NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2015-2016

MA4872 – AIRCRAFT RELIABILITY AND MAINTAINABILITY

April/May 2016 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 3 questions and comprises 5 pages.
2. Answer all 3 questions.
3. Marks for each question are as indicated.
4. This is a **Restricted Open-Book** examination. One double-sided A4 reference sheet is allowed.

---

1 (a) A company bought 100 trucks recently. Each truck has a decreasing failure rate characterized by a Weibull distribution with $\theta = 180$ years and $m = 0.5$. How many trucks are expected to continue working after 5 years? And how many trucks are likely to fail between the 5^{th} and the 6^{th} years?

(7 marks)

(b) Figure 1 illustrates the reliability block diagram of an electronic system which consists of 5 identical and independent components. The time to failure of the components is exponentially distributed. Each component has a MTTF of 10000 hours. The system requires at least one path to exist between input point A and output point B for successful operation. Determine the system reliability for an operating period of 1000 hours.

(8 marks)

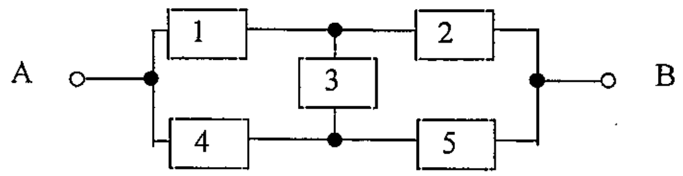


Figure1

Note: Question 1 continues on page 2.

(c) The power supply system for a hospital consists of two subsystems in a stand-by configuration. The primary subsystem is based on the utility grid and the stand-by subsystem is a power generator. The failure rate of the primary subsystem is always equal to λ_a. The failure rate of the stand-by subsystem is λ_{b1} when it is activated (after the failure of the primary subsystem) and λ_{b0} when it is not activated (i.e. the stand-by subsystem might fail due to some reasons when the primary subsystem is still functional). Draw a Markov state transition diagram and write down the state transition equations for the determination of the system reliability. Finally, express the system reliability in terms of the state probabilities P_i.

(8 marks)

(d) A system has an increasing failure rate characterized by a Weibull distribution with θ = 50 years and $m = 2$.

(i) Determine the maintenance time interval T for a preventive maintenance scheme so that the system reliability R_M with maintenance is no smaller than 0.98 at the end of a design life of 10 years.

(4 marks)

(ii) Using the maintenance time interval T determined in (i), calculate the final reliability R_M with maintenance for the design life of 10 years, taking into consideration a repair error probability $p = 0.01$.

(4 marks)

(e) A system consists of 3 components in parallel. All components have a same constant failure rate of $\lambda = 0.02$. The required stable availability for the system is $A_s = 0.98$. Determine the MTTR (Mean Time to Repair) for each component.

(7 marks)

2 (a) The following three techniques are widely used in a 2^k design in order to simplify the DOE (Design of Experiment) procedure:

- Single replicate
- Project
- Fractional design

Which of the above <u>THREE</u> techniques will reduce any of the following 3 parameters?

- K number of factors
- N total number of observations
- R number of runs

(6 marks)

Note: Question 2 continues on page 3

(b) A factorial experiment is conducted to study the influence of two factors (A: operator, B: machine) on the breaking strength of a synthetic fiber. The observations (in kPa) are displayed in Table 1.

(i) Determine the error ε_{ijk} for each observation y_{ijk}.

(3 marks)

(ii) Determine the test ratio F_0 for factor B (machine).

(5 marks)

Table 1

	Machine		
Operator	1	2	3
1	2, 3	2, 1	1, 3
2	3, 4	4, 5	3, 3

(c) A 2^3 design is carried out to study the effects of three factors A, B and C. The experiment layout and the observations are displayed in Table 2.

(i) Calculate the interaction ABC.

(3 marks)

(ii) Estimate the mean square error of MS_E.

(2 marks)

(iii) It has been found that the three main effects are A = 6.5, B = -6 and C = -2.5. Project this 2^3 design into a 2^2 design in terms of the two most effective factors.

(4 marks)

Table 2

Run	A	B	C	Observations
(1)	-	-	-	7
a	+	-	-	8
b	-	+	-	-2
ab	+	+	-	4
c	-	-	+	6
ac	+	-	+	3
bc	-	+	+	-12
abc	+	+	+	10

Note: Question 2 continues on page 4

(d) A researcher investigates five factors --- mold temperature (A), screw speed (B), holding time (C), cycle time (D), and gate size (E) --- each at two levels. The objective is to study how each factor affects the defective rate. The observations are displayed in Table 3.

