November / December 2021

a) $m\ddot{z} = f_{let} - mg$

b) Let $f = f_{let} - mg$

$m\ddot{z} = (mg + f) - mg$

$m\ddot{z} = f$

Laplace transform

$ms^2Z - ms\,z(0) - ms\,z'(0) = F$

Assume $z(0)$ and $z'(0) = 0$

$ms^2 Z = F$

$\frac{Z}{F} = \frac{1}{ms^2}$

c) CLTF: $\dfrac{\frac{k}{ms^2}}{\frac{k}{ms^2} + 1} = \dfrac{k}{k + ms^2}$

d) Let $m=1$, $k=9$,

$\frac{Z}{R} = \frac{9}{9+s^2} = \frac{3^2}{s^2+3^2}$

Stability check

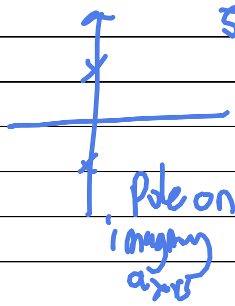
CE: $s^2 + 9 = 0$

① Routh Hurwitz

s^2	1	3
s	0	0
k		

← marginally stable

② Plot roots



Therefore the system is marginally stable.

Since it is marginally stable with no steady state output. Therefore there is no steady state error.

d) $\frac{Z}{R} = \frac{9}{s^2+4}$ Apply unit ramp $Z = \frac{9}{s^2+4}\left(\frac{1}{s^2}\right)$

Apply partial fraction

$$Z = \frac{1}{s^2} - \frac{1}{s^2+9}$$

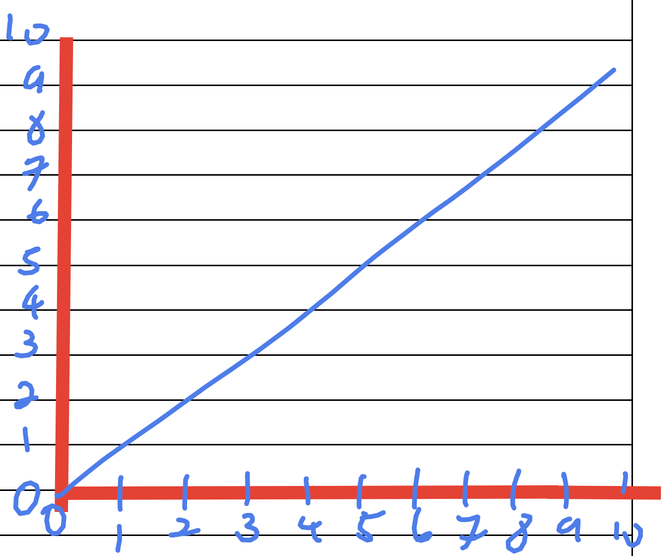
$$Z = \frac{1}{s^2} - \frac{1}{3}\left(\frac{3}{s^2+3^2}\right)$$

Apply Inverse Laplace

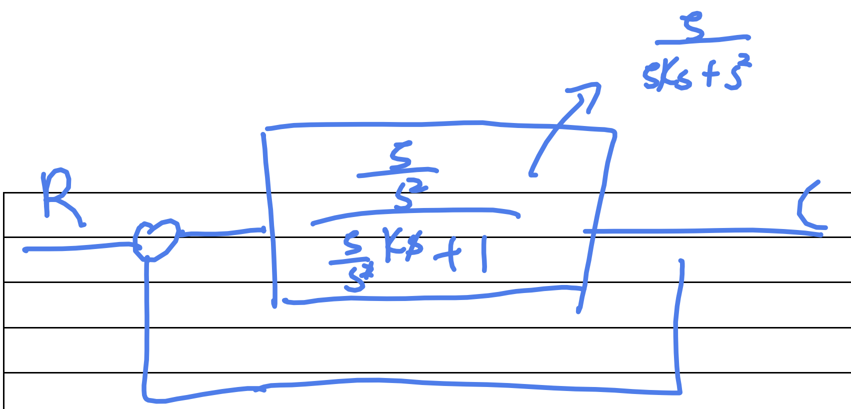
$$z(t) = t - \frac{1}{3}\sin 3t$$

e)

t	$z(t) = t - \frac{1}{3}\sin 3t$
0	0
1	0.95
2	2.09
3	2.86
4	4.17
5	4.78
6	6.25
7	6.72
8	8.3
9	8.68
10	10.324



