NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

MA2001 - MECHANICS OF MATERIALS

November/December 2017 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED-BOOK** Examination.
5. A list of equations can be found on page 5.

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1. A cantilever AD is subjected to a pure moment wa^2 at B and uniformly distributed load of intensity w along AB and BC and as shown in Figure 1. The beam has constant EI. Ignore the weight of the beam.

(a) Determine the reaction forces at the fixed end D.

(6 marks)

(b) Express the elastic curve of the beam in terms of EI, w, a and x.

(14 marks)

(c) Determine the allowable intensity w if the deflection at A is limited to 2 mm. Given: $E = 200$ GPa, $I = 2(10)^6$ mm^4 and $a = 1$ m.

(5 marks)

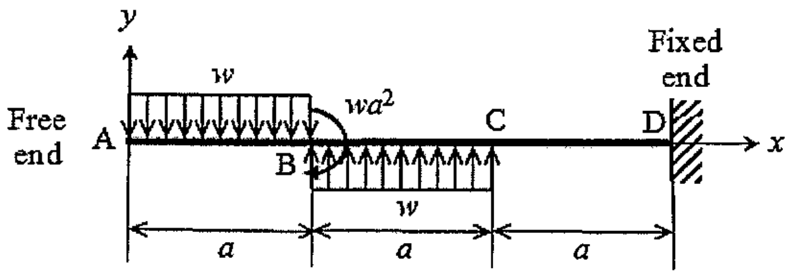

Figure 1

2. A continuous member ABCD is fixed at end A and subjected to a force at D as shown in Figure 2a. The force acts in the xz plane and is inclined at 30° to the z axis. The member has a constant solid circular cross section of radius r. AB, BC and CD have the same length L. Assume that the weight of the member is negligible and L is 25 times r.

(a) Express the normal and shearing stresses on an infinitesimal element at E (see Figure 2b) in terms of P and r.

(14 marks)

(b) Express the principal stresses and maximum shearing stress of the element at E in terms of P and r.

(6 marks)

(c) Using a factor of safety of 1.5, determine the minimum diameter of the member against yielding of the element at E according to Von Mises yield criterion. The yield stress of the material is 250 MPa and P is 6 kN.

(5 marks)

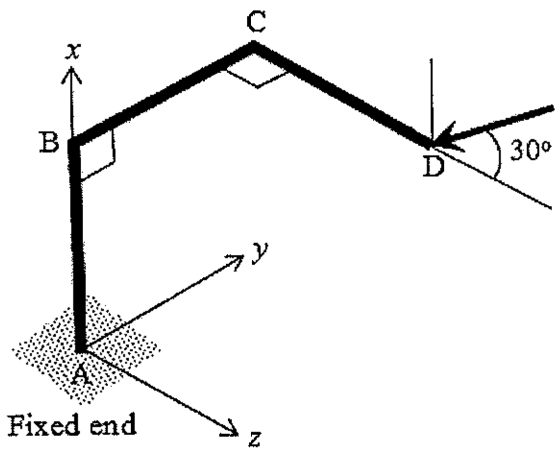

Figure 2a

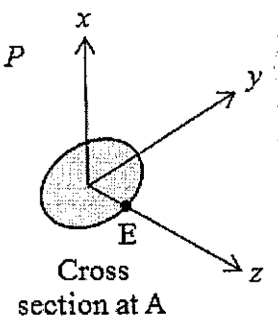

Figure 2b

3. Figure 3a shows a simply supported beam ABCDE with a uniformly distributed load q between B and D, a point load P at C, and concentrated moment M_o at A and C. Figure 3b shows the cross-section of the beam. Given: P = 1 kN, M_o = 1 kNm, q = 3 kN/m, L = 2 m, w = 100 mm, h = 60 mm and t = 10 mm.

(a) Draw the shear force and bending moment diagrams for the beam. Indicate all values at A, B, C, D and E.

(15 marks)

(b) Determine the maximum absolute normal stress and the largest in-plane shearing stress τ_{xy} on the beam.

(10 marks)

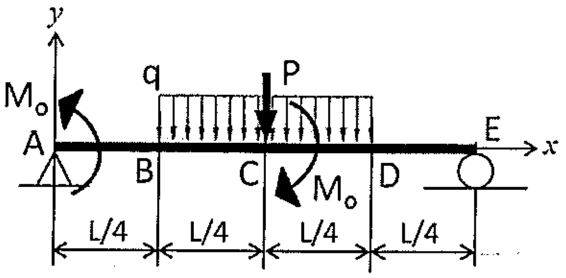

Figure 3a

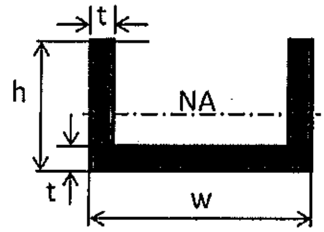

Figure 3b

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4. A thin-walled cylindrical column carries a weightless arch and is loaded as shown in Figure 4. The column has an internal diameter of 250 mm with a thickness of 2 mm. Given: E = 200 GPa and υ = 0.3.

The answer to each part of the question is to be used for subsequent parts.

(a) Determine the stresses on the outer surface of the cylinder at points A and B due to the 1.8 kN load only.

(8 marks)

(b) Determine the minimum internal pressure p required if the longitudinal/axial normal stress of the column cannot be less than +20 MPa (provide the answer up to 2 decimal places).

(6 marks)

(c) Determine the final axial and circumferential stresses at point A due to all the above load conditions and hence calculate the axial strain at point A.

(5 marks)

(d) A horizontal wind load causes the column to twist about its vertical axis. If the maximum principal stress at point A is 100 MPa, determine the torque created.

(6 marks)

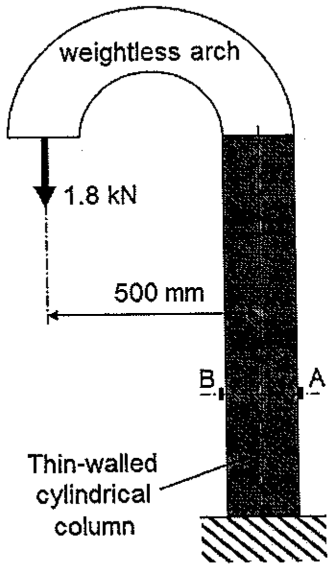

Figure 4

List of Equations

The symbols in the equations have the usual meaning as described in the course text book *Mechanics of Materials*, SI units, 5th Edition, McGraw Hill, 2009 by Beer F. P., Johnston E. R., Dewolf J. T., Mazurek D.

Deformation under axial loading

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Torsion

$$\tau = T\rho / J$$

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

Bending

$$\sigma = -My/I$$

Shear

$$\tau = VQ/(It)$$

$$\bar{y} = \frac{4r}{3\pi} \text{ for a semi circle}$$

Stress Transformation

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Generalized Hooke's Law

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

$$G = \frac{E}{2(1+\nu)}$$

Deflection

$$EIy'' = M(x)$$

Columns

$$P_{cr} = \pi^2 EI/L_e^2$$

END OF PAPER

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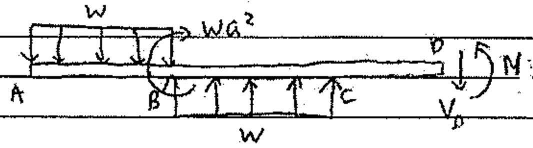
Q1

a)



$\Sigma F_y = 0$

$-w(a) + w(a) + V_D = 0$

$V_D = 0$

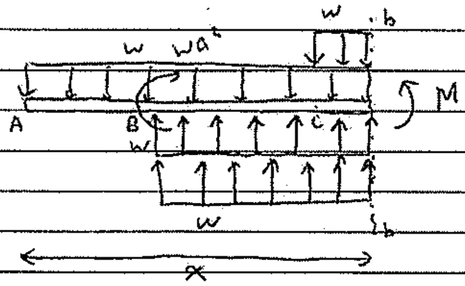
$\Sigma M_D = 0$

$M + wa^2 - w(a)(\frac{5}{2}a) + w(a)(\frac{3}{2}a) = 0$

$M + wa^2 - \frac{5}{2}wa^2 + \frac{3}{2}wa^2 = 0$

$M = 0$

b)



At section b-b

$M(x) = wa^2 \langle x-a \rangle^0 + (2)\left(\frac{w}{2}\right)\langle x-a \rangle^2 - \frac{w}{2}x^2 - \frac{w}{2}\langle x-2a \rangle^2$

$EI\,y'' = wa^2\langle x-a\rangle^0 + w\langle x-a\rangle^2 - \frac{w}{2}x^2 - \frac{w}{2}\langle x-2a\rangle^2$

$EI\,y' = wa^2\langle x-a\rangle + \frac{w}{3}\langle x-a\rangle^3 - \frac{w}{6}x^3 - \frac{w}{6}\langle x-2a\rangle^3 + C_1$ ---(1)

$EI\,y = \frac{wa^2}{2}\langle x-a\rangle^2 + \frac{w}{12}\langle x-a\rangle^4 - \frac{w}{24}x^4 - \frac{w}{24}\langle x-2a\rangle^4 + C_1x + C_2$ ---(2)

Conditions: $y'(3a) = 0$ and $y(3a) = 0$

Eqn.(1): $EI\,y'(3a) = wa^2(3a-a) + \frac{w}{3}(3a-a)^3 - \frac{w}{6}(3a)^3 - \frac{w}{6}(3a-2a)^3 + C_1$

$0 = 2wa^3 + \frac{8}{3}wa^3 - \frac{9}{2}wa^3 - \frac{1}{6}wa^3 + C_1$

$C_1 = 0$

Eqn (2):

$EI\,y(3a) = \frac{1}{2}wa^2(3a-a)^2 + \frac{w}{12}(3a-a)^4 - \frac{w}{24}(3a)^4 - \frac{w}{24}(3a-2a)^4 + C_2$

$0 = 2wa^4 + \frac{4}{3}wa^4 - \frac{27}{8}wa^4 - \frac{1}{24}wa^4 + C_2$

$C_2 = \frac{1}{12}wa^4$

Eqn (2):

$EI\,y = \frac{1}{2}wa^2\langle x-a\rangle^2 + \frac{1}{12}w\langle x-a\rangle^4 - \frac{1}{24}wx^4 - \frac{1}{24}w\langle x-2a\rangle^4 + \frac{1}{12}wa^4$

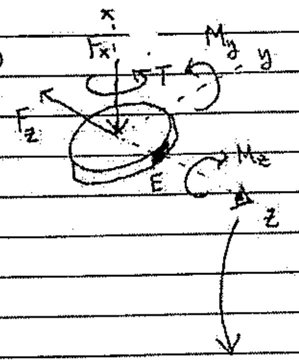
$y = \frac{1}{EI}\left(\frac{1}{2}wa^2\langle x-a\rangle^2 + \frac{1}{12}w\langle x-a\rangle^4 - \frac{1}{24}wx^4 - \frac{1}{24}w\langle x-2a\rangle^4 + \frac{1}{12}wa^4\right)$

c) $y(0) = \frac{1}{EI}\left(\frac{1}{12} w a^4\right)$

$$2\times10^{-3} = \frac{1}{(200\times10^9)(2\times10^6\times10^{-12})}\left(\frac{1}{12}\times w\times 1^4\right)$$

$$w = 9600 \text{ N/m}$$

Q2 a)

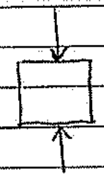


$$F_x = P\sin 30^\circ = \tfrac{1}{2}P$$
$$F_z = P\cos 30^\circ = \tfrac{1}{2}\sqrt{3}\,P$$
$$T = P\cos 30^\circ\,(\overline{BC})$$
$$= \tfrac{1}{2}\sqrt{3}\,P\,(25r) = \tfrac{25}{2}\sqrt{3}\,Pr$$
$$M_y = P\cos 30^\circ\,(\overline{AB}) - P\sin 30^\circ\,(\overline{CD})$$
$$= \tfrac{1}{2}\sqrt{3}\,P\,(25r) - \tfrac{1}{2}P\,(25r)$$
$$= 25\left(\tfrac{1}{2}\sqrt{3}-\tfrac{1}{2}\right)Pr$$
$$M_z = P\sin 30^\circ\,(\overline{BC})$$
$$= \tfrac{1}{2}P\,(25r) = \tfrac{25}{2}Pr$$

* Direction of stresses is illustrated if we are looking at element E in the $-z$ direction

1) Due to F_x : $\sigma_y = \frac{F_x}{A} = \frac{\frac{1}{2}P}{\pi r^2}$

$$= \frac{1}{2\pi}\frac{P}{r^2}$$



2) Due to M_y : $\sigma_y = \frac{M_y r}{I_y}$

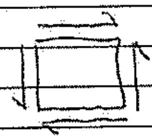
$$= \frac{25\left(\frac{1}{2}\sqrt{3}-\frac{1}{2}\right)Pr^2}{\frac{1}{4}\pi r^4}$$

$$= \frac{50(\sqrt{3}-1)}{\pi}\frac{P}{r^2}$$

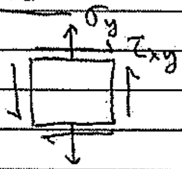
3) Due to T : $\tau_{xy} = \frac{Tr}{J}$

$$= \frac{\frac{25}{2}\sqrt{3}\,Pr^2}{\frac{1}{2}\pi r^4}$$

$$= \frac{25\sqrt{3}}{\pi}\frac{P}{r^2}$$



Overall



$\sigma_x = 0$

$$\sigma_y = \left(\frac{50\sqrt{3} - \frac{101}{2}}{\pi}\right)\frac{P}{r^2}$$

$$\tau_{xy} = \frac{25\sqrt{3}}{\pi}\frac{P}{r^2}$$

b)

$$\sigma_{max} = \frac{1}{2}\left(\frac{50\sqrt{3} - \frac{101}{2}}{\pi}\right)\frac{P}{r^2} + \sqrt{\left(-\frac{1}{2}\left(\frac{50\sqrt{3} - \frac{101}{2}}{\pi}\right)\right)^2 + \left(\frac{25\sqrt{3}}{\pi}\right)^2}\frac{P}{r^2}$$

$$= 20.6788\frac{P}{r^2}$$

$$\sigma_{min} = \frac{1}{2}\left(\frac{50\sqrt{3} - \frac{101}{2}}{\pi}\right)\frac{P}{r^2} - \sqrt{\left(-\frac{1}{2}\left(\frac{50\sqrt{3} - \frac{101}{2}}{\pi}\right)\right)^2 + \left(\frac{25\sqrt{3}}{\pi}\right)^2}\frac{P}{r^2}$$

$$= -9.1870\frac{P}{r^2}$$

$$\tau_{max} = \sqrt{\left(-\frac{1}{2}\left(\frac{50\sqrt{3} - \frac{101}{2}}{\pi}\right)\right)^2 + \left(\frac{25\sqrt{3}}{\pi}\right)^2}\frac{P}{r^2}$$

$$= 14.9329\frac{P}{r^2}$$

c)

$$\sigma_{max}^2 - \sigma_{max}\sigma_{min} + \sigma_{min}^2 < \left(\frac{\sigma_Y}{N}\right)^2$$