(i) Find all of the aliases of effect A.

(4 marks)

(ii) Calculate the joint effect ℓ_A.

(3 marks)

Table 3

No	A	B	C	$D=AB$	$E=AC$	No of defects
1	-	-	-	+	+	6
2	+	-	-	-	-	10
3	-	+	-	-	+	32
4	+	+	-	+	-	60
5	-	-	+	+	-	4
6	+	-	+	-	+	15
7	-	+	+	-	-	26
8	+	+	+	+	+	60

3 (a) The linked system in Figure 2 consists of 4 components with reliabilities $R_1 = R_2 = 0.9$, $R_3 = 0.81$, and $R_4 = 0.99$. You are **required to use component 3 as the key** to calculate the system reliability with all the steps presented (e.g. the decomposed block diagrams, the formulae for calculating R_S^+ and R_S^-).

(10 marks)

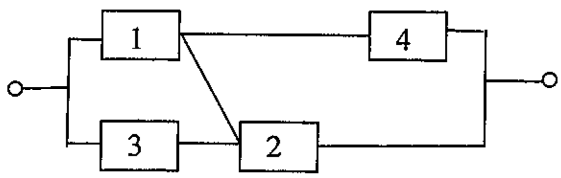


Figure2

(b) A fault tree consists of 2 minimum cut sets M_1 and M_2, and 4 independent basic events a, b, c and d.

$M_1 = a \cap b$, $M_2 = a \cap c \cap d$

The probabilities of occurrences of the basic events are enumerated below:

$P(a) = 0.90$, $P(b) = 0.95$, $P(c) = 0.80$, $P(d) = 0.85$

Calculate the probability P{T} that the top event of the fault tree will occur.

(6 marks)

Note: Question 3 continues on page 5

(c) Table 4 displays the observations of a 2^3 design in which the numbers of observations at different runs may not be the same. Calculate the main effects A and C.

(5 marks)

Table 4

Run	A	B	C	Observations
(1)	-	-	-	7
a	+	-	-	8
b	-	+	-	2, 3
ab	+	+	-	4
c	-	-	+	4, 6
ac	+	-	+	3
bc	-	+	+	2
abc	+	+	+	1

(d) Table 5 displays the observations of a one-factor-at-a-time experiment. Calculate the main effects A, B and C.

(6 marks)

Table 5

Run	A	B	C	Observations
ab	+	+	-	-8, -7
ac	+	-	+	3, 2
bc	-	+	+	-6, -5
abc	+	+	+	4, 3

(e) In a 2^{5-2} design, interactions ABD and ACE are used as the basic generators. They will always have plus signs in the plus-minus-sign table of the selected fraction. Determine whether any or both of the two runs (*ac*) and (*bc*) should be included in the selected fraction (quarter).

(5 marks)

End of Paper

~~MA4878~~ 15/16 Sem 2

1a i $R(5) = \exp\left(-\left(\frac{5}{100}\right)^{1.5}\right)$

$= 0.84648$

Number of trucks left $= 100\,R(5)$

$= 84.64817249$

≈ 84

1a ii $100\,[R(5) - R(6)]$

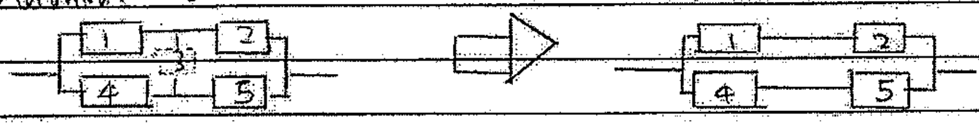
$= 100\left[\exp\left(-\left(\frac{5}{100}\right)^{1.5}\right) - \exp\left(-\left(\frac{6}{100}\right)^{0.5}\right)\right]$

$= 1.3358869$

≈ 2 (rounded up)

