{s^2} + \cancel{\frac{C}{J}s} + \frac{K_1 + K_0}{J}}$

$= \frac{K_1}{K_1 + K_0} = 0.5$ (because steady state response is $\frac{1}{2}$ of unit step input $\therefore K = 0.5$).

$t_p = \frac{\pi}{\omega_d}$ (from graph, $t_p = 1.405$)

$\omega_d = 2.236$ rads

$t_r = \frac{\pi - \cos^{-1}\zeta}{\omega_d}$ (from graph, $t_r = 0.914$)

$\zeta = \cos(\pi - t_r \cdot \omega_d) = 0.455$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$\omega_n = \frac{2.236}{\sqrt{1-0.455^2}} = 2.51$ rads

By comparison $\Rightarrow$

$0.5\omega_n^2 = \frac{K_1}{J}$

$K_1 = 0.5(2.51)^2(5)$

$= 15.75$ N/m

$\omega_n^2 = \frac{K_1 + K_0}{J}$

$K_0 = \omega_n^2(J) - K_1$

$= (2.51)^2(5) - 15.75$

$= 15.75$ N/m

$2\zeta\omega_n = \frac{C}{J}$

$C = 2(0.455)(2.51)(5)$

$= 11.42$ Ns/m.

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iii)

$\dot{\theta} = \frac{d}{dt}\theta$

$\mathcal{L}\{\dot{\theta}\} = \mathcal{L}\{\frac{d}{dt}\theta\} = s\Theta = \omega$

$\frac{\omega}{\Phi} = \frac{K_1 s}{Js^2 + Cs + K_1 + K_0}$

(iv)

$\Phi = \frac{1}{s^2}$

$\omega = \frac{1}{s^2} \cdot \frac{K_1 s}{Js^2 + Cs + K_1 + K_0}$

$\lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s^2}} \cdot \frac{K_1 J}{Js^2 + Cs + K_1 + K_0}$

$= \lim_{s \to 0} \frac{K}{Js^2 + \cancel{Cs} + K_1 + K_0}$

$= \frac{K_1}{K_1 + K_0}$

4a) consider CE : $KN(s) + D(s) = 0$

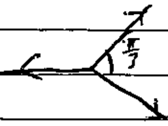
Zeros : 0 poles : roots of $D(s) = 0$

$(s+1)(s+2+j)(s+2-j)$

$\therefore s_1 = -1,\ s_2 = -2+j,\ \cancel{s_3 = -2+j}\ s_3 = -2-j$

b) $n_{\text{infinite branches}} = n_{poles} - n_{zeros}$

$= 3 - 0 = 3$. $\therefore$ 3 asymptotes.



$\theta = \frac{\pi}{3}$ (configuration)

$\sigma_a = \dfrac{\sum poles - \sum zeros}{n_{poles} - n_{zeros}} = \dfrac{(-2+j)(-2-j)+(-1) - 0}{3}$

$= -1$

c) $N(s) = 1$ $D(s) = (s+1)(s^2+4s+5) = s^3 + 5s^2 + 9s + 5$

$N'(s) = 0$ $D'(s) = 3s^2 + 10s + 9$

$N'(s)D(s) - D'(s)N(s) = 0$

$-(3s^2 + 10s + 9)(1) = 0$

$s_1 = -1.67 + 0.47j$ $s_2 = -1.67 - 0.47j$

Sub into $K = \dfrac{-D(s_1)}{N(s_1)}$ (check for real K).

$K = \dfrac{-(-1.67+0.47j+1)[(-1.67+0.47j)^2 + 4(-1.67+0.47j) + 5]}{1}$

$= 0.741 - 0.24j$ (not real K $\therefore$ no breakouts)

d) Let $s = j\omega$ $N(s) = 1$ $D(s) = (j\omega + 1)((j\omega)^2 + 4j\omega + 5)$