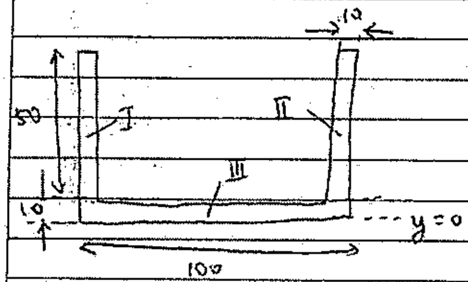
$$\left(20.6788^2 + 20.6788 \times 9.1870 + 9.1870^2\right)\frac{(6\times10^3)^2}{r^4} < \left(\frac{250\times10^6}{1.5}\right)^2$$

$$\frac{1}{r^4} < 1099168.185$$

$$r^4 > 9.0978\times10^{-7}$$

$$r > 0.03088\ m \rightarrow r_{min} = 30.88\ mm$$

(Q3)



Section	Area (mm²)	$\bar{y}$ (mm)	$A\bar{y}$
I	500	35	17500
II	500	35	17500
III	1000	5	5000

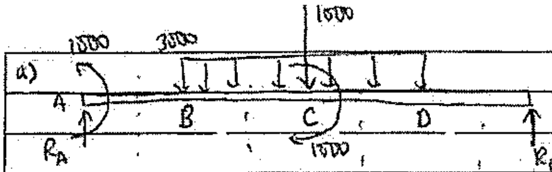
$$y_{NA} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{17500 + 17500 + 5000}{500 + 500 + 1000} = 20\ mm$$

$$I_{NA} = I_{I} + I_{II} + I_{III}$$

$$= \left(\tfrac{1}{12}\times10\times50^3 + 500\times(35-20)^2\right) + \left(\tfrac{1}{12}\times10\times50^3 + 500\times(35-20)^2\right)$$
$$+ \left(\tfrac{1}{12}\times100\times10^3 + 1000\times(5-20)^2\right)$$

$$= \frac{2}{3}\times10^6\ mm^4$$

a)



$\Sigma M_A = 0$

$R_E(L) - 1000\left(\frac{L}{2}\right) + 1000 - 1000 - 3000\left(\frac{L}{2}\right)\left(\frac{L}{2}\right) = 0$

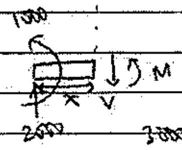
$2R_E - 1000 - 3000 = 0$

$R_E = 2000\ N$

$\Sigma F_y = 0$

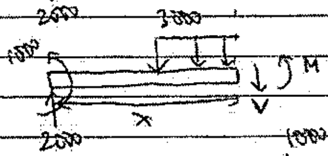
$R_A + R_E - 1000 - 3000\left(\frac{L}{2}\right) = 0$

$R_A = 2000\ N$



$V = 2000$

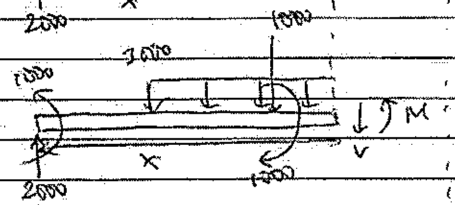
$M = 2000x - 1000$



$V = 2000 - 3000\left(x - \frac{1}{2}\right) = 3500 - 3000x$

$M = 2000x - \frac{3000}{2}\left(x - \frac{1}{2}\right)^2 - 1000$

$= -1500x^2 + 3500x - 1375$

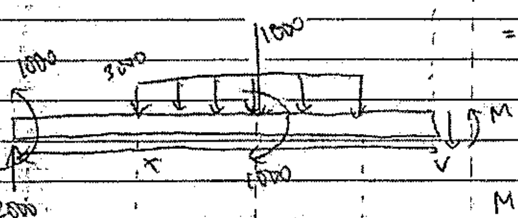


$V = 2000 - 1000 - 3000\left(x - \frac{1}{2}\right)$

$= -3000x + 2500$

$M = 2000x - 1000 + 1000 - 1000(x - 1) - \frac{3000}{2}\left(x - \frac{1}{2}\right)^2$

$= -1500x^2 + 2500x + 625$

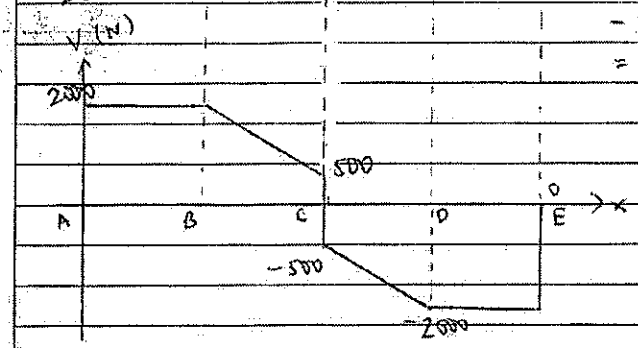


$V = 2000 - 1000 - 3000(1)$

$= -2000$

$M = 2000x - 1000 + 1000 - 1000(x - 1) - 3000(1)(x - 1)$

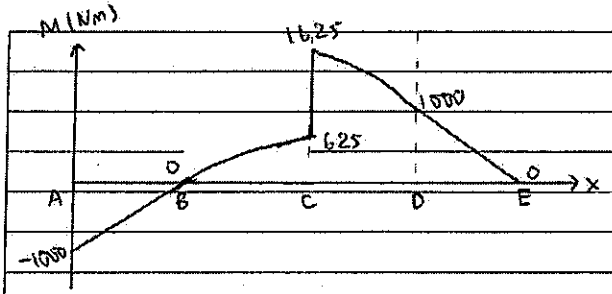
$= -2000x + 4000$



$|V|_{max} = 2000\ N$

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: http://bit.ly/2IW2C32



$|M|_{max} = 1625 \text{ Nm}$

b) Max normal stress is at the point furthest from NA → top part

$$\sigma_{max} = \frac{M_{max}\, y}{I_{NA}} = \frac{1625 \times (60-20)\times 10^{-3}}{\frac{2}{3}\times 10^{6} \times 10^{-12}} \times 10^{-6} \text{ MPa}$$

$$= 97.5 \text{ MPa}$$

Max shear stress is either at NA or junction

Since length of cut (t) is the same for both, max shear stress occur at NA ↳ $t = 20$ mm

$$Q_{NA} = 2 \times (60-20) \times 10 \times \left(\frac{60-20}{2}\right) = 16000 \text{ mm}^3$$

$$\tau_{max} = \frac{V_{max}\, Q}{I t} = \frac{2000 \times 16000 \times 10^{-9}}{\frac{2}{3}\times 10^{6} \times 10^{-12} \times 20 \times 10^{-3}} \times 10^{-6} \text{ MPa}$$

$$= 2.4 \text{ MPa}$$

(Q4)

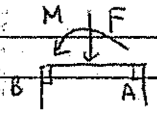
$r_i = 125$ mm ; $r_o = 125 + 2 = 127$ mm

$J = \frac{1}{2}\pi(127^4 - 125^4) = 8002008\pi \text{ mm}^4$

$I = \frac{1}{4}\pi(127^4 - 125^4) = 4001004\pi \text{ mm}^4$

$A = \pi(127^2 - 125^2)$
$= 504\pi \text{ mm}^2$

a)



$M = 1.8 \times 500$
$= 900$ Nm

$$\sigma_A = -\frac{F}{A} + \frac{M r_o}{I}$$

$$= \left(-\frac{1800}{504\pi} + \frac{900 \times 127 \times 10^{-3}}{4001004\pi \times 10^{-12}} \times 10^{-6}\right) \text{MPa}$$

$$= 7.9566 \text{ MPa}$$

$$\sigma_B = -\frac{F}{A} - \frac{M r_o}{I}$$

$$= \left(-\frac{1800}{504\pi} - \frac{900 \times 127 \times 10^{-3}}{4001004\pi \times 10^{-12}} \times 10^{-6}\right) \text{MPa}$$

$$= -10.2302 \text{ MPa}$$

* -ve is compression

b) Since without pressure, B is the most compressed part, we use B as reference

$$\sigma_B = -10.2302 + p\frac{r_i}{2t} = 20$$

$$p\left(\frac{125}{2\times 2}\right) = 30.2302$$

$$p = 0.97 \text{ MPa}$$

c) $$\sigma_A^{ax} = 7.9566 + p\frac{r_i}{2t}$$

$$= 7.9566 + 0.97 \times \frac{125}{2\times 2} = 38.27 \text{ MPa}$$

$$\sigma_A^{circ} = p\frac{r_i}{t}$$

$$= 0.97 \times \frac{125}{2} = 60.63 \text{ MPa}$$

$$\varepsilon_A^{ax} = \frac{\sigma_A^{ax}}{E} - \nu\frac{\sigma_A^{circ}}{E} = \frac{38.27}{200\,000} - 0.3\times\frac{60.63}{200\,000}$$

$$= 1.004 \times 10^{-4}$$

d) $$\sigma_A^{max} = \frac{\sigma_A^{ax} + \sigma_A^{circ}}{2} + \sqrt{\left(\frac{\sigma_A^{ax} - \sigma_A^{circ}}{2}\right)^2 + \tau^2}$$

$$100 = \frac{38.27 + 60.63}{2} + \sqrt{\left(\frac{38.27 - 60.63}{2}\right)^2 + \tau^2}$$

$$\tau = 49.298 \text{ MPa}$$

$$\frac{T r_o}{J} = \tau$$

$$T = \frac{49.298\times 10^6 \times 8002008\pi \times 10^{-12}}{125 \times 10^{-3}}$$

$$= 9914.47 \text{ Nm}$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

MA2001 - MECHANICS OF MATERIALS

April/May 2018 Time Allowed: 2 ½ hours

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1. A simply supported beam ABCDE is shown in Figure 1(a). The beam has a uniformly distributed load q between A and D, point loads P at B and D, and a concentrated moment M_o at C. Figure 1(b) shows the cross-section of the beam. Given: P = 500 N, M_o = 500 Nm, q = 1 kN/m, L = 2 m, a = 30 mm and b = 12 mm.

(a) Draw the shear force and bending moment diagrams for the beam. Indicate all values at A, B, C, D and E.

(12 marks)

(b) Determine the maximum moment between A and C (excluding point C).

(5 marks)

(c) Determine the maximum absolute normal stress and the largest absolute in-plane shearing stress τ_{xy} on the beam.

(8 marks)

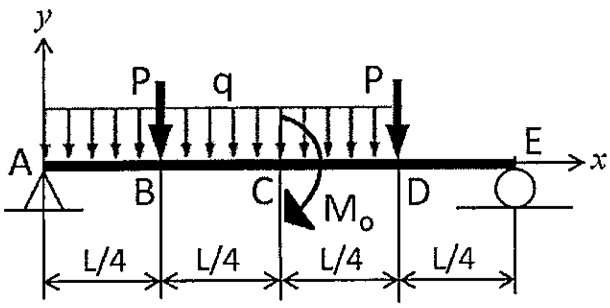

Figure 1(a)

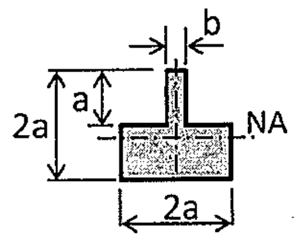

Figure 1(b)

2. Figure 2 shows a tall inflatable tubular structure. The structure is made of a fabric and requires a longitudinal stress at the bottom of the structure at section X to be at least 1.0 MPa for it to be kept upright. Assume thin-walled cylindrical pressure vessels theory is applicable at section X and the total weight of the structure is equivalent to 30 kg acting longitudinally at section X. Points A and B are on the external surface of the structure at section X. The structure has an internal diameter of 800 mm with a thickness of 1 mm. Assume acceleration of gravity, $g = 9.81 \text{ m/s}^2$.

The answer to each part of the question is to be used for subsequent parts.

(a) Determine the internal pressure required so that the structure can be kept upright (provide answer in kPa with at least 2 decimal places).

(6 marks)

(b) If the structure is modified resulting in an extra equivalent vertical offset load of 10 kg at 400 mm away from the axis of the tube (on the left hand side of Figure 2). What is the new minimum internal pressure required (provide answer in kPa with at least 2 decimal places)?

(9 marks)

(c) Determine and draw the stresses on an element at point B (on the right hand side of the structure).

(4 marks)

(d) Due to wind load, the structure is subjected to a torque. If the allowable maximum principal stress at point B is 6.0 MPa, determine the allowable torque.

(6 marks)

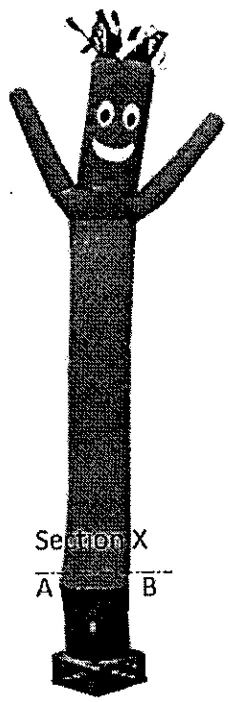

Figure 2

3. A cantilever ABC is subjected to two concentrated forces and uniformly distributed load as shown in Figure 3. Rigid members AD and BE are firmly fixed to the cantilever. The beam has constant *EI*. Ignore the weight of the cantilever and the rigid members.

