NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2015-2016

MA3006 – FLUID MECHANICS

April/May 2016

Time Allowed: $2^1/_2$ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL FOUR (4)** Questions.
3. All questions carry equal marks.
4. This is a **CLOSED BOOK** examination.

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1 (a) Figure 1 shows a lawn sprinkler with 4 symmetrical arms. Water enters the sprinkler through its base and exists through each of the nozzles equally.
Derive an expression to show that when nozzle angle θ = 90°, the rotational speed of the sprinkle is zero.

Water enters the sprinkle at a steady rate of 3.5×10^{-3} m^3/s. The nozzle exit area is 35×10^{-6} m^2 and the radius arm of the sprinkler is 0.4 m.

(i) If θ = 0°, what is the maximum rotational speed?
(ii) What is the torque required to reduce the rotational speed to half the maximum?
(iii) If θ = 15°, what is the torque required to hold the sprinkle rotational speed at 300 rpm?

(13 marks)

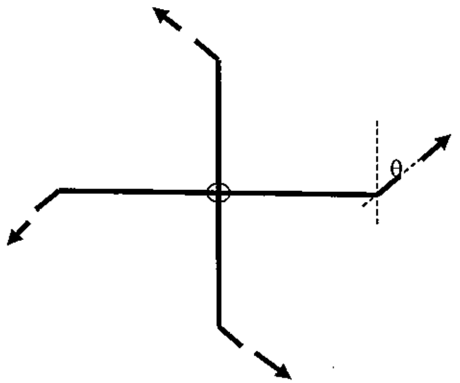

Figure 1

Note: Question 1 continues on page 2.

(b) Water is siphoned from a tank through a hose of diameter 2 cm and discharged to the atmosphere at point B, Figure 2. The total head loss from A to X and X to B is 0.2 m and 0.4 m respectively

(i) Determine the flow rate in the hose.

(ii) Determine the pressure at point X.

(iii) If a hole is punctured into the hose at point X, what will happen?

a. No changes to the flow
b. Water will leak out of the hose
c. Air will leak into the hose

Explain your answer.

(iv) The water tank is to be sealed and pressurized to increase the flow rate by two times. Assume that the water level in the tank remains constant, what is the required pressure in the tank?

(12 marks)

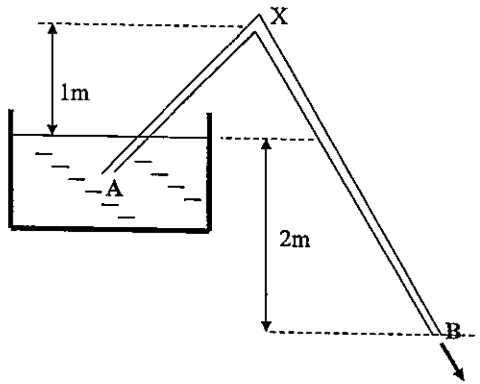

Figure 2

2 (a) An orifice plate diameter d is installed in a pipe diameter D to monitor water flow rate. Show that the pressure difference across the orifice plate can be expressed as:

$$p1 - p2 = \left(\frac{Q}{C_o A_2}\right)^2 \frac{(1-\beta^4)\rho}{2}$$

where Q = actual flow rate
p_1 = upstream pressure
p_2 = pressure at orifice
A_2 = area at orifice
ρ = density of fluid
C_o = coefficient of discharge of orifice
$\beta = \frac{d}{D}$

Figure 3 shows an orifice plate arrangement and variation of coefficient of discharge, C_o with Re No.
The diameter of the orifice is 40 mm and the pipe diameter is 80 mm. If the actual flow rate is 0.06 m^3/s, what should be the pressure difference between upstream pressure (p_1) and pressure at orifice (p_2).
Dynamic viscosity of water, $\mu = 1.002 \times 10^{-3}$ Ns/m^2.
Density of water, $\rho = 1000$ kg/m^3.

(13 marks)

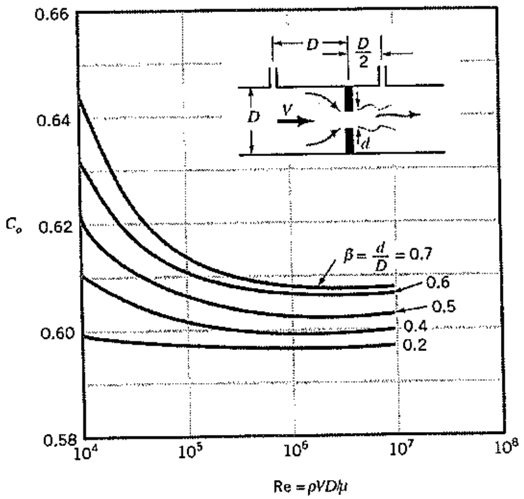

Figure 3

Note: Question 2 continues on page 4.