2)



OLTF: $\frac{5}{5Ks+s^2}$

$\frac{C}{R}: \frac{\frac{5}{5Ks+s^2}}{\frac{5}{5Ks+s^2}+1} = \frac{5}{5+5Ks+s^2}$

$= \frac{5}{s^2+5Ks+5}$

b) CE: $s^2+5Ks+5$

Routh Hurwitz

s^2	1	5
s	5K	0
k	5	

$5K>0$

$K>0$

c) OLTF: $\frac{5}{5(3)s+s^2} = \frac{5}{s^2+15s} = \frac{5}{s(s+15)}$

Type 1, unit step input

$e_{ss}=0$

d) Type 1, unit ramp input, $\rightarrow e_{ss}=\frac{1}{K_v}$

$K_v=\lim_{s\to0} s\,G(s)$ (OLTF) $=\lim_{s\to0} s\frac{5}{s(5K+s)} = \lim_{s\to0}\frac{5}{5K+s} = \frac{5}{5K} = \frac{1}{K}$

$e_{ss}=0.2=\frac{1}{\left(\frac{1}{K}\right)}$, $K=0.2$ #

e) $M_p = 2\%$ overshoot

$$\xi = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}} \quad (M_p \to 0.02)$$

$\xi = 0.7797$

$$\frac{C}{R} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\frac{C}{R} = \frac{5}{s^2 + 5Ks + 5}$$

$\omega_n^2 = 5$

$\omega_n = \sqrt{5}$

$5K = 2\xi\omega_n$

$$K = \frac{2(0.7797)(\sqrt{5})}{5}$$

$= 0.6973$ #

3) When $G_c(s) = K$

$$OLTF = \frac{Ks}{(s^2+4s+8)(s+2)}$$

Zero : $s = 0$

Poles : $s^2 + 4s + 8 = 0$

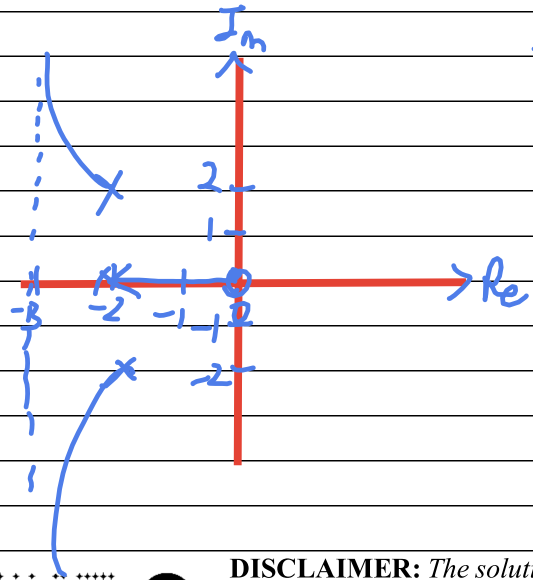
Pole 1 : $s = -2 + 2j$

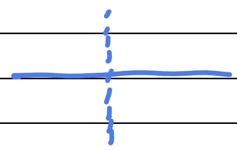
Pole 2 : $s = -2 - 2j$

Pole 3 : $s = -2$

pole − zero = asymptote

$3 - 1 = 2$





$$\sigma = \frac{-2-2-2}{2}$$

$= -3$

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. Campus supplies is not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

***Should there be any mistakes identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is:** http://bit.ly/2lW2C32* coefficient determine location of the desired pole

b) $A e^{-5t} \cos(t + \phi)$

↳ equals to $s = -5 \pm 1j$

PID controller

$G_C = K_D (s + Z_1) \frac{(s + Z_2)}{s}$

$OLTF = K_D \frac{(s + Z_1)(s + Z_2)}{s} \cdot \frac{s}{(s^2 + 4s + 8)(s + 2)}$

Let $Z_1 = 2$ (so that you can cross out the other pole)