(a) Determine the reaction forces at fixed end C.

(5 marks)

(b) Derive the elastic curve of the beam in terms of *EI* and *x*. The origin of *x* is at A.

(14 marks)

(c) Determine the deflection and slope at $x = 2.5$ m. Given: $E = 200$ GPa, $I = 4(10)^6$ mm^4.

(6 marks)

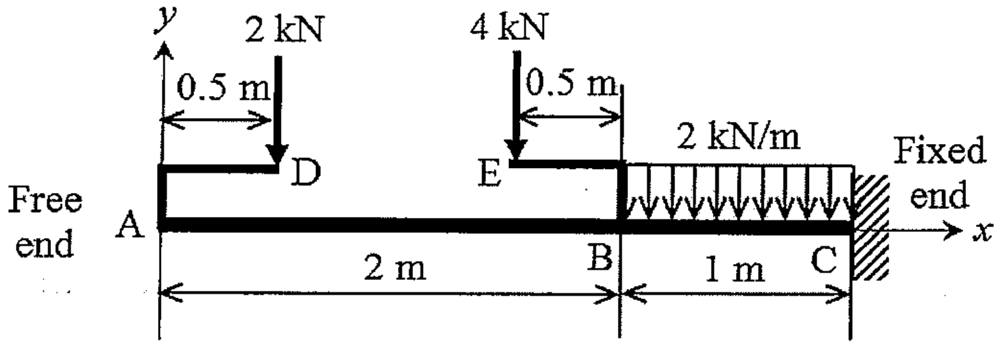

Figure 3

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4. Figure 4 shows a steel rod with one end fully fixed into a rigid wall. A rigid bar is welded centrally onto the other end of the rod. Five concentrated forces are applied to the rigid bar as shown. The diameter and Young's modulus of the rod are 50 mm and 200 GPa, respectively. Ignore the weight of the rod and bar.

(a) Determine the normal and shearing stresses of element "*a*" which is on the surface of the rod. Show the orientation of the stresses on a sketch.

(14 marks)

(b) Determine the principal stresses, principal planes and maximum shearing stress of element "*a*".

(5 marks)

(b) Using a factor of safety of 1.5, determine whether element "*a*" will yield according to von Mises and Tresca yield criteria. The yield stress of steel is 250 MPa.

(6 marks)

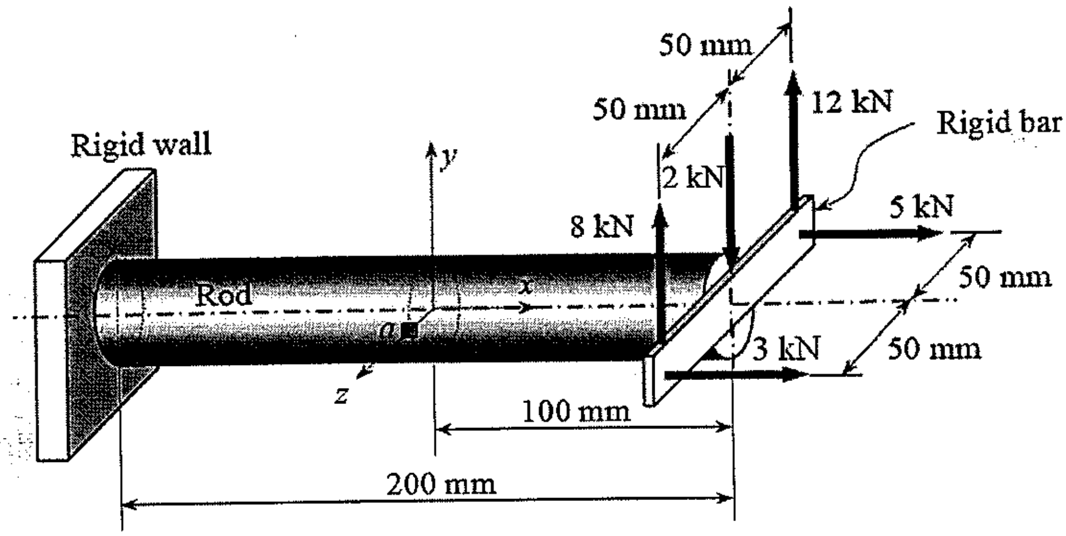

Figure 4

List of Equations

The symbols in the equations have the usual meaning as described in the course text book *Mechanics of Materials*, SI units, 5th Edition, McGraw Hill, 2009 by Beer F. P., Johnston E. R., Dewolf J. T., Mazurek D.

Deformation under axial loading

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Torsion

$$\tau = T\rho / J$$

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

Bending

$$\sigma = -My / I$$

Shear

$$\tau = VQ / (It)$$

$$\bar{y} = \frac{4r}{3\pi} \text{ for a semi circle}$$

Stress Transformation

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Generalized Hooke's Law

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

$$G = \frac{E}{2(1+\nu)}$$

Deflection

$$EI\,y'' = M(x)$$

Columns

$$P_{cr} = \pi^2 EI / L_e^2$$

END OF PAPER

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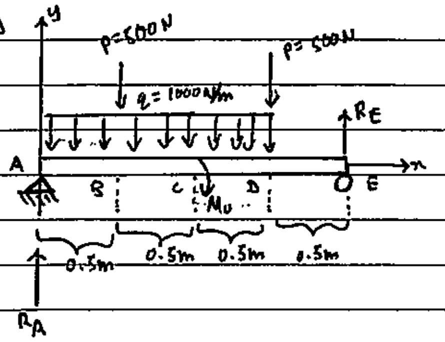
Q1)



$M_0 = 500$ Nm

(a) $\Sigma M_A = 0$

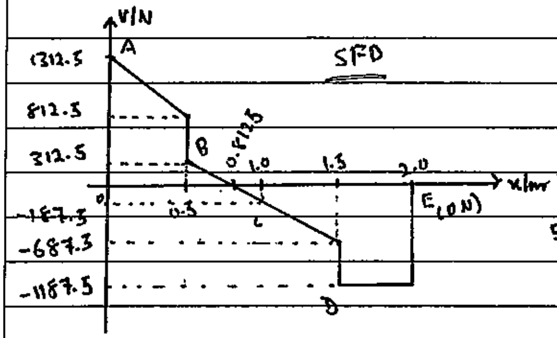
$500(0.5) + 500(1.5) + 500 + 1000(1.5)(0.75) - R_E(2) = 0$

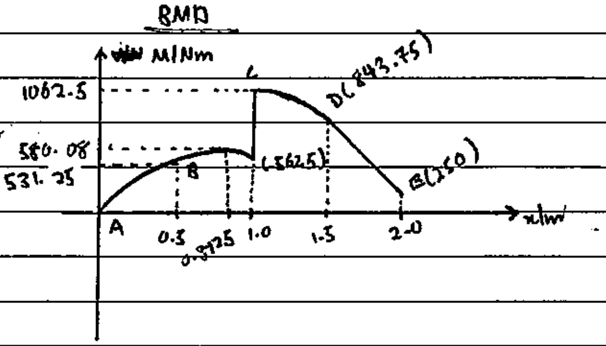
$R_E = 1312.5N$

$\Sigma F_y = 0$

$R_A + 1312.5 - 500 - 500 - 1000(1.5) = 0$

$R_A = 1187.5N$





$\frac{x}{312.5} = \frac{0.5-x}{187.5}$

$x = 0.3125$

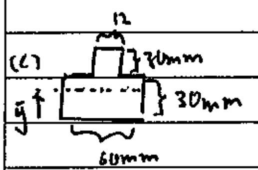
(b) Maximum moment between A and C occurs at point where $V=0$, as $\frac{dM}{dx} = V$

∴ we solve for x, $\frac{x}{312.5} = \frac{0.5-x}{187.5} \Rightarrow x = 0.3125$

max value of moment between A and C $= \frac{1312.5 + 812.5}{2}(0.5) + 0.5(0.3125)(312.5)$

$= 580.078 \approx 580.1$ Nm

(c)



∴ $\bar{y} = \frac{\Sigma yA}{\Sigma A} = \frac{(30)(60)(15) + (12)(30)(45)}{(12)(30) + (30)(60)} = 20mm$

$\approx 0.02m$

$I = \Sigma[I + Ad^2]$

$= [\frac{1}{12}(12)(30)^3 + (12)(30)(25)^2] + [\frac{1}{12}(60)(30)^3 + (60)(30)(5)^2]$

$= 432000 \text{ mm}^4 \approx 4.32 \times 10^{-7} \text{ m}^4$

Max absolute normal stress occurs at point furthest from Neutral axis, $\sigma_{max} = |\frac{My}{I}| = |\frac{1062.5(40 \div 1000)}{4.32 \times 10^{-7}}|$

$= 98.3796$ MPa

≈ 98.4 MPa

Q1(c) For shearing stress τ_{xy}, we will consider at both N.A and transition point. note, $V_{max} = 1312.5\,N$

Q at N.A $= (60)(20)(10) = 12000\,mm^3 = 1.2\times10^{-5}\,m^3$

Q at transition point $= (12)(30)(25) = 9000\,mm^3 = 9\times10^{-6}\,m^3$

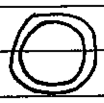
as $\tau = \frac{VQ}{It}$, at N.A, $\tau_{xy} = \frac{(1312.5)(1.2\times10^{-5})}{(4.32\times10^{-7})(0.06)}$

$= \sout{0.0607}\ 0.06076\,MPa$

at transition point, $\tau_{xy} = \frac{(1312.5)(9\times10^{-6})}{4.3\times10^{-7}(0.012)} = 2.278\,MPa$.

$\therefore$ largest shearing stress $= 2.28\,MPa$

Q2)



$d_i = 800\,mm$

$r_i = 400\,mm = 0.4\,m$

$r_o = 401\,mm = 0.401\,m$

$d_o = 802\,mm$

mass $= 30\,kg$

$I = \frac{\pi}{4}(0.401^4 - 0.4^4) = 2.01817\times10^{-4}\,m^4$

$J = 2I = 4.0363\times10^{-4}\,m^4$

thickness, $t = 1\,mm = 0.001\,m$

(a) ~~compress~~ normal stress due to weight $= \frac{30\times9.81}{\pi(0.401^2-0.4^2)} = -0.11645\,MPa$

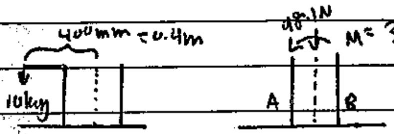
longitudinal stress due to internal pressure $= \frac{pr}{2t}$.

$\therefore \frac{pr}{2t} - \sigma_{weight} = 1.0$

$\frac{pr}{2t} - 0.11695 = 1.0$

$P = \frac{2(0.001)}{0.400}(1.116952) = 0.00558476\,MPa \approx 5.585\,kPa$

(b)



We can observe a moment causes compression at A and tension at B. We thus look at A to calculate Pressure.

$\sigma_m = \pm\left|\frac{My}{I}\right| = \pm\left|\frac{39.24(0.401)}{2.01817\times10^{-4}}\right|$

$= \pm 77.936\,kPa$

$\sigma_{weight\,load} = \frac{98.1}{\pi(0.401^2-0.4^2)}$

$= -38.984\,kPa$

new internal pressure, $\frac{P_1 r}{2t} = 1000 + 116.952 + 38.984 + 77.467$

$P_1 = \frac{2(0.001)}{0.400}(1233.4)$

$= 6.1643 \approx 6.17\,kPa$

(c) Hoop stress $= \frac{pr}{t}$

$= \frac{6.1695(0.4)}{0.001}$

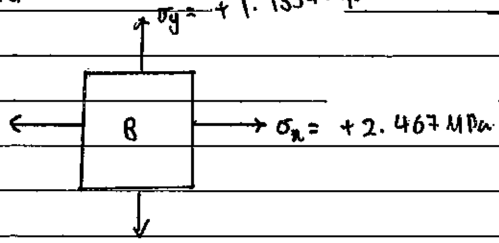
$= 2467.8\text{ kPa}$

$\approx +2.468\text{ MPa}$

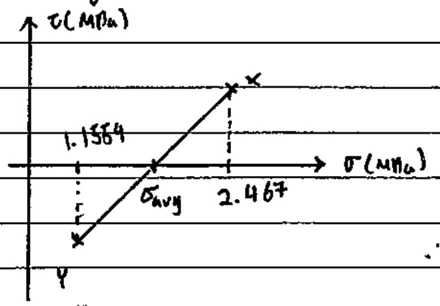
longitudinal stress $= \frac{2467.8}{2} - 116.452 - 38.484 + 77.967$

$= +1155.434\text{ kPa}$

$\approx +1.156\text{ MPa}$



(d) Let Torque be T.



$\sigma_{avg} = \frac{1.1569 + 2.4678}{2} = 1.81185\text{ MPa}$

$\sigma_{max} = \sigma_{avg} + R$

$6.0 = 1.81185 + R$

$R = 4.18815\text{ MPa}$

$\therefore 4.18818 = \sqrt{\tau^2 + (2.467 - 1.81185)^2}$

$\tau = 4.1365\text{ MPa}$

$\tau = \frac{Tc}{J} \Rightarrow T = \frac{\tau J}{c}$

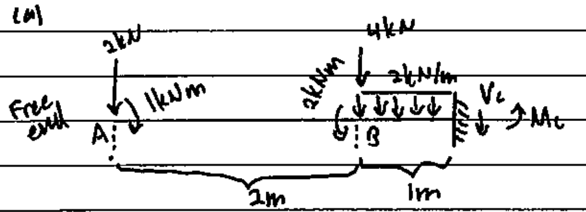
allowable torque $= \frac{(4.1365\times10^{6})(4.0363\times10^{-4})}{0.401}$

$= 4163.6\text{ Nm}$

$\approx 4164\text{ Nm}$

Q3) $E = 200\text{ GPa}$ $\quad I = 4(10)^6\text{ mm}^4 = 4\times10^{-6}\text{ m}^4$

(a)



$\Sigma F_y = 0$

$2 + 4 + 2(1) + V_c = 0$

$V_c = -8\text{ kN}$ (upwards)

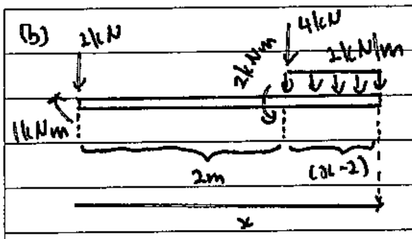
$\Sigma M = 0$

$M_c = -2(3) + 1 - 2 - 4(1) - 2(1)(0.5)$

$= -12\text{ kNm}$ (↺)

(2)

(b)



$M(x) = -2000\langle x\rangle + 1000\langle x\rangle^0 - 2000\langle x-2\rangle^0 - 4000\langle x-2\rangle - \frac{2000}{2}\langle x-2\rangle^2$

$EIy'' = -2000\langle x\rangle + 1000\langle x\rangle^0 - 2000\langle x-2\rangle^0 - 4000\langle x-2\rangle - 1000\langle x-2\rangle^2$

Hence, $EIy' = -1000\langle x\rangle^2 + 1000\langle x\rangle - 2000\langle x-2\rangle - 2000\langle x-2\rangle^2 - \frac{1000}{3}\langle x-2\rangle^3 + C_1$

$EIy = -\frac{1000}{3}\langle x\rangle^3 + 500\langle x\rangle^2 - 1000\langle x-2\rangle^2 - \frac{2000}{3}\langle x-2\rangle^3 - \frac{1000}{12}\langle x-2\rangle^4 + C_1x + C_2$ (in Nm)

we know that $y' = 0$ at $x = 3$.