1b ① $\boxed{3}$ is chosen as the key component $r = e^{-\frac{100}{1000}}$

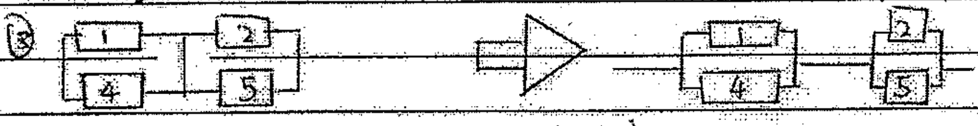
② Calculate R_s^-



$R_s^- = R_1R_2 + R_4R_5 - R_1R_2R_4R_5$

$= 2r^2 - r^4$

③



$R_s^+ = (R_1 + R_4 - R_1R_4)(R_2 + R_5 - R_2R_5)$

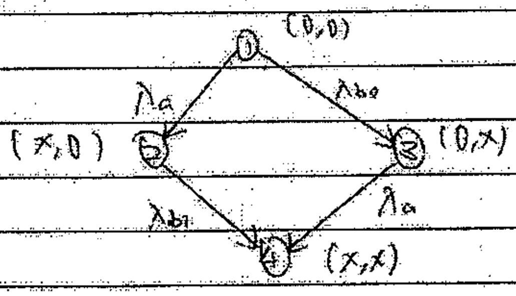
$= (2r - r^2)^2$

④ $R_{sys} = (1-r)(R_s^-) + (R_s^+)\,r$

$= 0.980559036$

1c

Component	State 1	2	3	4
Primary	0	X	0	X
Secondary	0	0	X	X
	0	0	0	X



$\frac{d}{dt}P_1(t) = -\lambda_a P_1(t) - \lambda_{b0} P_1(t)$

$\frac{d}{dt}P_2(t) = \lambda_a P_1(t) - \lambda_{b1} P_2(t)$

$\frac{d}{dt}P_3(t) = \lambda_{b0} P_1(t) - \lambda_a P_3(t)$

$\frac{d}{dt}P_4(t) = \lambda_a P_3(t) + \lambda_{b1} P_2(t)$

$R = P_1(t) + P_2(t) + P_3(t)$ $\quad P_1(t) + P_2(t) + P_3(t) + P_4(t) = 1$

Date No.

1di $R_m(t) = [R(T)(1-p)]^N$ T_d = design life

$= [\exp -(\frac{T_d}{N\theta})^m]^N$

$N = -\left[\frac{\ln R_m}{(T_d/\theta)^m}\right]^{\frac{1}{1-m}} = -\left[\frac{\ln 0.98}{(10/50)^2}\right]^{\frac{1}{1-2}}$

$= 1.979932158$

$T = T_d/N = 10/1.9799$

$= 5.0504768$

≈ 5 years

1dii $R_m(10) = [R(5)]^2(1-p)$

$= [\exp[-(5/50)^2]]^2 (1-0.01)$

$= 0.970396686$

≈ 0.970 (to 3sf)

1e unavailability of system = 0.02

unavailability of component = $\sqrt[3]{0.02}$

$= 0.2714417$

$A_{component} = 1 - 0.27144 = 0.728558238$

$\nu = \frac{A(\infty)\lambda}{1-A(\infty)} = \frac{(0.728)(0.02)}{1-0.72853} = 0.053680629$

$MTTR = 1/\nu = 18.6286$

≈ 18.6 years

2a Number of factors, k, is reduced by projecting

Total number of observations, N, is reduced by using single replicate

Number of runs, R, is reduced by using fractional design

2bi

	Machine		
Operator	1	2	3
1	-0.5, 0.5	0.5, -0.5	-1, 1
2	-0.5, 0.5	-0.5, 0.5	0, 0

Date No.

2bii $DOF_C = 1$ $DOF_B = 2$ $DOF_{CB} = 2$ $DOF_T = 11$

$DOF_E = 6$

$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$

$= 2^2+3^2+3^2+4^2+2^2+1^2+4^2+5^2+1^2+3^2+3^2+3^2 - \frac{34^2}{2\times3\times2}$

$= 112 - 96\frac{1}{3} = 15\frac{2}{3}$

$SS_B = \sum_{j=1}^{b} \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn} = \frac{1}{2\times2}(12^2+12^2+10^2) - \frac{34^2}{2\times3\times2} = 2/3$

$MS_B = \frac{SS_B}{DOF_B} = 2/3 \div 2 = 1/3$

$SS_A + SS_B + SS_{AB} = \sum_{i=1}^{a}\sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn}$

$= \frac{1}{2}(5^2+7^2+3^2+9^2+4^2+6^2) - \frac{34^2}{2\times3\times2} = 11\frac{2}{3}$

$SS_E = SS_T - (SS_A + SS_B + SS_{AB}) = 15\frac{2}{3} - 11\frac{2}{3} = 4$

$MS_E = \frac{SS_E}{DOF_E} = 4/6 = 2/3$

$F_0(B) = \frac{MS_B}{MS_E} = 1/3 \div 2/3 = 1/2$

2ci $C_{ABC} = -7+8+(-2)-4+6-3-(-12)+10 = 20$

$ABC = \frac{C_{ABC}}{2N} = \frac{20}{2\times8} = 5$

2cii $MS_E = MS_{ABC} = \frac{C_{ABC}^2}{N} = \frac{1}{8}(20^2) = 50$

2ciii Factor C is least effective

Run	A	B	observations
(1)	-	-	7 , 6
a	+	-	8 , 3
b	-	+	-2 , -12
ab	+	+	4 , 10

2di Generators : ABD, ACE

Induced generator : BCDE

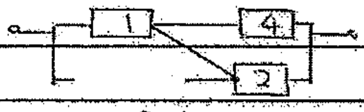
$A \hat{=} BD \hat{=} CE \hat{=} ABCDE$

2dii $\ell_A = -6+10-32+60-4+15-28+60$

$= 77$

3a ① Component 3 is chosen as key component

② Calculate R_s^-:

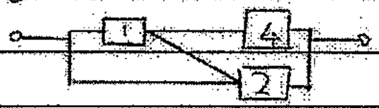


$R_s^- = R_1(R_2 + R_4 - R_2 R_4)$

$= 0.9(0.9 + 0.99 - 0.9 \times 0.99)$

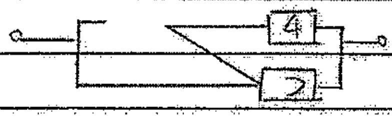
$= 0.8991$

③ Calculate R_s^+:



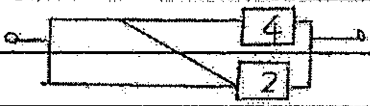
① Choose ① as key component

② Calculate R_G^-:



$R_G^- = R_2 = 0.9$

③ Calculate R_G^+:



$R_G^+ = R_2 + R_4 - R_2 R_4$

$= 0.9 + 0.99 - 0.9 \times 0.99$

$= 0.999$

④ $R_G = R_1 R_G^+ + (1-R_1) R_G^-$

$= 0.9 \times 0.999 + 0.1 \times 0.9$

$= 0.9891 = R_s^+$

④ $R_{sys} = (1 - R_3) R_s^- + R_3 R_s^+$

$= (1 - 0.81)(0.8991) + 0.81 \times 0.9891$

$= 0.972$

Date No.

3b $P(T) = P(M_1 \cup M_2)$

$= P(M_1) + P(M_2) - P(M_1 \cap M_2)$

$= P_a P_b + P_a P_c P_d - P_a P_b P_c P_d$

$= 0.8856$

3c $A = \frac{1}{4}(8+4+3+1) - \frac{1}{6}(7+2+3+4+1+2)$

$= 0$

$C = \frac{1}{5}(4+6+3+2+1) - \frac{1}{5}(4+2+3+8+7)$

$= -1.6$

3d

A	Observations
+	4, 3
−	−6, −5

$A = \frac{1}{2}(4+3) - \frac{1}{2}(-6-5)$

$= 9$

B	Observations
+	4, 3
−	3, 2

$B = \frac{1}{2}(4+3) - \frac{1}{2}(3+2)$

$= 1$

C	Observations
+	4, 3
−	−8, −7

$C = \frac{1}{2}(4+3) - \frac{1}{2}(-8-7)$

$= 11$

3e

Run	A	B	C	D = AB	E = AC
de	−	−	−	+	+
a	+	−	−	−	−
be	−	+	−	−	+
abd	+	+	−	+	−
cd	−	−	+	+	−
ace	+	−	+	−	+
bc	−	+	+	−	−
abcde	+	+	+	+	+

(bc) is included but (ac) is not included

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

MA4872 – AIRCRAFT RELIABILITY AND MAINTAINABILITY

April/May 2017 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **THREE (3)** questions and comprises **SEVEN (7)** pages including **TWO (2)** pages of Appendices.
2. Answer **ALL** questions.
3. Marks for each question are as indicated.
4. This is a **RESTRICTED OPEN-BOOK** examination (1 sheet of double-sided A4 reference paper is allowed).
5. Relevant data to answer the questions may be extracted from the Tables in the Appendices.

---

1 (a) The time to failure, t, of a component follows a Weibull distribution with the two parameters of ($m = 1$) and ($\theta = 100$).

(i) Calculate the failure rate λ, and find the relationship between λ and θ.

(3 marks)

(ii) Calculate the mean time to failure (*MTTF*).

(2 marks)

(b) A system has an m/N configuration. The total number, N, of components is equal to 20. All of the components have a same reliability of 0.40. Determine the system reliability for each of the THREE cases corresponding to the required minimum number, m, of the functional components equal to 1, 2, and 20, respectively (**the cumulative binomial probability can be found from a table in the Appendix.**).

(8 marks)

Note: Question 1 continues on page 2.

(c) Figure 1 displays the reliability block diagram of a system. Each block represents a component. The reliability of the i^{th} block is R_i. Express the system reliability R_S in terms of the component reliabilities R_i.