$OLTF = K_D \frac{(s+2)(s + Z_2)}{(s^2 + 4s + 8)(s+2)}$

$CLTF = \frac{K_D (s + Z_2)}{(s^2 + 4s + 8) + K_D (s + Z_2)}$

$CE = (s^2 + 4s + 8) + K_D (s + Z_2) = 0$

Subs desired pole $s = -5 + j$

$(12 - 6j) + K_D(-5 + j + Z_2) = 0$

$12 - 6j - 5K_D + K_D j + K_D Z_2 = 0$

$(12 - 5K_D + K_D Z_2) + (-6 + K_D)j = 0$

$12 - 5(6) + (6) Z_2 = 0$ $\quad K_D = 6$

$Z_2 = 3$

$CLTF = \frac{6(s+3)}{(s^2 + 4s + 8)}$

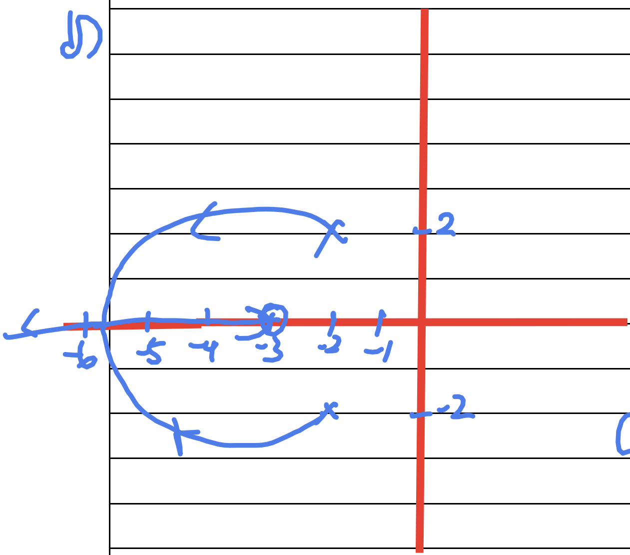
c) No, both conditions cannot be fully satisfied. Based on the OLTF of the PID controller, it is a type 0 system, $e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+0.75} = 0.8$

$K_p = \lim_{s \to 0} \frac{6(s+3)}{s^2+4s+8}$

$= \frac{6(3)}{8} = 0.75$

You can have the desired transient response but not the steady state response.

d)



Zero: $s=-3$

Pole: $P1: s=-2+2j$

$P2: s=-2-2j$

Asymptote

$\sigma = \frac{(-2-2)-(-3)}{1}$

$= -1$

Break in point

$(s^2+4s+8)+K(s+3)=0$

$K = -\frac{(s+3)}{(s^2+4s+8)}$

$\frac{dK}{ds} = 0$ $\quad 0 = (2s+4)(s+3) - (1)(s^2+4s+8)$

$0 = 2s^2+6s+4s+12-(s^2+4s+8)$

$0 = s^2+6s+4$

$s = -5.236$, $s = -0.764$ (rejected)