$0 = -1000\langle 3\rangle^2 + 1000\langle 3\rangle - 2000\langle 3-2\rangle - 2000\langle 3-2\rangle^2 - \frac{1000}{3}\langle 3-2\rangle^3 + C_1$

$0 = -9000 + 3000 - 2000 - 2000 - \frac{1000}{3} + C_1$

$C_1 = \frac{31\,000}{3}$

when $x=3$, $y=0$.

$0 = -\frac{1000}{3}\langle 3\rangle^3 + 500\langle 3\rangle^2 - 1000\langle 3-2\rangle^2 - \frac{2000}{3}\langle 3-2\rangle^3 - \frac{1000}{12}\langle 3-2\rangle^4 + \frac{31\,000}{3}(3) + C_2$

$C_2 = -24750$

$\therefore$ elastic curve, $y = \frac{1}{EI}\left[-\frac{1000}{3}\langle x\rangle^3 + 500\langle x\rangle^2 - 1000\langle x-2\rangle^2 - \frac{2000}{3}\langle x-2\rangle^3 - \frac{1000}{12}\langle x-2\rangle^4 + \frac{31\,000}{3}x - 24750\right]$ (m)

(c) deflection at $x = 2.5$m,

$y = \frac{1}{(200\times10^9)(4\times10^{-6})}\left[-\frac{1000}{3}\langle 2.5\rangle^3 + 500\langle 2.5\rangle^2 - 1000\langle 0.5\rangle^2 - \frac{2000}{3}\langle 0.5\rangle^3 - \frac{1000}{12}\langle 0.5\rangle^4 + \frac{31\,000}{3}(2.5) - 24750\right]$

$= -0.001673\text{m} \approx -1.67\text{m}$

slope at $x = 2.5$m,

$y' = \frac{1}{(200\times10^9)(4\times10^{-6})}\left[-1000\langle 2.5\rangle^2 + 1000\langle 2.5\rangle - 2000\langle 0.5\rangle - 2000\langle 0.5\rangle^2 - \frac{1000}{3}\langle 0.5\rangle^3 + \frac{31\,000}{3}\right]$

$= 0.006302$

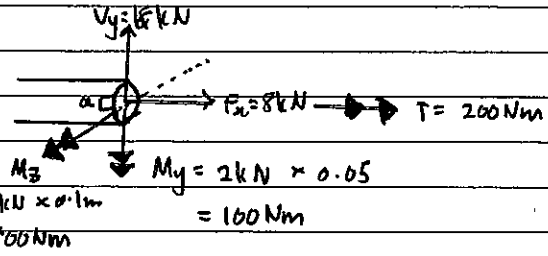
Q4) $d = 50mm$ $E = 200\ GPa$ $r = 25mm = 0.025m$ $I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.025)^4$

(a) at end of rod: $V_y = 12+8-2 = 8kN(\uparrow)$ $= 3.06740 \times 10^{-7} m^4$

$F_x = 5+3 = 8kN(\rightarrow)$ $J = 2I = 6.1359 \times 10^{-7} m^4$

$T = 12\times0.05 - 8\times0.05 = 0.2kNm\ (\twoheadrightarrow)$

We now move force to A.



$\therefore \sigma$ due to $F_x = \frac{8000}{\pi(0.025)^2} = +4.0743MPa$ Q of circle $= \frac{4r}{3\pi} \times \frac{1}{2}A$

M_y causes compression at a along x-axis, $\sigma_m = -\frac{My}{I}$ $= \frac{4(0.025)}{3\pi} \times \frac{1}{2}\pi(0.025)^2$

$= -\frac{100(0.025)}{3.0674\times10^{-7}}$ $= 1.04166\times10^{-5} m^3$

$= -8.148MPa.$

note that M_z does not cause normal stress as its along z-z axis.

$\Sigma\sigma_x = +4.0743 - 8.1487 = -4.0744MPa$ (compression)

$\sigma_y = 0.$

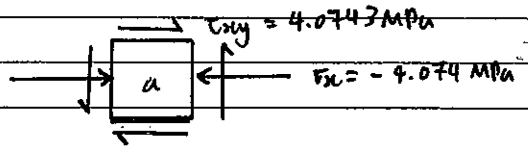
τ due to torque T, $\tau = \frac{Tc}{J} = \frac{200(0.025)}{6.1359\times10^{-7}}$

$= 8.148MPa(\downarrow)$

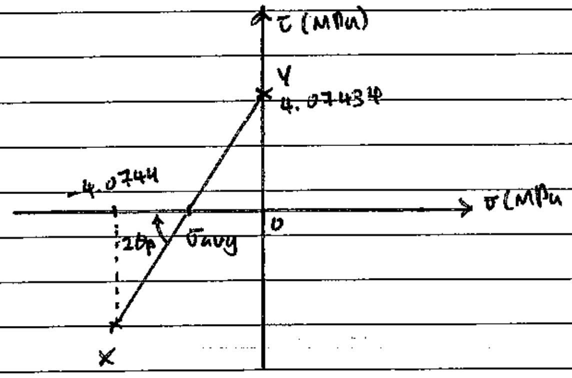
τ due to shear force, $\tau = \frac{VQ}{It} = \frac{(18\times10^3)(1.04166\times10^{-5})}{(3.0674\times10^{-7})(0.05)}$

$= 12.223MPa(\uparrow)$

$\therefore \tau_{xy} = 12.223 - 8.148 = 4.0743MPa(\uparrow)$



(b)



$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = -\frac{4.0744}{2}$

$= -2.03715 \text{ MPa}$

Maximum shearing stress,

$\tau = R = \sqrt{4.07434^2 + 2.03715^2}$

$\tau_{max} = 4.555 \text{MPa}$

$\approx 4.56 \text{MPa}$

$\sigma_{max} = \sigma_{avg} + R = -2.03715 + 4.555$

$= 2.5184$

$\approx +2.52 \text{MPa}$

$\sigma_{min} = \sigma_{avg} - R$

$= -2.03715 - 4.555$

$= -6.4523$

$\approx -6.45 \text{MPa}$

principal planes, $\theta_p = -\frac{1}{2}\tan^{-1}\left(\frac{4.07434}{2.03715}\right)$

$= -31.7177°$

$\therefore \theta_p = -31.72°, \ 58.28°$ $\theta_s = 13.28°, -76.71°$ (θ_s differ $45°$ from θ_p)

(c) Factor of safety = 1.5

By Tresca, $\tau_{max} < \frac{\sigma_y}{2}$

LHS = 4.56 MPa RHS = $\frac{250 \div 1.5}{2}$

$= 83.33 \text{MPa}$.

As $\tau_{max} < \frac{\sigma_y}{2}$, element will not yield according to Tresca yield criteria.

By Von Mises, $\sigma_{max}^2 - \sigma_{max}\sigma_{min} + \sigma_{min}^2 < \sigma_y^2$

LHS = $2.518^2 - (2.518)(-6.542) + 6.542^2$

$= 66.393 \text{ MPa}$

RHS = $(250 \div 1.5)^2 = 27777 \text{ MPa}$.

Hence, element will not yield according to Von Mises criteria.

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2018-2019

MA2001 - MECHANICS OF MATERIALS

November/December 2018 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED-BOOK** examination.
5. A list of equations can be found on Page 5.

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1. A cantilever beam ABCD is loaded as shown in Figure 1(a). The beam has a uniformly distributed load q between A and C, point loads P at C and 2P at D, and concentrated moments M_1 at B and M_2 at D. Figure 1(b) shows the cross-section of the beam. Given: P = 600 N, M_1 = 400 Nm, M_2 = 600 Nm, q = 1 kN/m, L = 2 m, a = 60 mm and t = 10 mm.

(a) Draw the shear force and bending moment diagrams for the beam. Indicate all values at A, B, C and D.

(12 marks)

(b) Determine the maximum moment between A and C, and thus calculate the corresponding maximum bending stress (between AC). Determine the largest in-plane shearing stress τ_{xy} for the whole beam.

(13 marks)

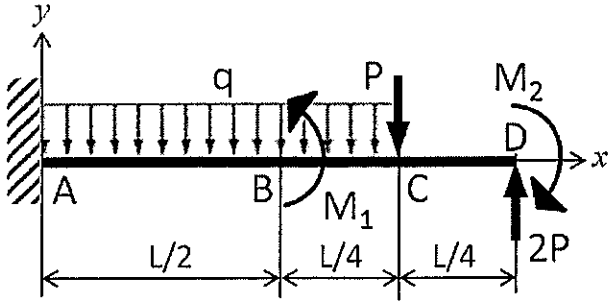

Figure 1(a)

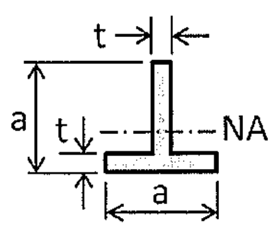

Figure 1(b)

2. A 5 kN rigid weight is welded onto the top end of an upright thin-walled cylindrical pressure vessel as shown in Figure 2. The outer diameter of the vessel is 200 mm and it has a thickness of 1 mm. Given: E = 200 GPa and ν = 0.3.

(a) Determine the pressure required in the vessel in order to achieve a -2 MPa (compressive) longitudinal stress on the outer surface of the vessel. Give your answer in kPa.

(6 marks)

(b) The pressure vessel is welded at an angle of 60° to the vertical axis. Determine the normal stress perpendicular to the weld and the corresponding shearing stress on the outer surface of the weld.

(9 marks)

(c) If the rigid weight is welded offset by 40 mm to the right of the vertical axis (viewed from Figure 2) and an extra 200 kNm torque was added on the vessel, determine the principal stresses on the outer surface of the vessel at element A.

(10 marks)

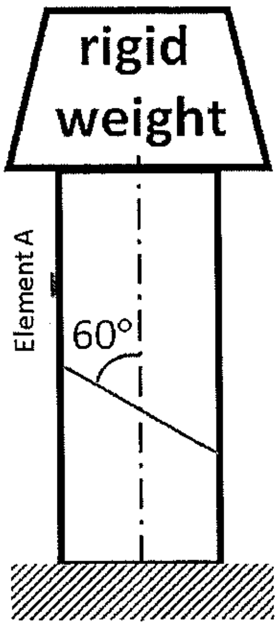

Figure 2

3. A simply supported member AB is loaded as shown in Figure 3. The member has a uniform 50×120 mm rectangular cross section. Ignore the weight of the member.

(a) Determine the magnitude and orientation of the vertical and horizontal components of the reaction force at supports A and B.

(6 marks)

(b) Determine the normal and shearing stresses of element H which lies on the surface of the member. Show the orientation of the stresses on a sketch.

(14 marks)

(c) Using a factor of safety of 1.5, determine if element H will yield according to von Mises yield criterion. The yield stress of the material is 50 MPa.

(5 marks)

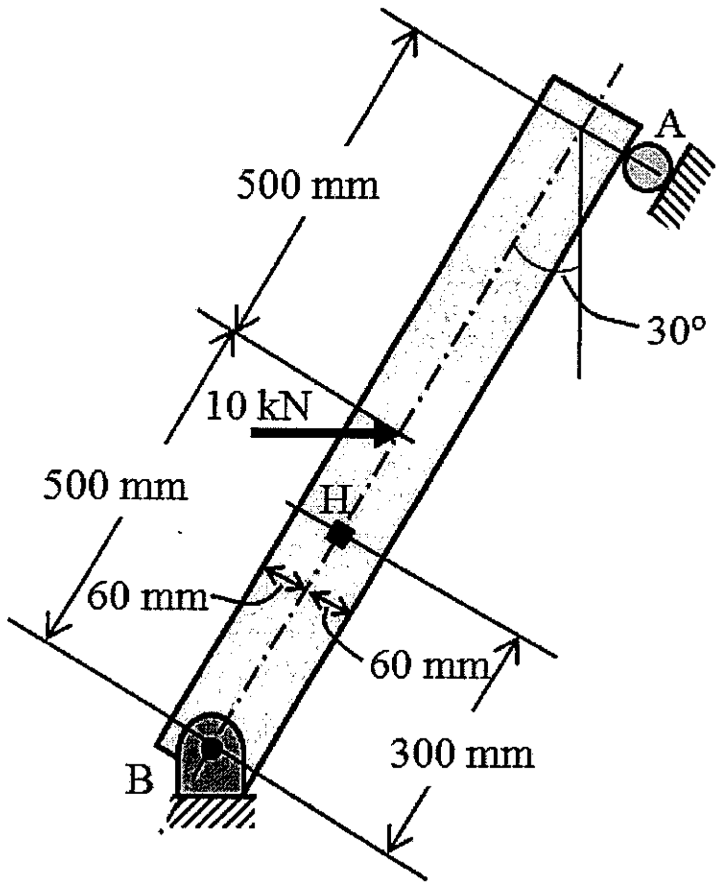

Figure 3

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4. A beam ABCD of constant circular cross section is simply supported at B and D as shown in Figure 4. It is subjected to uniformly distributed load of intensity w along AB. A concentrated load P, of magnitude wa, is applied at E of a rigid bracket CE which is firmly fixed to the beam at C. The beam has constant EI. Length of AB = BC = CD = a. Ignore the weight of the beam and bracket.