(8 marks)

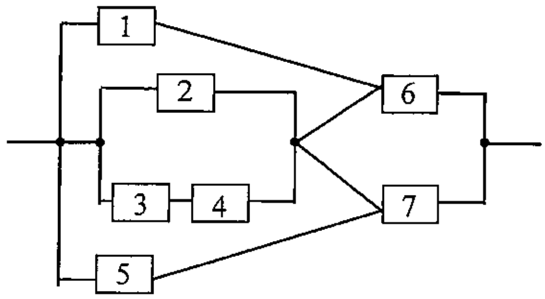


Figure 1

(d) A 30-hour replacement test is carried out on 10 resistors. During the 30 experimental hours, 5 failures have been observed. The failure times (in hours) are: 12, 14, 20, 22 and 25. It is known that the failure time follows an exponential distribution.

(i) Estimate the Mean Time To Failure (MTTF) and the failure rate of the resistor.

(3 marks)

(ii) Calculate the reliability and unreliability of the resistor for a specified period of time of 20 hours.

(3 marks)

(e) A system has two components in a parallel connection. Work out the Markov state table, transition diagram and transition equations for the evaluation of the system **availability**, and express this system availability in terms of the state probabilities P_i. The failure rate and repair rate of the first component are λ_1 and ν_1, and the failure rate and repair rate of the second component are λ_2 and ν_2. All of λ_1, ν_1, λ_2, and ν_2 are constant.

(9 marks)

2 (a) An ANOVA project studies the effect of the carbon ingredient (x) on the strength (y) of a new material. The records of two observations y_{12} and y_{31} are lost as shown in Table 1. However, one of the researchers remembers that the grand total $y_{..}$ of the data is equal to 94 and the effect τ_3 is equal to 6.33. Try to recover the values for y_{12} and y_{31}.

(6 marks)

Table 1

No.	Carbon ingredient, x (%)	Strength, y (kg/cm^2)	
1	10	9	?
2	15	15	14
3	20	?	22

(b) Three factors A, B and C were studied in a full factorial experiment using a replicate of (n = 3). The numbers of levels of the factors are $a = 3$, $b = 2$, and $c = 5$.

(i) Find the degrees of freedom of all the main effects, interactions and errors.

(3 marks)

(ii) Find the total degrees of freedom from the total number (N) of observations. Is the total degrees of freedom equal to the sum of all the degrees of freedom found in (i)?

(2 marks)

(c) A 2^3 design is carried out to study the effects of three factors A, B and C. The experiment layout and the observations are displayed in Table 2.

(i) Use ANOVA to find whether interaction AC has significant impact on the output (use $\alpha = 0.05$).

(8 marks)

(ii) Project this 2^3 design into a 2^1 design in terms of factor A.

(3 marks)

Table 2

Run	A	B	C	Observations
(1)	-	-	-	7
a	+	-	-	8
b	-	+	-	-2
ab	+	+	-	4
c	-	-	+	6
ac	+	-	+	3
bc	-	+	+	-12
abc	+	+	+	10

Note: Question 2 continues on page 4.

(d) An experiment is carried out in hope to increase the productivity of a production line. The experiment involves five factors: A (speed, m/s); B (clamp time, min); C (closing time, min); D (temperature, °C); and E (concentration, %). Due to the limitation on time and resources, only eight observations can be taken.

(i) Design the experiment with the help of Table 3, and work out the plus-and-minus-signs table for this fractional design.

(8 marks)

(ii) Determine the aliases of interaction ABC.

(2 marks)

Table 3

Number of factors	Fraction	Number of runs	Basic generators
3	2_{III}^{3-1}	4	*ABC*
4	2_{IV}^{4-1}	8	*ABCD*
5	2_{V}^{5-1}	16	*ABCDE*
	2_{III}^{5-2}	8	*ABD, ACE*

3 (a) In a single factor experiment, the error ε_{ij} is calculated by ($\varepsilon_{ij} = y_{ij} - \bar{y}_{i.}$). Verify that the sum of the errors ε_{ij} ($i = 1, 2, \dots a,\ j = 1, 2, \dots n$) is equal to zero.

(5 marks)

(b) A 2^1 experiment studies the effect of the text book on the studying results (see Table 4).

(i) What is the effect of the text if Text 1 is taken as the low level and Text 2 as the high level?

(2 marks)

(ii) What is the effect of the text if Text 1 is taken as the high level and Text 2 as the low level?

(2 marks)

(iii) Interpret the results obtained in (i) and (ii).