(b) Figure 4 shows a viscosity meter design with an inner rotating cylinder. The outer cylinder is fixed and the gap between the cylinders is filled with testing liquid.

The torque required to rotate the inner cylinder depends on rotating speed ω, dynamic viscosity of liquid μ, diameter D, gap a and length l.

Using ω, μ and D as repeating variables, determine suitable dimensionless group relating the torque.

If the diameter D is increased 3 times and the gap a is increased 2 times, how many times will the torque increase?

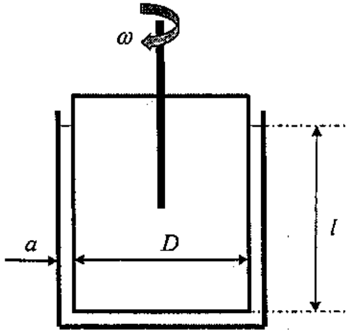

Figure 4

(12 marks)

3 (a) The velocity profile of a turbulent flow in a horizontal pipe of radius R is given as:

$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

Where $\bar{u}$ is the velocity at radius r, V_c is the centerline velocity and n is a constant. Show that the volume flow rate, Q, can be expressed as:

$$Q = 2\pi R^2 V_c \frac{n^2}{(n+1)(2n+1)}$$

If the radius of the pipe is 10 cm, the average flow velocity is 2 m/s and the value of n is 8, determine the value of V_c.

(10 marks)

(b) Water flows from reservoir A into reservoirs B and C as shown in Figure 5. The elevations of the free surface of reservoirs A, B and C are 40 m, 10m and 0 m respectively. Given that the flow rate in pipe 1 is 1 m^3/s and that flow is equally divided between pipes 2 and 3, determine the diameters of pipes 2 and 3. You may make use of the following information:

Description	Diameter (m)	Length (m)	Friction factor	ΣK_L
Pipe 1	0.5	200	0.04	0
Pipe 2	?	200	0.04	0
Pipe 3	?	150	0.04	0

Sketch the energy grade line (EGL) and hydraulic grade line from reservoir A to reservoir B, taking into account the effects of frictional loss, entry loss, junction loss and exit loss.

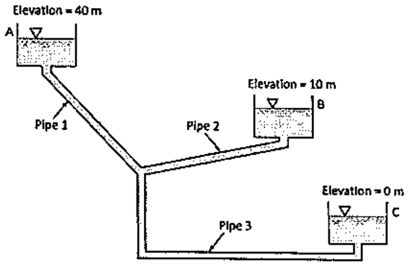

Figure 5

(15 marks)

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4 Water is pumped from reservoir A to reservoir C as shown in Figure 6. The elevations of the free surface of reservoirs A and C are 0 m and 40 m respectively. The diameter of the pump impeller is 0.5 m and it is operating at a rotational speed of N rpm. The non-dimensional pump characteristics is given as:

$$C_H = 1 - 2000 C_Q^2$$

(i) Determine the total head loss for flow through the system if the flow velocity is 2 m/s.
(ii) What is the rotational speed of the pump impeller?
(iii) If the minimum pressure at point B is -90 kPa (gauge), what is the maximum elevation of point B?
(iv) If a higher volume flow rate is required and the impeller diameter remains the same, how should the pump be operated? Explain your answer.

You may make use of the following information.

Description	Diameter (m)	Length (m)	Friction factor	ΣK_L
Pipe 1	0.2	400	0.025	5
Pipe 2	0.2	100	0.025	2

$$C_Q = \frac{Q}{\omega D^3} \quad \text{and} \quad C_H = \frac{g\, h_p}{\omega^2 D^2}$$

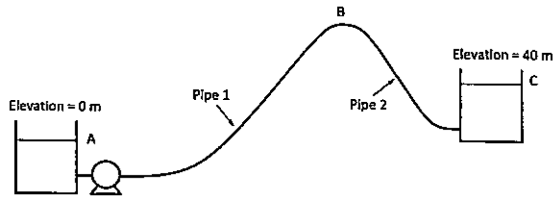


Figure 6

(25 marks)

End of Paper

15

1a) $(\dot{m}\,\vec{r} \times \vec{v})_{out} = \dot{m}\left(\frac{Q}{4A}\cos\theta - R\omega\right)$