(a) Determine the magnitude (in terms of w and a) and orientation of the reaction force at supports B and D.

(6 marks)

(b) Express the elastic curve of the beam in terms of EI, w, a and x. The origin of x is at A.

(14 marks)

(c) Determine the minimum diameter (in mm) of the cross section if the deflection at C is limited to 2 mm. Given: E = 200 GPa, P = 5 kN and a = 1 m.

(5 marks)

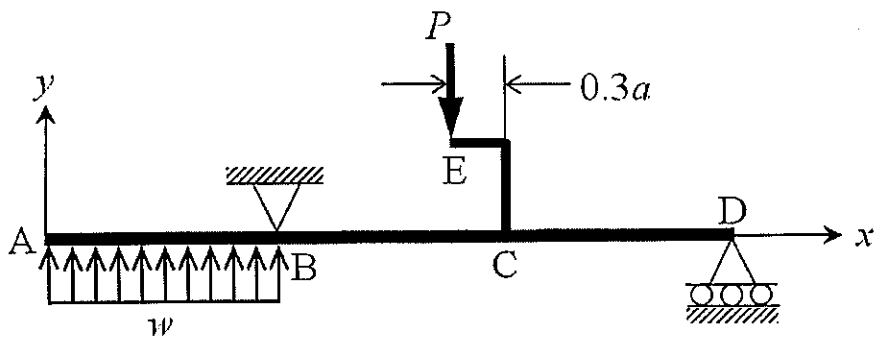


Figure 4

List of Equations

The symbols in the equations have the usual meaning as described in the course text book *Mechanics of Materials*, SI units, 5th Edition, McGraw Hill, 2009 by Beer F. P., Johnston E. R., Dewolf J. T., Mazurek D.

Deformation under axial loading

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Torsion

$$\tau = T\rho / J$$

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

Bending

$$\sigma = -My/I$$

Shear

$$\tau = VQ/(It)$$

$$\bar{y} = \frac{4r}{3\pi} \text{ for a semi circle}$$

Stress Transformation

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Generalized Hooke's Law

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

$$G = \frac{E}{2(1+\nu)}$$

Deflection

$$EI\,y'' = M(x)$$

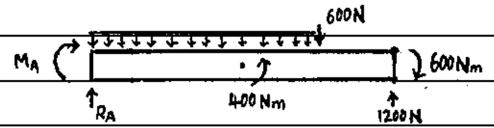
Columns

$$P_{cr} = \pi^2 EI/L_e^2$$

End of Paper

MA 2001 Mechanics Of Materials (Semester 1 Examination 2018-19)

1) (a)



$\Sigma F_{vertical}$: $R_A + 1200 = 600 + 1000(1.5)$

$R_A = 900N$

Don't forget this!!

Σ Moment = 0 : Method 1 : Clockwise * Counter Clockwise ↓

$M_D = 0$; $0 = 900(2) + M_A - 400 - 600(0.5) - 1000(1.5)(0.5 + 1.5 \div 2) + 600$

$M_A = 175\ Nm$

* Distance indicated by method 1 and 2 are different because of different point of reference.

Method 1 → D

Method 2 → A

Method 2:

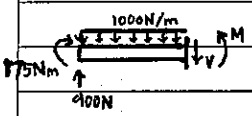
1) Draw moment direction at point of interest
2) Let all moments that have same direction as indicated to have negative sign and the opposite positive sign
3) Equate sum of these moments to moment at point of interest

Eg. For this question

1) ↻ M_A indicated clockwise
2) $-1000(1.5)(0.75) + 400 + 1200(2) - 600 - 600(1.5)$
3) $= M_A$

$M_A = 175\ Nm$

For $0 \leq x < 1$:

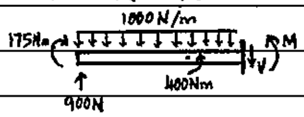


$V = 900 - 1000x$

$M = 175 + 900x - 1000x(\frac{1}{2}x)$

$= 175 + 900x - 500x^2$

For $1 \leq x < 1.5$:

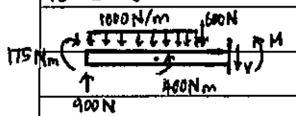


$V = 900 - 1000x$

$M = 175 + 900x - 1000x(\frac{1}{2}x) - 400$

$= -225 + 900x - 500x^2$

For $1.5 \leq x < 2$:

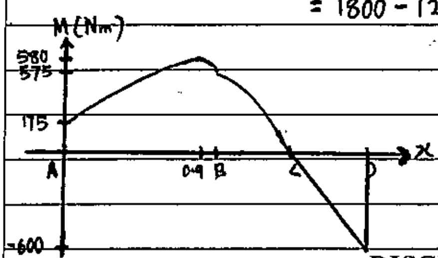


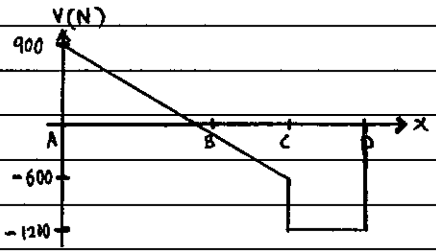
$V = 900 - 1000(1.5) - 600$

$= -1200N$

$M = 175 + 900x - 1000(1.5)(x - 1.5/2) - 400 - 600(x - 1.5)$

$= 1800 - 1200x$





①*

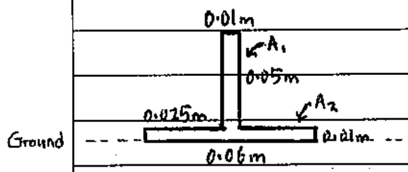
(b) $\frac{dM}{dx} = 0 = 900 - 1000x$

$x = 0.9\,m$

$M_{max} = 175 + 900(0.9) - 500(0.9)^2$

$= 580\,Nm$

Remember to sub values to correct equation.

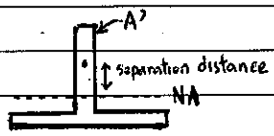


$\tilde{y} = \frac{\overbrace{(0.01)(0.05)}^{\text{Area } A_1}\overbrace{(0.05 \div 2 + 0.01)}^{\text{Centroid of } A_1 \text{ to ground}} + \overbrace{(0.01)(0.06)}^{\text{Area } A_2}\overbrace{(0.01 \div 2)}^{\text{Centroid of } A_2 \text{ to ground}}}{(0.01)(0.05) + (0.01)(0.06)}$

$= 0.018636363\,m$ from ground as reference

$I_{NA} = \left[\underbrace{\frac{(0.05)^3(0.01)}{12}}_{I \text{ of } A_1} + \underbrace{(0.05)(0.01)}_{\text{Area } A_1}\underbrace{(0.01 + 0.05 \div 2 - \tilde{y})^2}_{\text{Centroid } A_1 \text{ and NA separation distance}}\right] + \underbrace{\left[\frac{(0.01)^3(0.06)}{12} + (0.01)(0.06)(\tilde{y} - 0.01 \div 2)^2\right]}_{\text{" for } A_2}$

$= 3.546212121 \times 10^{-7}\,m^4$

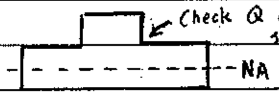


$Q_{@NA} = \underbrace{(0.01)(0.06 - \tilde{y})}_{\text{Area } A'}\underbrace{(0.06 - \tilde{y})(0.5)}_{\text{Centroid of } A' \text{ to NA separation distance}}$

$= 8.554752066 \times 10^{-6}\,m^3$

✱ For unsymmetrical section with NA not at section with smallest t, check transition point.

Eg.



Check Q at this cutting section and use smaller t for shear force calculation.

That's usually the section with maximum shear.

(Refer PYP 2014/15 Sem 2 3(b))

$|\sigma_{max}| = \frac{M_{max}\,c}{I}$

$= \frac{(580)(0.06 - \tilde{y})}{I_{NA}}$

$= 67\,652\,211.07\,Pa$

$\approx 67.7\,MPa$ ✱

$|\tau_{max}| = \frac{V_{max}\,Q}{It}$

$= \frac{1200\,(Q)}{I_{NA}\,(0.01)}$

$= 2\,894\,835.991\,Pa$

$\approx 2.89\,MPa$ ✱

2) $P = 5000N$

$d_{out} = 0.2\,m$ $r_{out} = 0.1\,m$

$t = 0.001\,m$ $r_{in} = 0.099\,m$

$E = 200\,GPa$

$\nu = 0.3$

(a) $-2\,000\,000 = -\frac{5000}{\pi(0.1^2 - 0.099^2)} + \frac{p(0.099)}{2(0.001)}$ (Inner radius)

$p = 121\,166.3805\,Pa$

$\approx 121\,kPa$ //

(b) $\sigma_x = -2\,000\,000\,Pa$

$\sigma_y = \frac{pr}{t}$ (Caused by pressure)

$= \frac{p(0.099)}{0.001}$

$= 11\,995\,471.67$

$\approx 11.995\,MPa$

$\tau_{xy} = 0\,Pa$

$\sigma' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos[2(-30)] + \tau_{xy}\sin[2(-30)]$ (Clockwise 30°; $\tau_{xy} = 0$)

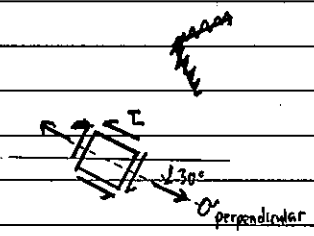
$= 1\,498\,867.917\,Pa$

$\approx 1.50\,MPa$ //

$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin[2(-30)] + \tau_{xy}\cos[2(-30)]$

$= -6\,060\,217.001\,Pa$

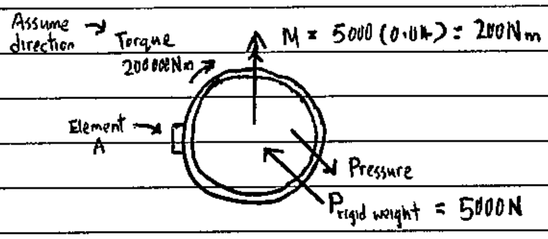
$\approx -6.06\,MPa$ //



(2)*

(c) Offset 0.04m to the right

Extra 200 kNm torque



$$\sigma_x = -\frac{5000}{\pi(0.1^2-0.099^2)} + \frac{P(0.099)}{2(0.001)} + \frac{200\,(0.1)}{\frac{\pi}{4}(0.1^4-0.099^4)}$$

(← Outer radius)

$= 4\ 462\ 490.447\ Pa$

$\approx 4.46\ MPa$ #

$$\sigma_y = \frac{P(0.099)}{(0.001)} = 11\ 995\ 471.67\ Pa$$

$\approx 11.995\ MPa$ #

$$\tau_{xy} = \frac{(200\ 000)(0.1)}{\frac{\pi}{2}(0.1^4-0.099^4)}$$

(← Outer radius)

$= 3\ 231\ 245\ 223\ Pa$

$\approx 3.23\ GPa$ #

$$\sigma_{max} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$= 3\ 239\ 476\ 400\ Pa$

$\approx 3.24\ GPa$ #

* Note: $\frac{\sigma_x+\sigma_y}{2}$ gives center of Mohr's circle

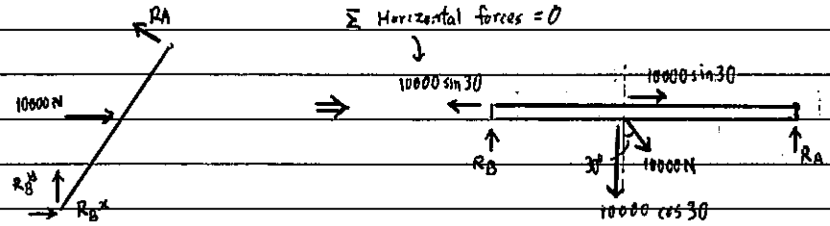
$\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$ gives radius of Mohr's circle

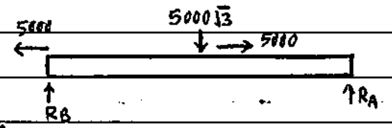
(↑ Base: $\frac{\sigma_x-\sigma_y}{2}$; ↑ Height: τ_{xy})

3) 50 X 120mm $\quad$ * Pin joint no moment!!