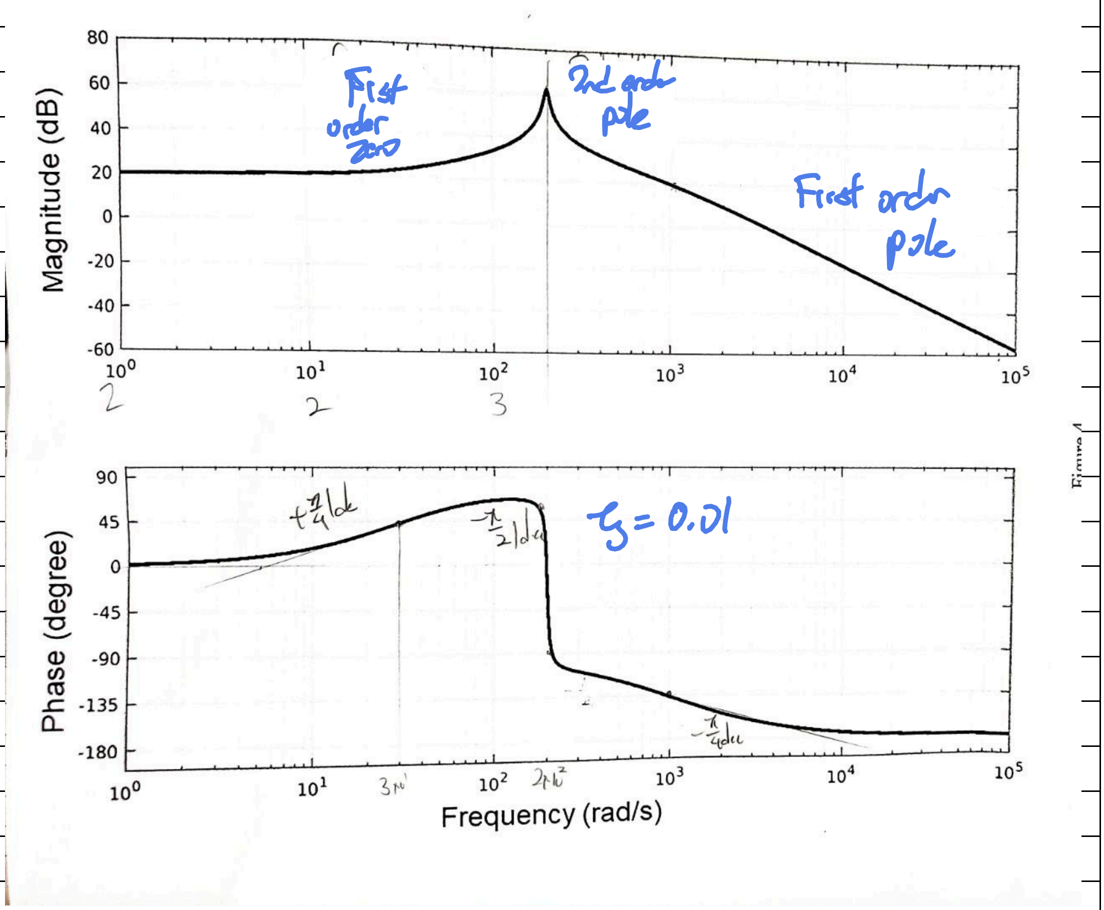
e) i) As K increases, the ξ damping ratio increases, the system stop oscillating, the overshoot of the system decreases.

ii) All roots in LHP, therefore stable

iii) Large K, Larger K_p, smaller e_{ss}, e_{ss} tends to 0

iv) As K increases, the ξ damping ratio increase, system becomes overdamped, therefore 2% settling time increases

4)



Check intercept

a) $20\log\left|\lim_{s\to 0} G(s)\right| = 20$ $\lim_{s\to 0} G(s) = 10^1$

$K = 10$

No slope

First order zero At $\omega = 30$

Second order pole at $\omega = 2\times10^2 = 200$

First order pole at $\omega = 1\times10^3 = 1000$

$$G(s) = 10\left(\frac{s+30}{30}\right)\left(\frac{200^2}{s^2+(0.01)(200)s+200^2}\right)\left(\frac{1000}{s+1000}\right)$$

$$= 13\ 333\ 333.33\ \frac{(s+30)}{(s^2+2s+200^2)(s+1000)}$$

b) i) System is stable because all poles in RHP

$s^2+2s+200^2$, $s = -1 \pm j199.9975$, (Pole 1, pole 2)

$s+1000$, $s = -1000$ (Pole 3)

ii) System type 0, $e_{ss} = \frac{1}{1+K_p}$

$$\lim_{s\to 0} G(s) = \lim_{s\to 0} 13\ 333\ 333.33\ \frac{(s+30)}{(s^2+2s+200^2)(s+1000)} = 9.9999 \approx 10$$

iii) System type 0, unit ramp, $e_{ss} = \infty$

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$$G_2(s) = \frac{(s+40)^2}{100\,(s^2+1)(s+100)} = \frac{(s+40)(s+40)}{100\,(s^2+2(10)(0.005)s+10^2)}$$

$$= \frac{(40^2)}{(100)(10^2)}\left(\frac{s+40}{40}\right)\left(\frac{s+40}{40}\right)\frac{10^2}{s^2+2(10)(0.005)s+10^2}$$

$$= 0.1\left(\frac{s+40}{40}\right)\left(\frac{s+40}{40}\right)\frac{100}{s^2+2(10)(0.005)s+10^2}$$