$(\dot{m}\,\vec{r} \times \vec{v})_{in} = 0$

$T = (\dot{m}\,\vec{r} \times \vec{v})_{out} - (\dot{m}\,\vec{r} \times \vec{v})_{in}$

$= \dot{m}\left(\frac{Q}{4A}\cos\theta - R\omega\right)$

$T = 0, \quad \theta = 90°, \quad \omega = 0$

ai) $R\omega = \frac{Q}{4A}\cos\theta$

$0.4\,\omega = \frac{3.5\times10^{-3}}{4\times35\times10^{-4}}$

$\omega = 62.5\ \text{rad/s}$

aii) $T = \dot{m}\left(\frac{Q}{4A}\cos\theta - R\omega\right)$

$= (1000\times3.5\times10^{-3}) \times \left(\frac{3.5\times10^{-3}}{4\times35\times10^{-4}} - \frac{1}{2}\times62.5\times0.4\right)$

$= 43.75\ \text{Nm}$

aiii) $T = \dot{m}\left[\frac{Q}{4A}\cos\theta - R\omega\right]$

$= (1000\times3.5\times10^{-3})\left(\frac{3.5\times10^{-3}}{4\times35\times10^{-4}}\cos 15° - 0.4\times\frac{300\times2\pi}{60}\right)$

$= 40.53621265\ \text{Nm}$

1bi) $\cancel{\frac{P_A}{\rho g}} + z_A + \cancel{\frac{V_A^2}{2g}} = \cancel{\frac{P_B}{\rho g}} + z_B + \frac{V_B^2}{2g} + h_L$

$2 = \frac{V_B^2}{2g} + 0.2 + 0.4$

$V_B = 5.240992272\ \text{m/s}$

1bii) $\cancel{\frac{P_A}{\rho g}} + z_A + \cancel{\frac{V_A^2}{2g}} = \frac{P_x}{\rho g} + z_x + \frac{V_x^2}{2g} + h_L\ (A\ \text{to}\ x)$

$-1 = \frac{P_x}{\rho g} + \frac{5.24^2}{2g} + 0.2$

$P_x = -2.6\times1000\times9.81$

$= -25\,506\ \text{Pa}$

1biii) (c)

Air will leak into the hose as the gauge pressure is negative.

1b iv) $Q' = 2Q \Rightarrow V = \sqrt{2}\, V$

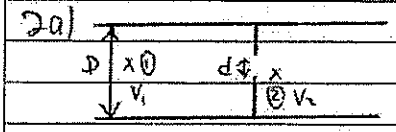
Assume head loss remains constant,

$$\frac{P_A}{\rho g} + \cancel{\frac{V_A^2}{2g}} + z_A = \cancel{\frac{P_B}{\rho g}} + \frac{V_B^2}{2g} + z_B + h_L$$

$$\frac{P_A}{\rho g} + 2 = \frac{(\sqrt{2} \times 5.241)^2}{2g} + (0.27\ 0.4)$$

$$P_A = 13\ 734\ Pa \text{ (gauge)}$$

2a)



$$D^2 V_1 = d^2 V_2$$

$$V_1 = (d/D)^2 V_2$$

$$= \beta^2 V_2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$P_1 - P_2 = \rho g \left(\frac{V_2^2 - V_1^2}{2g} \right)$$

$$= \frac{(1-\beta^4) V_2^2 \rho}{2}$$

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho (1 - \beta^4)}} \quad \text{———} \quad ①$$

$$Q = A_2 V_2 C_0$$

$$V_2 = \frac{Q}{A_2 C_0} \quad \text{———} \quad ②$$

sub ① into ②

$$\left(\frac{Q}{A_2 C_0} \right)^2 = \frac{2(P_1 - P_2)}{\rho (1 - \beta^4)}$$

$$P_1 - P_2 = \left(\frac{Q}{A_2 C_0} \right)^2 \frac{(1-\beta^4)\rho}{2}$$

$$V_1 = \frac{4Q}{\pi D^2} = \frac{4 \times 0.06}{\pi \times (80/1000)^2} = 11.93662\ m/s$$

$$Re = \frac{\rho V D}{\mu} = \frac{11.93662 \times 80/1000 \times 1000}{1.002 \times 10^{-3}}$$

$$= 953\ 023.6113$$

$$\beta = d/D = 40/80 = 0.5$$

$$\Rightarrow C_0 = 0.602$$

$$P_1 - P_2 = \left(\frac{0.06}{\pi/4 \times (80/1000)^2 \times 0.602} \right)^2 \frac{(1 - 0.5^4)(1000)}{2}$$

$$= 184\ 293.9542\ Pa$$

2b) $T\left[\frac{ML^2}{T^2}\right]$ $\textcircled{\omega}\left[1/T\right]$ $\textcircled{\mu}\left[\frac{M}{LT}\right]$ $\textcircled{D}\left[L\right]$ $a\left[L\right]$ $\ell\left[L\right]$