(a)

Σ Horizontal forces = 0





$R_A = 2500\sqrt{3} \angle 150°$ // $\qquad R_A = R_B = 2500\sqrt{3}$ N

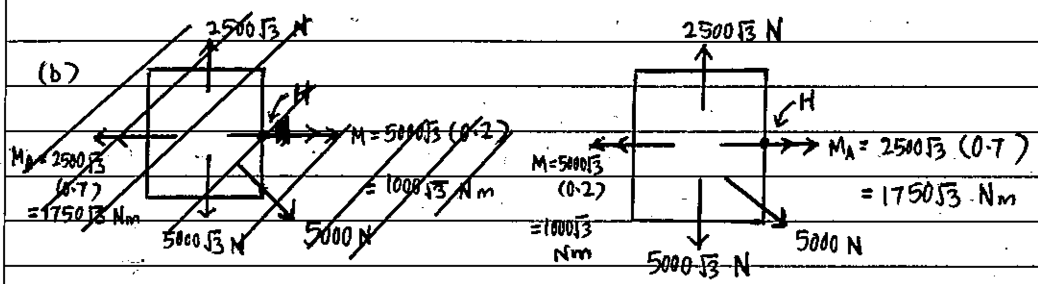
$|R_B| = \sqrt{(10000\sin 30)^2 + (2500\sqrt{3})^2} = 6614.378278$

$\theta_B = \tan^{-1}\left(\frac{2500\sqrt{3}}{10000\sin 30}\right) \qquad R_B = 6614 \angle (180 - 40.89 + 60)$

$= 6614 \angle 199.11°$ //

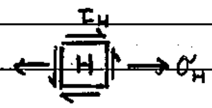
$= 40.89°$

(b)



$\sigma_H = \frac{5000}{(0.05)(0.12)} = 833\ 333.3333$ Pa //

$\tau_H = \frac{(2500\sqrt{3})((0.06)(0.05)(0.06 \div 2))}{\frac{(0.12)^3(0.05)}{12}(0.05)} = 1\ 082\ 531.755$ Pa //



(c) $\sigma_{max, min} = \frac{\sigma_H + 0}{2} \pm \sqrt{\left(\frac{\sigma_H - 0}{2}\right)^2 + (\tau_H)^2}$

$= 1\ 576\ 617.576$ Pa , $-743\ 284.2423$ Pa

$\left(\frac{\sigma_y}{1.5}\right)^2 = 1.11 \times 10^{15}\ Pa^2 > \sigma_{max}^2 - \sigma_{max}\sigma_{min} + \sigma_{min}^2 = 4.21 \times 10^{12}\ Pa^2$

Nope it will not yield!! //

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

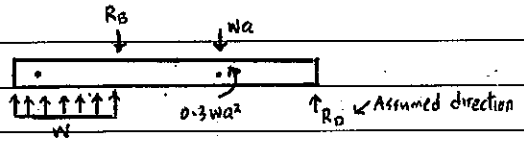
Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: http://bit.ly/2lW2C32

(3) *

4) $P = wa$

$AB = BC = CD = a$

(a)



$\Sigma F_{vertical}$: $wa + R_D - R_B - wa = 0$

$R_D = R_B$

Σ Moment : $M_A = 0 = -wa\left(\frac{a}{2}\right) + R_B a - 0.3wa^2 + wa(2a) - R_D(3a)$

$= 1.2wa^2 - 2R_D a$

$R_B = R_D = 0.6wa$ //

(b) $EIy'' = \frac{w}{2}\langle x^2\rangle - \frac{w}{2}\langle x-a\rangle^2 - 0.6wa\langle x-a\rangle - 0.3wa^2\langle x\rangle^0 - wa\langle x-2a\rangle$

$EIy' = \frac{w}{6}\langle x\rangle^3 - \frac{w}{6}\langle x-a\rangle^3 - 0.3wa\langle x-a\rangle^2 - 0.3wa^2\langle x\rangle - \frac{wa}{2}\langle x-2a\rangle^2 + C_1$

$EIy = \frac{w}{24}\langle x\rangle^4 - \frac{w}{24}\langle x-a\rangle^4 - 0.1wa\langle x-a\rangle^3 - \frac{0.3}{2}wa^2\langle x\rangle^2 - \frac{wa}{6}\langle x-2a\rangle^3 + C_1\langle x\rangle + C_2$

When $x = a$, $y = 0$:

$0 = \frac{w}{24}a^4 - \frac{0.3}{2}wa^4 + C_1 a + C_2$

$\frac{13}{120}wa^4 = C_1 a + C_2$

When $x = 3a$, $y = 0$:

$0 = 3.375wa^4 - \frac{2}{3}wa^4 - 0.8wa^4 - 1.35wa^4 - \frac{1}{6}wa^4 + 3aC_1 + C_2$

$-\frac{47}{120}wa^4 = 3aC_1 + C_2$

$\frac{1}{2}wa^4 = -2C_1 a$

$C_1 = -\frac{1}{4}wa^3$ $\quad C_2 = \frac{43}{120}wa^4$

$\therefore y = \frac{1}{EI}\left(\frac{w}{24}\langle x\rangle^4 - \frac{w}{24}\langle x-a\rangle^4 - 0.1wa\langle x-a\rangle^3 - \frac{0.3}{2}wa^2\langle x\rangle^2 - \frac{wa}{6}\langle x-2a\rangle^3 - \frac{1}{4}wa^3\langle x\rangle + \frac{43}{120}wa^4\right)$ //

Remember modulus as it could be positive or negative deflection.

(c) $|0.0021| = \frac{1}{EI}\left(-1083\frac{1}{3}\right)$

$P = 5000N$ $\quad I = 2.70833 \times 10^{-6} m^4 = \frac{\pi d^4}{64}$

$w = 5000 N/m$

$a = 1m$ $\quad d = 86.1852715 mm$ //

$E = 200 GPa$

$x = 2m$

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

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NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

MA2001 - MECHANICS OF MATERIALS

April/May 2019 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED-BOOK** Examination.
5. A list of equations can be found on Page 5.

---

1. A cantilever AB is subjected to a uniformly distributed load along CD and a moment at A as shown in Figure 1. Rigid member ACD is firmly fixed to the cantilever at A. The cantilever has constant EI. Ignore the weight of the cantilever and rigid member.

(a) Determine the reaction forces at support B. (5 marks)

(b) Derive the elastic curve of the beam in terms of EI and x. The origin of x is at A. (14 marks)

(c) Determine the slope and deflection at A. Given: $E = 200$ GPa and $I = 10\times10^6$ mm^4. (6 marks)

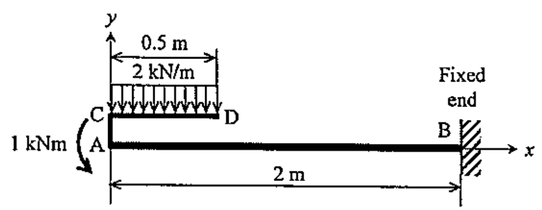

Figure 1

2. Two concentrated forces are applied through the cross section centroid at the free end of a cantilever as shown in Figure 2. The strain gauge located at A registers a longitudinal strain of -150×10^{-6}. The cross section has a constant thickness of 8 mm. Young's modulus of the beam is 200 GPa.

(a) Determine the magnitude of P. (8 marks)

(b) Determine the principal stresses, maximum shear stress and principal planes at point A. (14 marks)

(c) Using Tresca yield criterion and a factor of safety of 1.5, determine if yielding will occur at point A. The yield stress in simple tension is 250 MPa. (3 marks)

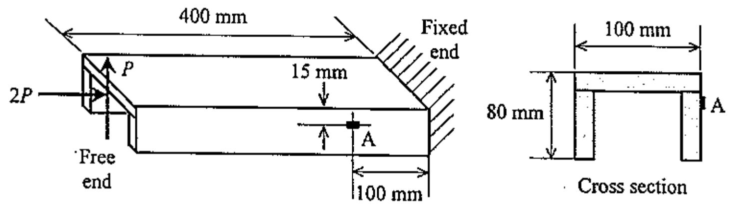

Figure 2

3. Figure 3(a) shows a simply supported beam ABCDE with uniformly distributed load q from C to E, point loads P acts at B and C, and a concentrated moment M at B. Figure 3(b) shows the cross-section of the beam. Given: P = 500 N, M = 400 Nm, q = 1 kN/m, L = 2 m, a = 30 mm, b = 40 mm and t = 4 mm.

(a) Draw the shear force and bending moment diagrams for the beam. Indicate all values at A, B, C, D and E.

(12 marks)

(b) Determine the distance from E where the shear force on the beam between points C and D (excluding points C and D) is zero. Calculate the bending moment at that point.

(4 marks)

(c) Determine the absolute maximum bending moment for the whole beam and hence calculate its absolute maximum bending stress. Determine the magnitude of the largest in-plane shearing stress τ_{xy} for the whole beam.

(9 marks)

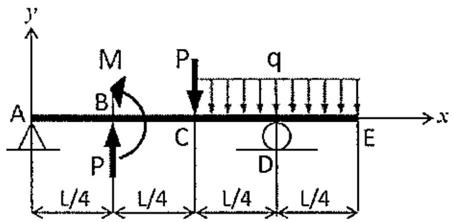

Figure 3(a)

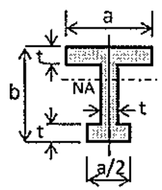

Figure 3(b)

4. A thin-walled cylindrical pressure vessel has an inner diameter of 800 mm and a wall thickness of 2 mm. The longitudinal strain on the outer surface of the pressure vessel is $+1 \times 10^{-4}$ mm/mm. Given: E = 200 GPa and $\upsilon = 0.3$.

(a) Determine internal pressure of the vessel.

(8 marks)

For questions 4(b) and 4(c), a longitudinal/axial external force P acts on the vessel in additional to the internal pressure. A strain gauge on the outer surface of the vessel now measures a normal strain of $+1.5 \times 10^{-4}$ mm/mm in the longitudinal direction.

(b) Determine the new longitudinal/axial stress on the outer surface of the vessel. Hence determine the axial force P.

(8 marks)

(c) A torque is created on the pressure vessel due to abnormal support conditions. Determine the allowable torque value if the allowable maximum principal stress (on the outer surface) is 120 MPa.

(9 marks)

List of Equations

The symbols in the equations have the usual meaning as described in the course text book *Mechanics of Materials*, SI units, 5th Edition, McGraw Hill, 2009 by Beer F. P., Johnston E. R., Dewolf J. T., Mazurek D.

Deformation under axial loading

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Torsion

$$\tau = T\rho / J$$

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Bending

$$\sigma = -My/I$$

Shear

$$\tau = VQ/(It)$$

$$\bar{y} = \frac{4r}{3\pi} \text{ for a semi circle}$$

Stress Transformation

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Generalized Hooke's Law

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

$$G = \frac{E}{2(1+\nu)}$$

Deflection

$$EI\,y'' = M(x)$$

Columns

$$P_{cr} = \pi^2 EI/L_e^2$$

End of Paper

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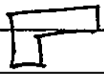
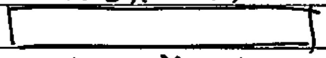
1a)

FBD 

↑ ~~(1b-5)~~ (0.5)(2000)

↪ (2000)(0.5)(0.25)

F = ↓ (0.5)(2000)   B (0.5)(2000) = F

 ↑ (0.5)(2000)(2 − 0.25) ↓ = M

M = ↷ (2000)(0.5)(0.25)

∴ At B

F = 0.5(2000) upwards

M = 0.5(2000)(1.75) Clockwise

1b) $y'' = \frac{1}{EI}\left(2000(0.5)(0.25) - 0.5(2000)x\right)$

$y' = \frac{1}{EI}\left(250x - 500x^2 + C_1\right)$

$y = \frac{1}{EI}\left(125x^2 - \frac{500}{3}x^3 + C_1x + D\right)$

When $x = 2$ $y = 0$ $y' = 0$ as is cantilever

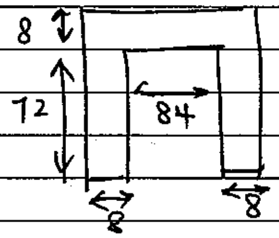
$C_1 = 1500$ $D = -2166.67$

1c) $I = 10\times10^6 \times (10^{-3})^4$

$y = \frac{(-2166.67)}{10\times10^6 \times 10^{-12} \times 200\times10^9} = -0.001083\text{ m}$

$y' = \frac{1500}{10\times10^6 \times 10^{-12} \times 200\times10^9} = 0.00075$

2a)



Centre of mass / ~~Centr~~ Neutral Axis

$$= \frac{72(8)(36)\times 2 + 8\times 100\times 76}{72\times 8\times 2 + 800} = 52.3934\text{mm}$$

$c = 80 - 52.3934 - 15 = 12.6066\text{mm}$

$$I \text{ of shape} = \frac{1}{12}(0.008)(0.072)^3\times 2 + (0.072)(0.008)(0.0523934 - 0.036)^2\times 2$$

$$+ \frac{1}{12}(0.1)(0.008)^3 + 0.008(0.1)(0.076 - 0.0523934)^2$$

$$= 1.25734\times 10^{-6}$$

$\text{Area} = 0.072\times 0.008\times 2 + 0.008\times 0.1$

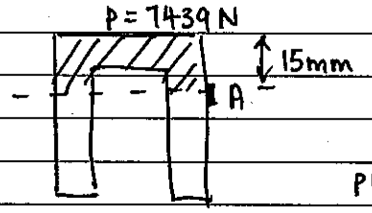
$= 0.001952$

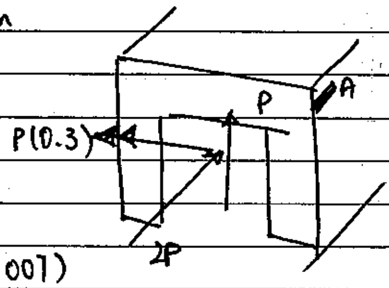
$$\frac{-2P}{0.001952} - \frac{P(0.3)(0.0126066)}{1.25734\times 10^{-6}} = 200\times 10^9 \times -150\times 10^{-6}$$

$$-4032.5P = 200\times 10^9\times -150\times 10^{-6}$$

$P = 7439\text{ N}$

2b) Finding J at A





$\text{Area} = 0.008(0.1) + 2\times 0.007\times 0.008$

$= 0.000912$

Centre of mass / centroid =

$$\frac{0.008(0.1)(0.076) + (0.0685)(0.008)(2)(0.007)}{0.000912}$$

$= 0.075078\text{m}$

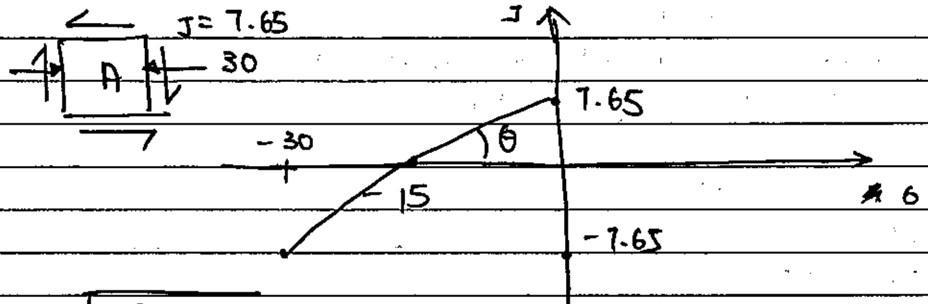
$Q = (0.075078 - 0.0523934)(0.000912) =$

$$J = \frac{VQ}{It} = \frac{7439(0.075078 - 0.0523934)(0.000912)}{1.25734\times 10^{-6}\times 2\times 0.008}$$

$= 7650112 = 7.650\text{ MPa}$

$\sigma_x = 200\times 10^9 \times -150\times 10^{-6} = -30\text{MPa}$

$\sigma_y = 0$



$R = \tau_{max} = \sqrt{15^2 + 7.65^2}$

$= 16.83 MPa$

Max $\sigma = -15 + 16.83$ Min $\sigma = -15 - 16.83$

$= 1.838 MPa$ $= -31.84 MPa$

$\tan^{-1}\left(\frac{7.65}{15}\right) = \theta = 27.02°$

$\frac{\theta}{2} = 13.51°$ since need to rotate clockwise, should be negative

Therefore principal plane = ~~−13.5~~ −13.51°, 76.48°

2c) Safety factor = 1.5

$\frac{\sigma_{yield}}{1.5} = 166.67 MPa$

Tresca criterion:

$\tau_{max} \leq \frac{\sigma_{yield}}{2}$

$\frac{\sigma_{yield}}{2} = 83.335 MPa$

Since $16.83 < 83.335$, the material will not yield.