$= -j\omega^3 - 5\omega^2 + 9j\omega + 5$

consider CE : $KN(j\omega) + D(j\omega) = 0$

$K - j\omega^3 - 5\omega^2 + 9j\omega + 5 = 0$

$K - 5\omega^2 + 5 + (9\omega - \omega^3)j = 0$

$\left.\right\}$ $K - 5\omega^2 + 5 = 0$

$9\omega - \omega^3 = 0$

$\omega_1 = 0,\ \omega_{2,3} = \pm 3$

$K_1 = -5,\ K_{2,3} = 4$

$\therefore$ with $K = 4$ at $\omega = \pm 3$ $\therefore$ $0 < K < 4$

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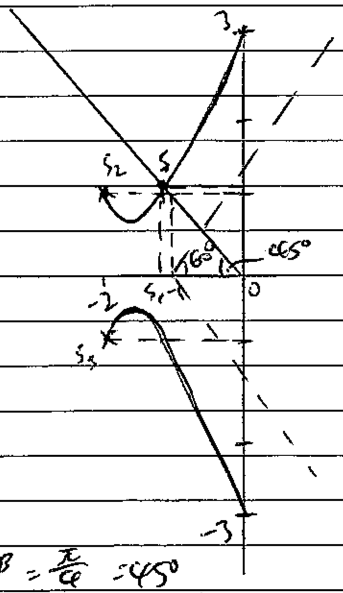
e)



Can use α_{dep} to find out direct^n
but because there is pole of $S_1 = -1$,
α_{dep} will move towards it before
moving towards asymptotes.

f) $\zeta = \cos\beta$, $\beta = \frac{\pi}{4} = 45°$

∴ graph should sketch closely to scale.

$S \approx -1.2 \pm j1.2$ ∴

g) Sub $s = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$

$s = -0.707\omega_n + j0.707\omega_n$ in CE:

$K + (-0.707\omega_n + j0.707\omega_n + 1)((-0.707\omega_n + j0.707\omega_n)^2 + 4(-0.707\omega_n + j0.707\omega_n) + 5) = 0$

$= K + 0.707\omega^3 - 6.36\omega + 5 + j(0.707\omega^3 - 5\omega^2 + 6.36\omega) = 0$

$K + 0.707\omega^3 - 6.36\omega + 5 = 0$

$0.707\omega^3 - 5\omega^2 + 6.36\omega = 0$ → $\omega_n = 0, \omega_n = 5.4$ rad/s, $\omega_n = 1.66$ rad/s

$K = -5, K = -81.98, K = 2.32$.

$\therefore s = -(0.707)(1.66) \pm j(0.707)(1.66)$

$= -1.17 \pm j1.2$ at $K = 2.32$

lower order approx: $\frac{K}{(s+1)(s^2+4s+5)+K} = \frac{2.32}{s^3+5s^2+9s+5+2.32}$

$= \frac{2.32}{s^3+5s^2+9s+7.32}$

$= \frac{2.32}{(s+2.64)(s-1.18+j1.18)(s+1.18-j1.18)}$

$\approx \frac{2.32}{(s-1.18+j1.18)(s+1.18-j1.18)}$ → has sim pole s. valid & quite accurate.

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use wolfram alpha
to check if root locus
is drawn properly.
Type "root locus plot".