(2 marks)

Table 4

	Students' marks
Text 1	76, 74, 78
Text 2	48, 50, 52

Note: Question 3 continues on page 5

(c) Table 5 displays the results of a single-replicate 2^3 design. Calculate the main effect B, the conditional effect $B_{(A+)}$, and the conditional effect $B_{(A+ \text{ and } C-)}$.

(8 marks)

Table 5

No	A	B	C	response
1	-	-	-	1
2	+	-	-	3
3	-	+	-	0
4	+	+	-	2
5	-	-	+	-1
6	+	-	+	4
7	-	+	+	-2
8	+	+	+	4

(d) A brand of TV has a constant failure rate $\lambda = 0.001/\text{hr}$. If a TV works satisfactorily up to the end of 100 hours, what is the probability that this TV will fail in the next 10 hours?

(5 marks)

(e) Convert the fault tree in Figure 2 into a reliability block diagram, and then find all of the minimum cut sets.

(8 marks)

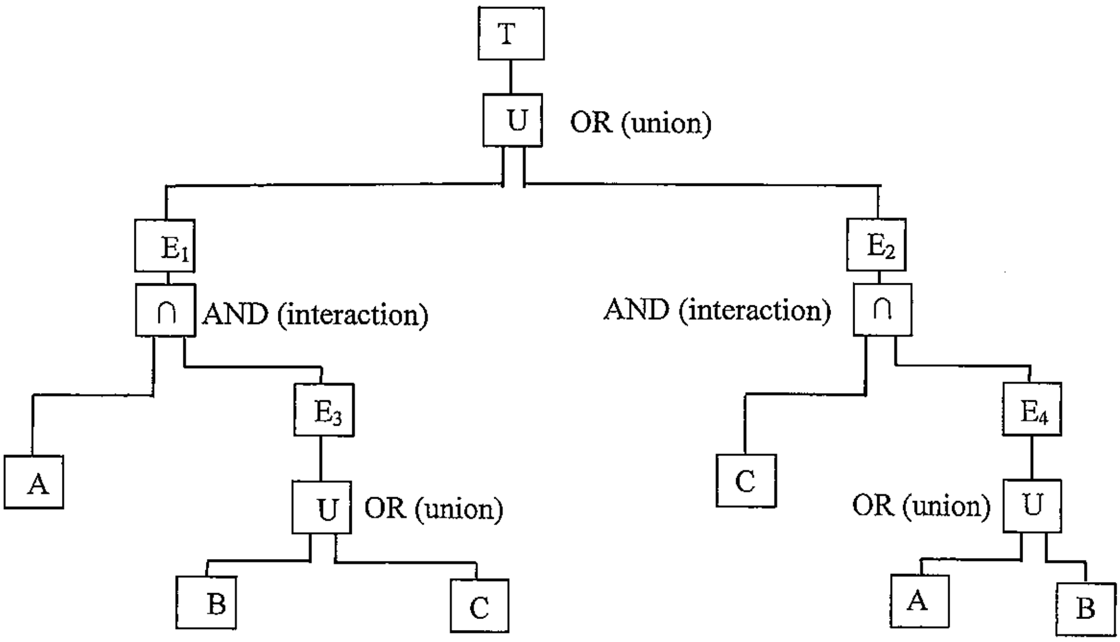


Figure 2

Table: Percentage Points of the F Distribution ($\alpha = 0.05$)

ν_1 degrees of freedom of the numerator
ν_2 degrees of freedom of the denominator

$F_{0.05,\nu_1,\nu_2}$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211

Cumulative Binomial probabilities

$$P[X \leq c] = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$

		p					
	c	0.05	0.10	0.20	0.30	0.40	0.50
n = 20	0	0.358	0.122	0.012	0.001	0.000	0.000
	1	0.736	0.392	0.069	0.008	0.001	0.000
	2	0.925	0.677	0.206	0.035	0.004	0.000
	3	0.984	0.867	0.411	0.107	0.016	0.001

END OF PAPER

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

MA4872 – AIRCRAFT RELIABILITY AND MAINTAINABILITY

April/May 2018 Time Allowed: $2^1/_2$ hours

INSTRUCTIONS

1. This paper contains **THREE (3)** questions and comprises **SEVEN (7)** pages which include **TWO (2)** pages of Appendices.
2. Answer **ALL** questions.
3. Marks for each question are as indicated.
4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided A4 reference sheet is allowed.
5. Question paper not to be removed from the examination hall.

---

1 (a) The probability density function $f(t)$ (where t is the time to failure) of a component is given below:

$$f(t) = \begin{cases} 0.1 - 0.005t, & 0 \le t \le 20 \\ 0, & t > 20 \end{cases}$$

Calculate the unreliability $F(t)$, reliability $R(t)$ and failure rate $\lambda(t)$ at $t = 5$.