3a) Summing moment about D

~~P and R ca~~ The distributed load cancel out

$$\frac{500(0.5)+400-500(1)}{1.5} = R_A$$

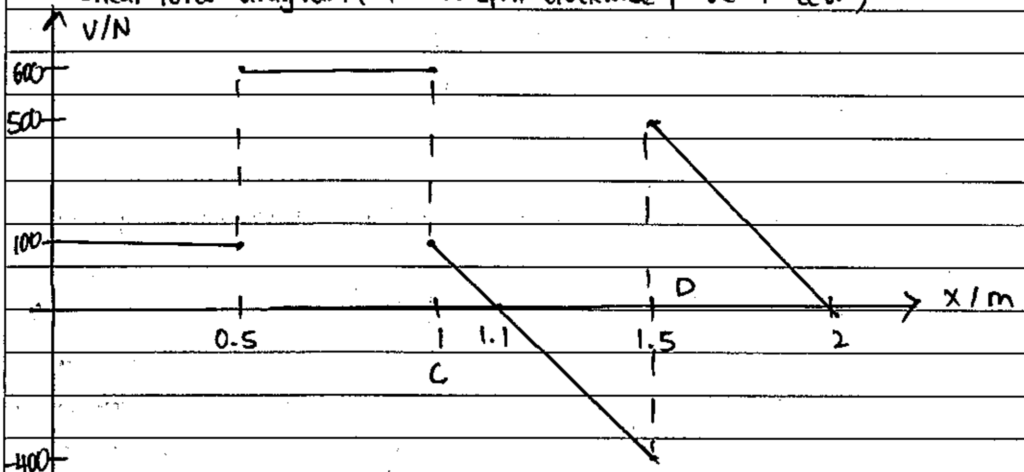
$R_A = 100N$

~~Fao~~ Summing force

$R_D = P - P + 1000 - 100$

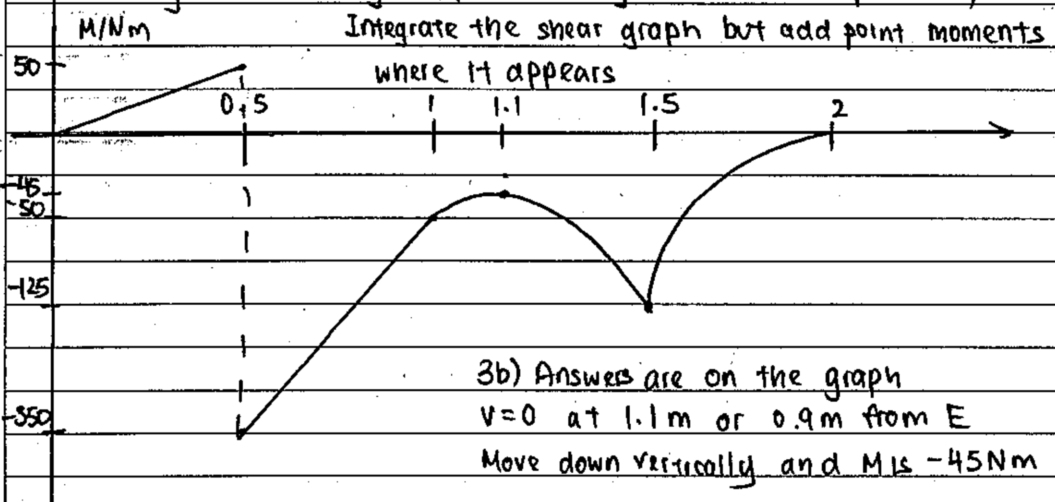
$= 900N$

Shear force diagram (+ve if spin clockwise , -ve if ccw)



Bending Moment Diagram (☹ is negative ☺ is positive)

Integrate the shear graph but add point moments where it appears



3b) Answers are on the graph

V=0 at 1.1m or 0.9m from E

Move down vertically and M is -45Nm

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

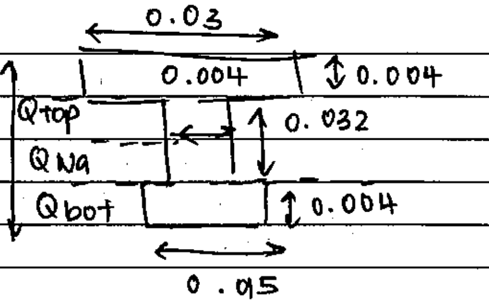
Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: http://bit.ly/2lW2C32

3c)

~~3b)~~ Area =

$0.004(0.015)+0.032(0.004)+0.03(0.004)$

$= 0.000308 m^2$



centroid = $\frac{0.004(0.015)(0.002)+0.032(0.004)(0.020)+0.03(0.004)(0.038)}{0.000308}$

$= 0.023506 m$

To find maximum τ, we need check 3 place, at N.A, at the top line when area goes from fat to thin, and bottom thin to fat.

$Q_{top} = 0.004(0.03)(0.038-0.023506) = 0.0000017392 8$

$Q_{bot} = 0.015(0.004)(0.023506 - ~~0.00~~\ 0.002) = 0.000012903$

To find Q_{NA}, find centroid of top area above NA — 0.0000012903

$= \frac{0.03(0.004)(~~0.038~~) + (0.036-0.023506)(0.004)\left(\frac{0.036-0.023506}{2}+0.023506\right)}{(0.038-0.023506)}$

$\frac{}{0.03 \times 0.004 + (0.~~036~~\ 032 - 0.023506)(0.004)}$ (036)

= ~~0.0355752~~ 0.01898042

since Q NA biggest shear biggest there since t = 0.004 for all 3.

~~τmax = VQ = 600(0.0355752)~~

$I = \frac{1}{12}(0.03)(0.004)^3 + 0.03(0.004)(0.038-0.023506)^2 + ~~\frac{1}{12}(0.032)(0~~$

$+ \frac{1}{12}(0.004)(0.032)^3 + (0.004)(0.032)(0.023506-0.02)^2$

$+ \frac{1}{12}(0.015)(0.004)^3 + (0.004)(0.015)(0.023506-0.002)^2$

$= 0.000\ 00006569565$

$\tau_{max} = \frac{V_{max}\ Q_{NA}}{It}$ (↑ V_{max} from graph)

$= \frac{600(0.01898042)}{0.0000000 6569565(0.004)} = 4.337 \times 10^{10}$ Pa

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Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: http://bit.ly/2lW2C32

$\sigma_{max} = \frac{Mc}{I}$ Since NA is 0.023506 m from bottom and it is bigger than half of 0.4 c should be 0.023506

$= \frac{350(0.023506)}{0.00000006569565}$ Max M = 350Nm from graph

$= 125.23 MPa$

4a) Hoop pressure = $\frac{pr}{t}$ Longitudinal pressure = $\frac{pr}{2t}$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E}$$

$$1\times10^{-4} = \frac{\frac{p(0.4)}{2(0.002)} - \frac{0.3(p)(0.4)}{0.002}}{200\times10^9}$$

Solving for p

$p = 500\,000\,Pa$

4b) Since we know initial pressure cause 1×10^{-4} strain and with additional P is now 1.5×10^{-4}, we can conclude that P is causing 0.5×10^{-4} strain.

$$0.5\times10^{-4} = \frac{\sigma}{E}$$

$$\sigma = 0.5\times10^{-4} \times 200\times10^9 = 10\,MPa$$

Force = σA

$$= 10\times10^6 \times \left(\pi\left(\frac{0.804^2}{4}\right) - \pi\left(\frac{0.8^2}{4}\right)\right)$$

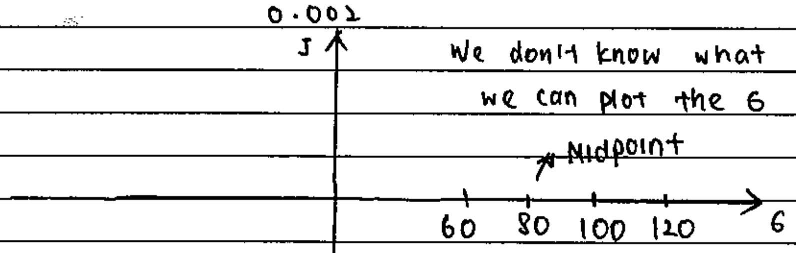
$= 50391\,N$

4c) σ_x = P + longi pressure

$$= 10\times10^6 + \frac{500000(0.4)}{2(0.002)} = 60\,MPa$$

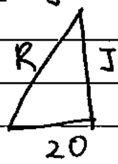
$$\sigma_y = \frac{500\,000(0.4)}{0.002} = 100\,MPa$$

We don't know what τ is at 0°, but we can plot the σ first



Since max can't be 120 and max is when the line formed by Mohr's circle is horizontal we can conclude R = 40MPa and τ_{max} = 40MPa.

Jxy when σx at original and σy at original value is found with pythagoreas theorem



$$J = \sqrt{40^2 - 20^2}$$
$$= 34.64 \text{MPa}$$

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = \frac{\pi}{2}\left(0.402^4 - 0.4^4\right)$$
$$= 0.000810299$$

$$T = \frac{Jc}{J} = \frac{34.64 \times 10^6 \times 0.000810299}{0.402}$$
$$= 69047 \text{NM}$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

MA2001 - MECHANICS OF MATERIALS

November/December 2019 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED-BOOK** examination.
5. A list of equations can be found on Page 5.

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1. The simply supported beam AB is subjected to a uniformly distributed load and two concentrated forces P and Q as shown in Figure 1. The bending moment is +0.8 kNm at D and +1.3 kNm at E. The length of AC = CD = DE = EB = 0.3 m. Ignore the weight of the beam.

(a) Determine the forces P and Q and the reaction force at supports C and B.
(8 marks)

(b) Derive the elastic curve of the beam in terms of EI and x. The origin of x is at A.
(14 marks)

(c) Determine the deflection at D. The flexural rigidity EI is 23 kNm^2.
(3 marks)

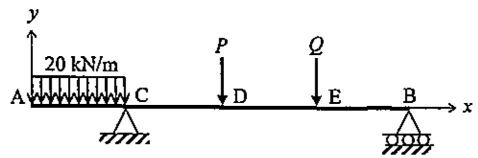

Figure 1

2. A machine component is fully fixed at A is subjected to the four concentrated forces, each of which is parallel to one of the coordinate axes as shown in Figure 2(a). The 300 N, 250 N and 120 N forces act through the centroid of their respective cross sections. The location of elements G and H on the cross section at A is shown in Figure 2(b). Ignore the weight of the component.

(a) Determine the normal and shearing stresses of element G and show the orientation of the stresses on a clearly labelled sketch.
(12 marks)

(b) Determine the normal and shearing stresses of element H and show the orientation of the stresses on a clearly labelled sketch.
(9 marks)

(c) Determine the principal stresses and maximum shearing stress of element H.
(4 marks)

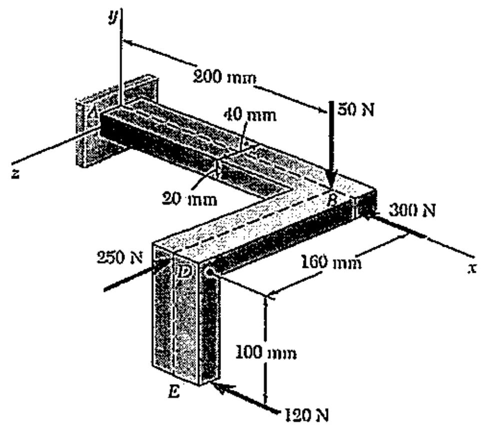

Figure 2(a)

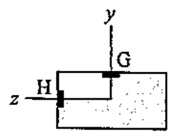

Figure 2(b)

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3. Figure 3(a) shows a simply supported beam ABCDE with a uniformly distributed load q from A to C, point loads P at C and D, and a concentrated moment M_o at D. Figure 3(b) shows the cross-section of the beam. Given: P = 400 N, M_o = 300 Nm, q = 2 kN/m, L = 2 m, a = 50 mm, b = 30 mm and c = 10 mm.

(a) Draw the shear force and bending moment diagrams for the beam. Indicate all values at A, B, C, D and E.

(12 marks)

(b) Determine the maximum bending moment in the beam.

(4 marks)

(c) Determine the absolute maximum bending stress and the largest absolute in-plane shearing stress τ_{xy} for the whole beam.

(9 marks)

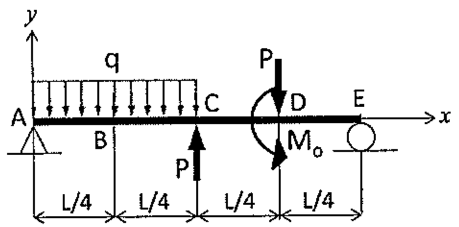


Figure 3(a)

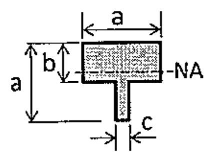


Figure 3(b)

4. A thin-walled cylindrical column carries an offset load F supported by a weightless horizontal bar as shown in Figure 4. Point A is an infinitesimal element on the outer radius of the column at section X-X. The inner diameter of the column is 120 mm and the thickness is 1 mm. Given: E = 200 GPa and ν = 0.3.

(a) Determine the forces acting at section X-X due to F only.