5. $G(s) = \frac{1}{s^2+5s+100}$

a) $\omega_n = \sqrt{100} = 10\ (\omega_{min}, \omega_{max})$

$0.1\omega_{min} = 1\,rads \qquad 100\omega_{max} = 100\,rads$

left asymptote : $G(s) = \frac{N(s)}{s^k D(s)}$ where $k=0$

$\left|\frac{N(0)}{D(0)}\right| = \frac{1}{100} = 0.01$

$\therefore y = 20\log\left|\frac{N(0)}{D(0)}\right| - 20k\log(\omega)$

intercept $(0,-40)$

phase shift : $\frac{N(0)}{D(0)} > 0 \quad \therefore y = 0$

Right asymptote : $G(s) = \frac{N(s)}{D(s)}$

$n = \deg(D) - \deg(N) = 2-0 = 2$

$|A| = 1$

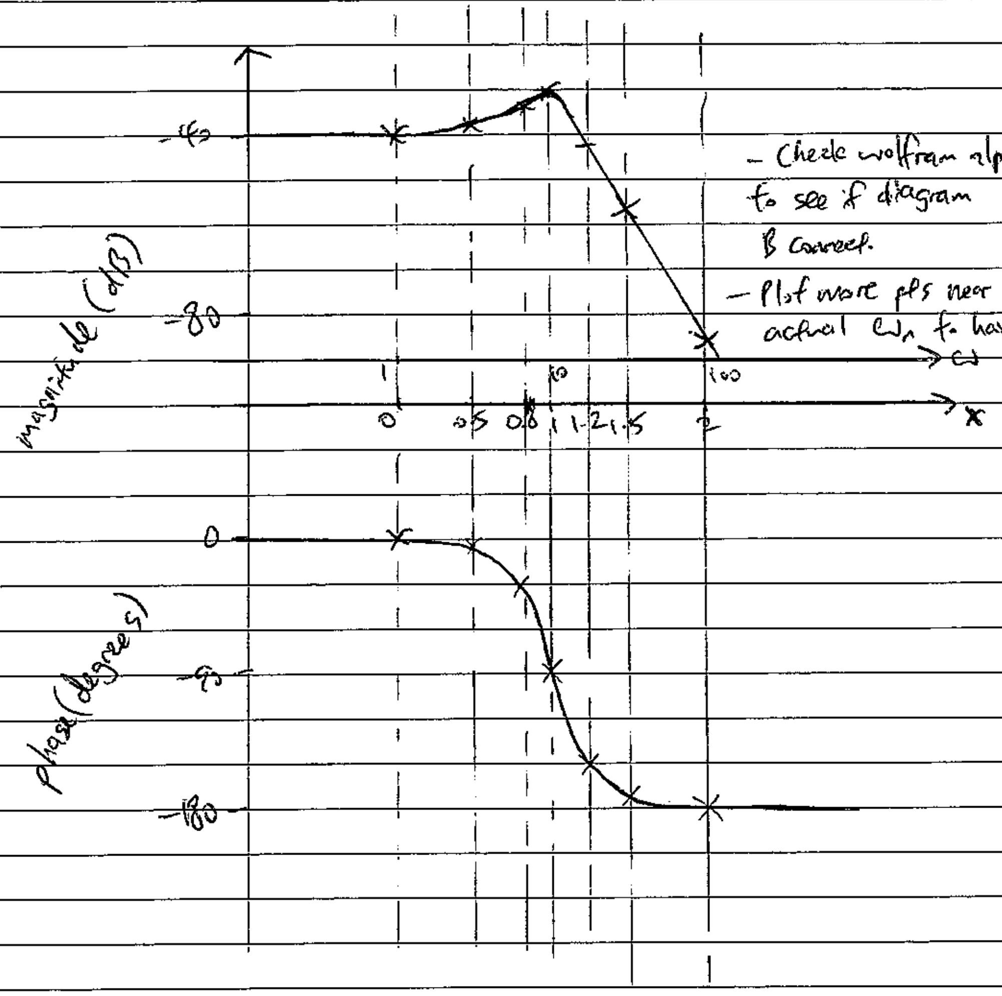
$\therefore y = 20\log|A| - 20n\log(\omega)$

$y = -40\log(\omega) \qquad$ intercept $(0,0)$.

phase shift : $A>0 \qquad \therefore y = -180$

$G(j\omega) = \frac{1}{(j\omega)^2+5(j\omega)+100}$

x	ω	$20\log\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	1	-40	-0.03
0.5	3.2	-39.20	-10.11
0.8	6.3	-36.66	-27.58
1	10	-33.98	-90
1.2	15.9	-44.72	-152.51
1.5	31.6	-59.20	-170.03
2	100	-79.92	-177.12



- Check wolfram alpha to see if diagram is correct.
- Plot more pts near actual ω_n to have more accurate diagram.

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c)

$$\frac{1}{|G(j\omega)|} = |(j\omega)^2 + 5(j\omega) + 100|$$

$$= |-\omega^2 + 100 + 5j\omega|$$

$$= \sqrt{(-\omega^2+100)^2 + (5\omega)^2}$$

$$(-\omega^2+100)^2 + (5\omega)^2$$

$$\omega^4 - 175\omega^2 + 10'000$$

when minimum, $\frac{d}{d\omega}(\omega^4 - 175\omega^2 + 10'000) = 0$

$$3\omega^2 - 350\omega = 0$$

$$\omega^2 = 116.67$$

$$\omega_p = 10.8 \text{ rad/s}$$

$|G(j\omega_p)| = 0.0177$

$|G(j\omega_p)| = 0.02$

the results are similar.
the peak usually happens close to ω corner frequency.