(5 marks)

(b) Figure 1 displays the reliability block diagram of a system. Each block represents a component. The reliability of the ith component is R_i. Express the system reliability R_S in terms of the component reliabilities R_i.

(8 marks)

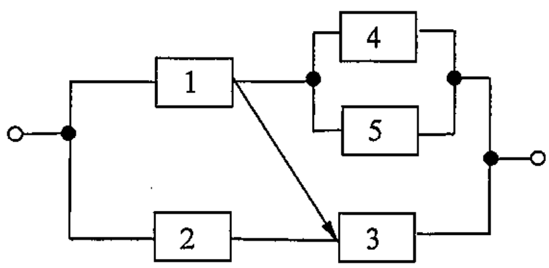


Figure 1

Note: Question 1 continues on page 2.

(c) The system in Figure 2 consists of three components. Components 1 and 2 constitute a standby configuration with component 1 as the primary unit and component 2 as the backup unit. The failure rates of components 1 and 3 are λ_1 and λ_3, respectively. The failure rate of component 2 is equal to λ_2 when component 1 fails, but equal to zero when component 1 is functional. Establish the state table, transition diagram, transition equations for a Markov analysis, and express the system reliability R_s as a function of the probabilities of the states (i.e., p_1, p_2, ...) **(DO NOT SOLVE THE TRANSITION EQUATIONS)**.

(8 marks)

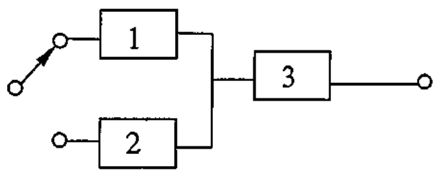


Figure 2

(d) From the following 10 observed values of t (time to failure, in hours)

1.5, 2.7, 3.3, 4.2, 5.1, 5.5, 5.9, 6.5, 7.2, 8.8

find the numbers of components that fail in each of the three intervals as shown in Table 1. Then, use the group data method to determine

(i) the reliability at $t = 6$;

(2 marks)

(ii) the failure rate in the time interval of ($6 \le t < 9$);

(3 marks)

(iii) the mean time to failure (MTTF).

(3 marks)

Table 1

Time interval	Number of failures
$0 \le t < 3$	
$3 \le t < 6$	
$6 \le t < 9$	

(e) A system consists of 3 components in a parallel connection. The failure rates of all three components are constant. The MTTF of components 1 and 2 are equal to 100 and 50, respectively, in hours. The system reliability R_s is equal to 0.993 at t = 10. Find the MTTF of the third component.

(6 marks)

2 (a) You are assigned to study the effect of a single factor A (spindle speed) on the surface roughness. However, you only have the data of an early study in which factor A was investigated together with other two factors B (feed rate) and C (machine) as shown in Table 2. Calculate the effect τ_i (i = 1, 2, 3) at each of the three levels for factor A (spindle speed).

(6 marks)

Table 2

No	A (rpm)	B (μm/s)	C	Surface roughness
1	1000	0.1	USA	4
2	2000	0.3	UK	5
3	3000	0.2	USA	6
4	2000	0.1	Jap	3
5	1000	0.3	UK	2
6	3000	0.2	USA	4
7	3000	0.2	Jap	3
8	2000	0.3	UK	2
9	1000	0.1	Jap	5

(b) A factorial experiment studies two factors: flow rate (A) and deposition time (B). Two replicates are run. The response (layer thickness) is measured in μm, as shown in Table 3. Does the interaction AB have significant effect on the response (given that the sum of squares of error SS_E = 6.5 and α = 0.05)?

(7 marks)

Table 3

	Factor B	
Factor A	0.01	0.03
10	14, 13	15, 13
25	13, 15	12, 14

(c) Students A and B will conduct, separately and independently, an experiment to study the effect of three factors. Each factor is investigated at 2 levels. Each student can take at most 10 observations. While student A uses the one-factor-at-a-time method, student B carries out a 2^k design. What is the value of m (m is the number of pairs of observations used to study an effect) that can be obtained by the experiment of each of the two students? Which experiment will achieve higher accuracy and reliability in experiment results?

(6 marks)

(d) A standard run order is required by some experiment method (e.g., the Yates' algorithm). Write down the run notations of a full 2^4 design according to the standard run order.

(3 marks)

(e) A researcher carries out a 2^{5-2} design to investigate five factors --- mold temperature (A), screw speed (B), holding time (C), cycle time (D) and gate size (E) --- each at two levels. The objective is to study how these factors affect the defective rate. The observations are displayed in Table 4.