Intercept:

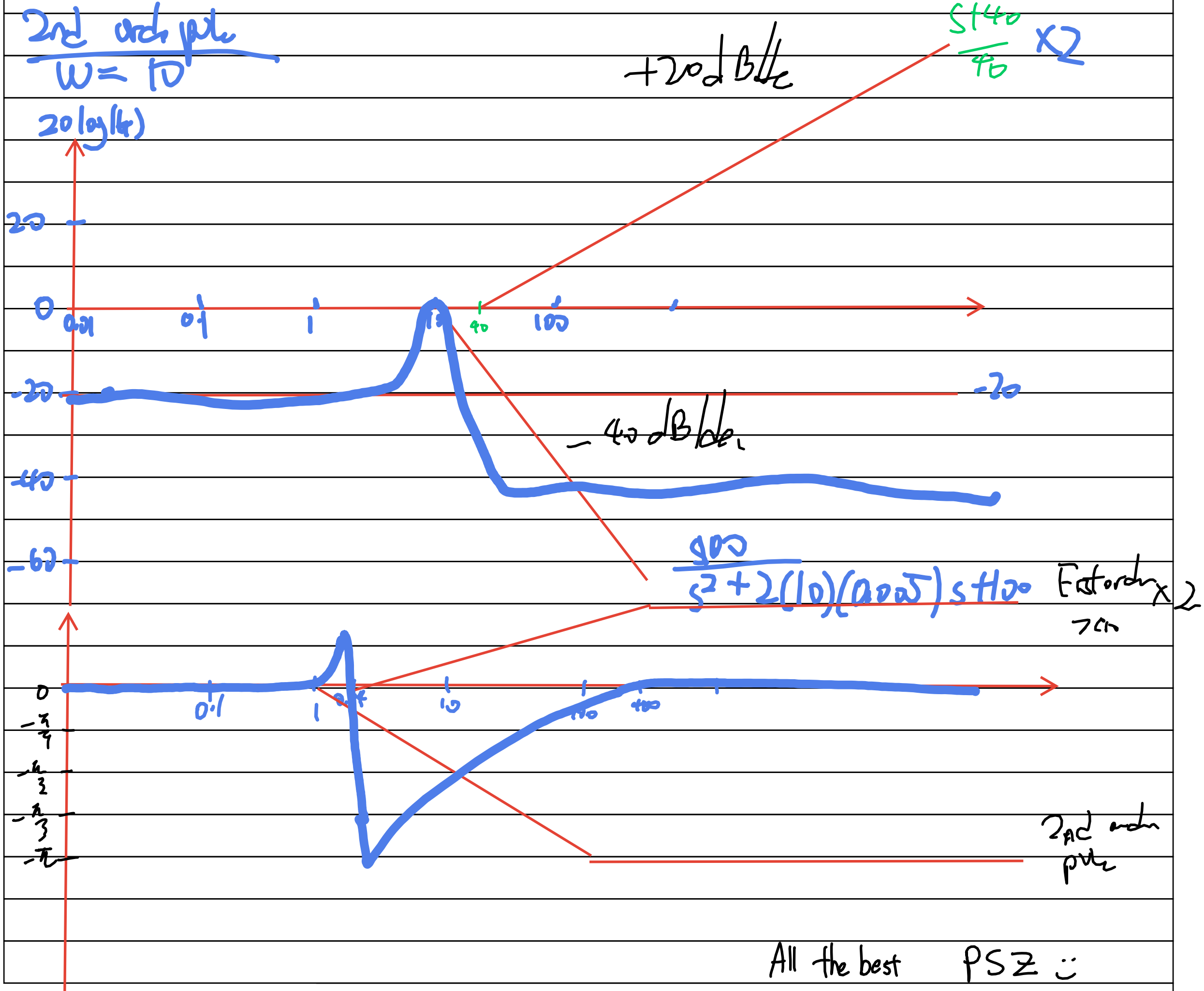
$20\log|0.1| = -20$

First order zero

$\frac{s+40}{40}$, $\omega = 40$

2nd order pole

$\omega = 10$



All the best PSZ :)