(7 marks)

For part (b) and (c), assume the thin-walled cylindrical column is pressurized.

(b) Calculate the internal pressure *p* required to cause the longitudinal stress of the column at element A to be zero.

(9 marks)

(c) If the allowable minimum principal stress at element A is -5 MPa, determine the allowable angle of twist in degrees on the column (provide answer up to 3 decimal places). The total length of the column is 1200 mm.

(9 marks)

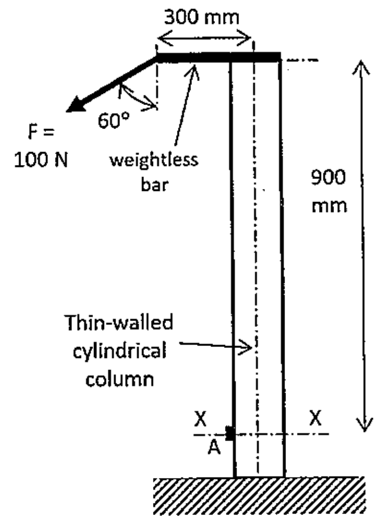
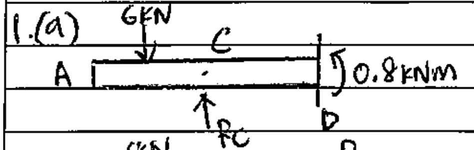


Figure 4

MA2001 Nov/Dec 2019

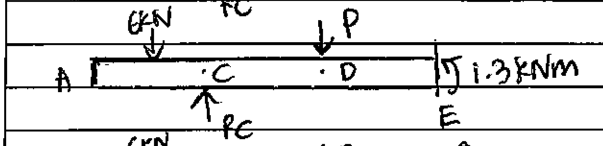
1.(a)



$\Sigma M_D = +0.8\,\text{kNm}$,

$0.8 + (6)(0.45) = (R_C)(0.3)$

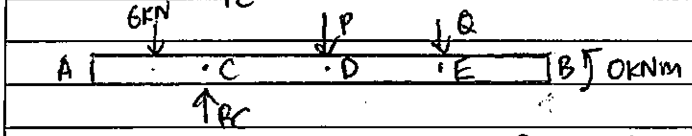
$\Rightarrow R_C = \frac{35}{3}\,\text{kN}$ //



$\Sigma M_E = +1.3\,\text{kNm}$,

$1.3 + (6)(0.75) + (P)(0.3) = (\frac{35}{3})(0.6)$

$\Rightarrow P = 4\,\text{kN}$ //



$\Sigma M_B = 0$, $(6)(1.05) + (4)(0.6) + (Q)(0.3) = (\frac{35}{3})(0.9)$

$\Rightarrow Q = 6\,\text{kN}$ //

$\Sigma F_y = 0$, $6 + P + Q = R_C + R_B \Rightarrow R_B = \frac{13}{3}\,\text{kN}$ //

(b) $EIy'' = M = -10\langle x\rangle^2 + 10\langle x-0.3\rangle^2 + \frac{35}{3}\langle x-0.3\rangle - 4\langle x-0.6\rangle$
$- 6\langle x-0.9\rangle$

$EIy' = -\frac{10}{3}x^3 + \frac{10}{3}\langle x-0.3\rangle^3 + \frac{35}{6}\langle x-0.3\rangle^2 - 2\langle x-0.6\rangle^2 - 3\langle x-0.9\rangle^2 + C_1$

$EIy = -\frac{5}{6}x^4 + \frac{5}{6}\langle x-0.3\rangle^4 + \frac{35}{18}\langle x-0.3\rangle^3 - \frac{2}{3}\langle x-0.6\rangle^3 - \langle x-0.9\rangle^3 + C_1x + C_2$

At C, $y=0$ & $x=0.3$,
we have $-\frac{5}{6}(0.3)^4 + 0.3C_1 + C_2 = 0$ —①

At B, $y=0$ & $x=1.2$,
we have $-\frac{5}{6}(1.2)^4 + \frac{5}{6}(0.9)^4 + \frac{35}{18}(0.9)^3 - \frac{2}{3}(0.6)^3 - (0.3)^3 + 1.2C_1 + C_2 = 0$ ②

solving ① & ②, $C_1 = -\frac{2}{25}$, $C_2 = \frac{123}{4000}$

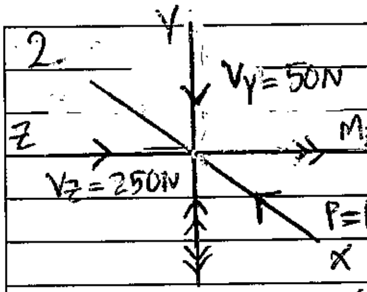
$\Rightarrow y = \frac{1}{EI}\left[-\frac{5}{6}x^4 + \frac{5}{6}\langle x-0.3\rangle^4 + \frac{35}{18}\langle x-0.3\rangle^3 - \frac{2}{3}\langle x-0.6\rangle^3 - \langle x-0.9\rangle^3 - \frac{2}{25}x + \frac{123}{4000}\right]$

(in kN & m) //

(c) At D, $x=0.6$, $y = \frac{1}{23}\left[-\frac{5}{6}(0.6)^4 + \frac{5}{6}(0.3)^4 + \frac{35}{18}(0.3)^3 - \frac{2}{25}(0.6) + \frac{123}{4000}\right]$

$= -0.0028695 = -0.00287\,\text{m}$ //

2.



$M_z = (50N)(200mm) + (120N)(100mm)$

$P = 420N$ (compressive)

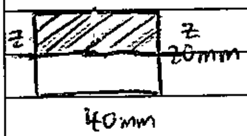
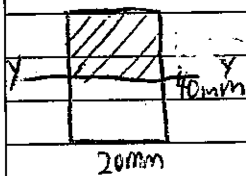
$V_y = 50N$ (↓)

$V_z = 250N$ (→)

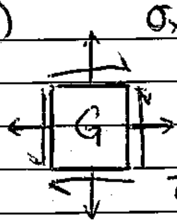
$M_y = 30800 Nmm = 30.8 Nm$ (↑)

$M_z = 22000 Nmm = 22 Nm$ (→→)

$M_y = (250N)(200mm) - (120N)(160mm)$

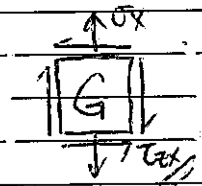
	A	$\bar{Y}$	Q	I
	$10 \times 40 = 400 mm^2$	5mm	$2000 mm^3$	$26666\frac{2}{3} mm^4$
	$20 \times 20 = 400 mm^2$	10 mm	$4000 mm^3$	$106666\frac{2}{3} mm^4$

(a) $\sigma_x = -\frac{420}{800 \times 10^{-6}} + \frac{(22)(10 \times 10^{-3})}{(26666\frac{2}{3} \times 10^{-12})} = 7.725 MPa$ //

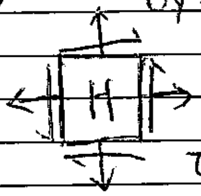


$\sigma_z = 0$

$\tau_{zx} = \frac{(250)(4000 \times 10^{-9})}{(106666\frac{2}{3} \times 10^{-12})(20 \times 10^{-3})} = -0.46875 MPa$ //

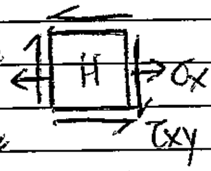


(b) $\sigma_y = 0$



$\sigma_x = -\frac{420}{800 \times 10^{-6}} + \frac{(30.8)(20 \times 10^{-3})}{(106666\frac{2}{3} \times 10^{-12})} = 5.25 MPa$ //

$\tau_{xy} = -\frac{(50)(2000 \times 10^{-9})}{(26666\frac{2}{3} \times 10^{-12})(40 \times 10^{-3})} = -0.09375 MPa$ //



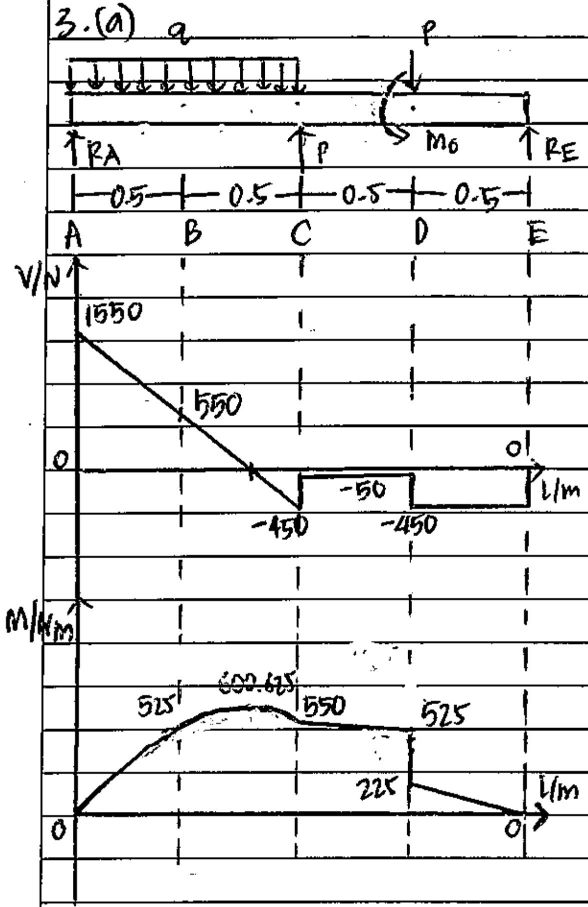
//

(c) $\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{5.25}{2} + \sqrt{\left(\frac{5.25}{2}\right)^2 + (-0.09375)^2} = 5.2517 MPa$ //

$\sigma_{min} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{5.25}{2} - \sqrt{\left(\frac{5.25}{2}\right)^2 + (-0.09375)^2} = -0.0016736 MPa$ //

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2.6267 MPa$ //

3.(a)



$P = 400N$, $M_o = 300Nm$, $q = 2000N/m$

$\Sigma M_E = 0$,

$(R_A)(2) + (400)(1) = (2000)(1.5) + 300 + (400)(0.5)$

$\Rightarrow R_A = 1550N\ (\uparrow)$

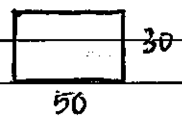
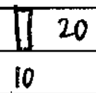
$\Sigma F_y = 0$, $R_E = 450N\ (\uparrow)$

(b) At $V = 0$, $L = 0.775m$ (from A)

max bending moment $= \frac{1}{2}(1550)(0.775)$

$= 600.625Nm$ //

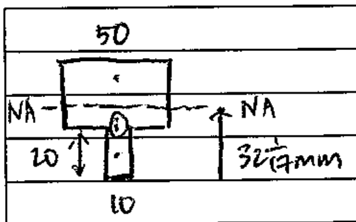
(c)

	A	$\bar{y}$	Q	I
	$1500mm^2$	$35mm$	$52500mm^3$	$112500mm^4$
	$200mm^2$	$10mm$	$2000mm^3$	$6666\frac{2}{3}mm^4$

$$\bar{y}_{NA} = \frac{(52500 + 2000)}{1500 + 200} = 32\tfrac{1}{17}\ mm$$

$$I_{NA} = 112500 + (1500)(35 - 32\tfrac{1}{17})^2 + 6666\tfrac{2}{3} + (200)(32\tfrac{1}{17} - 10)^2$$

$$= 229460.8\ mm^4$$



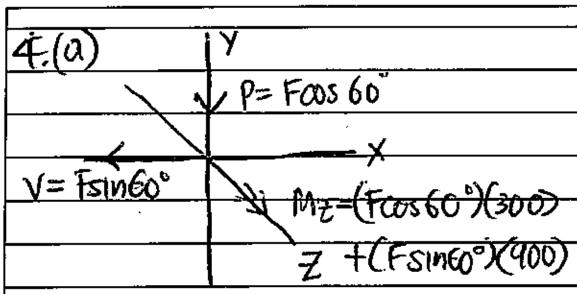
@ NA, $\frac{Q}{t} = \frac{(50-32\frac{1}{17})(\cancel{50})(\frac{50-32\frac{1}{17}}{2})}{\cancel{50}} = 160.943\,mm^2$

@ ①, $\frac{Q}{t} = \frac{(10)(20)(\frac{32\frac{1}{17}-10}{2})}{10} = 441.176\,mm^2$

$$\sigma_{max} = \frac{(600.625)(32\frac{1}{17})(10^{-3})}{229460.8\times10^{-12}} = 83.9155 = 83.92\,MPa$$

$$\tau_{max} = \frac{(1550)(441.176\times10^{-6})}{229460.8\times10^{-12}} = 2.98013 = 2.98\,MPa$$

4.(a)



$P = 50N$ (compressive) //

$V = 86.6025N$ (←) //

$M_z = 92942.3\ Nmm = 92.942 Nm$ //

$d_i = 120mm \quad d_o = 122mm$

$t = 1mm$

$r_i = 60mm \quad r_o = 61mm$

$A = \pi(61^2 - 60^2) = 121\pi\ mm^2$

$Q = \frac{2}{3}(61^3 - 60^3) = 7320\frac{2}{3}\ mm^3$

$I = \frac{\pi}{4}(61^4 - 60^4) = 695737.9 mm^4$

(b) $\sigma_y = 0 \Rightarrow \frac{p(60)}{2(1)} - \frac{50}{(121\pi)(10^{-6})} - \frac{(92.942)(61\times10^{-3})}{695737.9\times10^{-12}} = 0$

$p = 276013.5 Pa = 276\ kPa$ //

(c)

← [A] → $\sigma_z = \frac{pr}{t} = 16.5608\ MPa$

$\sigma_{min} = \frac{\sigma_z + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_z - \sigma_y}{2}\right)^2 + \tau_{zy}^2} = -5MPa$

$68.5651 + \tau_{zy}^2 = 176.369$

$\tau_{zy} = 10.3828\ MPa$

$\tau_{zy} = \frac{Tc}{J} \qquad \phi = \frac{TL}{JG}$

$\phi = \frac{\tau_{zy} L}{cG} = \frac{(10.3828)(10^6)(1.2)}{(61\times10^{-3})(200\times10^9)}$

$= 0.00102126\ rad$

$= 0.059°$ //