Note: Question 2 continues on page 4.

(i) Find all of the aliases of effect A.

(4 marks)

(ii) Project this fractional design into a full 2^3 design in terms of factors B, D and E.

(3 marks)

(iii) Using the 2^3 design obtained in (ii), find whether interaction DE is significant (use $\alpha = 0.05$).

(4 marks)

Table 4

No	A	B	C	$D=AB$	$E=AC$	No of defects
1	-	-	-	+	+	1
2	+	-	-	-	-	3
3	-	+	-	-	+	1
4	+	+	-	+	-	2
5	-	-	+	+	-	4
6	+	-	+	-	+	5
7	-	+	+	-	-	2
8	+	+	+	+	+	3

3 (a) There are three blocks in the system shown in Figure 3.

Block a is a single component with a reliability of ($R_a = 0.85$).

Block B is a subsystem with n identical components in parallel connection. The reliability of each component is ($R_b = 0.70$).

Block C is another subsystem with an m/N configuration. The reliability of each of the N identical components is ($R_c = 0.10$).

(i) Suppose $n = 3$ in block B, and $m = 2$ and $N = 20$ in block C. Whether the requirement of ($R_s \geq 0.9$) for the system reliability R_s can be met?

(4 marks)

(ii) Suppose $m = 2$ and $N = 20$ in block C. Determine the value of n in block B, so that the system reliability R_s is no smaller than 0.93.

(4 marks)

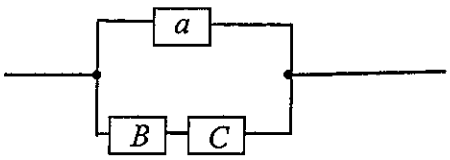


Figure 3

Note: Question 3 continues on page 5.

(b) In a test for estimating the repair rate v of a machine part, five failed units are fixed in 12, 14, 15, 17, 22 hours, respectively. Suppose that the repair rate is a constant. What is the estimated repair rate? And what is the probability that a failed part can be fixed between 13 and 15 hours?

(5 marks)

(c) Efforts should be paid to eliminate the minimum cut sets that include small number of basic events (or units), because their probabilities to take place are very high. There are five events (or units) a, b, c, d and e in the reliability block diagram of a linked system as shown in Figure 4. Find all of the minimum cut sets that include one or two units.

(7 marks)

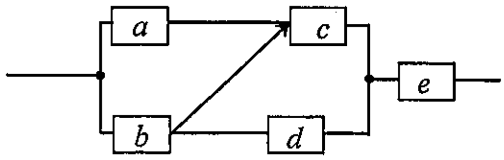


Figure 4

(d) A pharmaceutical manufacturer investigates the effect of the dosage of a new drug on the bioactivity. An experiment was conducted using 2 dosage levels, and the results are displayed in Table 5.

Table 5

Dosage	Bioactivity index	
20 g	20	30
40 g	40	50

(i) Calculate the contrast C_A of the single factor A (dosage).

(2 marks)

(ii) Based on the result of C_A, calculate the effect and sum of squares of factor A.

(2 marks)

(iii) Determine the degrees of freedom of factor A and the degrees of freedom of error.

(2 marks)

(e) Due to some reasons, a 2^{k-1} design may use another interaction (rather than the highest order interaction) as the generator. In a DOE study, the following four runs — (1), a, bc and abc — from a full 2^3 design are selected to constitute a one-half 2^{3-1} fractional design. What is the generator in this fractional design and what is the alias of the main effect A?

(6 marks)

Table: Percentage Points of the F Distribution ($\alpha = 0.05$)

ν_1 degrees of freedom of the numerator
ν_2 degrees of freedom of the denominator

$F_{0.05,\nu_1,\nu_2}$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211

Cumulative Binomial probabilities

$$P[X \leq c] = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$

	c	p: 0.05	0.10	0.20	0.30	0.40	0.50
n = 20	0	0.358	0.122	0.012	0.001	0.000	0.000
	1	0.736	0.392	0.069	0.008	0.001	0.000
	2	0.925	0.677	0.206	0.035	0.004	0.000
	3	0.984	0.867	0.411	0.107	0.016	0.001
	...	...	...	...	...	...	...
	11	0.000	0.000	0.000	0.012	0.071	0.160
	12	0.000	0.000	0.000	0.004	0.035	0.120
	13	0.000	0.000	0.000	0.001	0.015	0.074
	14	0.000	0.000	0.000	0.000	0.005	0.037
	15	0.000	0.000	0.000	0.000	0.001	0.015

END OF PAPER