$\pi_1 : \left[\frac{ML^2}{T^2}\right]\left[1/T\right]^a \left[\frac{M}{LT}\right]^b \left[L\right]^c$

M: $1+b=0$ $\quad b=-1$

T: $-2-a-b=0$ $\quad a=-1$ $\qquad \frac{T}{\omega\mu D^3}$

L: $2-b+c=0$ $\quad c=-3$

$\pi_2 : \left[L\right]\left[1/T\right]^a \left[M/LT\right]^b \left[L\right]^c$ $\qquad a/D$

$a=0,\ b=0,\ c=1$

π_3: same as π_2 $\Rightarrow$ $\qquad \ell/D$

$$\frac{T}{\omega\mu D^3} = f\left(a/D,\ \ell/D\right)$$

$a_2 = 2a_1$

$D_2 = 3D_1$

$$\left(a/D\right)_1 = 1.5\left(a/D\right)_2$$

$$\left(\frac{T}{\omega\mu D^3}\right)_1 = 1.5\left(\frac{T}{\omega\mu D^3}\right)_2$$

$$T_1 = 1.5\ \frac{T_2}{D_2^3} \times D_1^{\ 3}$$

$$T_2 = \frac{1}{1.5} \times \left(D_2/D_1\right)^3\ T_1$$

$$= \frac{1}{1.5} \times 3^3\ T_1 = 18\,T_1$$

3a) $dQ = \left(2\pi r\ dr\right)\bar{u}$

$$= 2\pi r\ V_c \left(1-r/R\right)^{1/n}\ dr$$

$$Q = \int dQ = \int_0^R \left(2\pi r V_c\right)\left(1-r/R\right)^{1/n}\ dr$$

$$= 2\pi V_c \int_0^R r\left(1-r/R\right)^{1/n}\ dr \longrightarrow \text{Integration by parts needed}$$

$$= 2\pi V_c\left\{\left[\frac{-\left(1-r/R\right)^{1/n+1} Rr}{1/n+1}\right]_0^R - \int_0^R \frac{-\left(1-r/R\right)^{1/n+1} R}{\left(1/n+1\right)}\,dr\right\}$$

$$= 2\pi V_c\left\{-\left(0-0\right) - \left[\frac{\left(1-r/R\right)^{1/n+2} R^2}{\left(1/n+1\right)\left(1/n+2\right)}\right]_0^R\right\}$$

$$= 2\pi V_c\left\{-\left[0-\frac{R^2}{\left(1/n+1\right)\left(1/n+2\right)}\right]\right\}$$

$$= 2\pi V_c\ \frac{R^2}{\left(1/n+1\right)\left(1/n+2\right)}$$

$$= \frac{2\pi V_c\ R^2 n^2}{\left(n+1\right)\left(2n+1\right)}$$

$Q = \pi R^2 V_{avg}$

$= \pi (0.1)^2 (2)$

$= 0.02\pi$ m/s

$Q = 2\pi R^2 V_c \frac{n^2}{(n+1)(2n+1)}$

$0.02\pi = 2\pi (0.1)^2 V_c \frac{8^2}{(8+1)(2\times 8+1)}$

$V_c = 2.390625$ m/s

3b) Given $Q_1 = 1 m^3/s$

$Q_2 = Q_3 = 0.5 m^3/s$

$V_A = \frac{Q_A}{\pi (D^2/4)} = \frac{1}{0.25^2 \pi}$

$= 5.092958$

A to B: $\cancel{\frac{P_A}{\rho g}} + \cancel{\frac{V_A^2}{2g}} + z_A = \cancel{\frac{P_B}{\rho g}} + \cancel{\frac{V_B^2}{2g}} + z_B + H_{L_1} + H_{L_2}$

$30 = f_1 (l_1/D_1) \frac{V_1^2}{2g} + f_2 (l_2/D_2) \frac{V_2^2}{2g}$

$30 = 0.04 \times \frac{200}{0.5} \times \frac{5.092^2}{2g} + 0.04 \times \frac{200}{D_2} \times \frac{0.5^2}{2g(\pi/4 \times D_2^2)^2}$

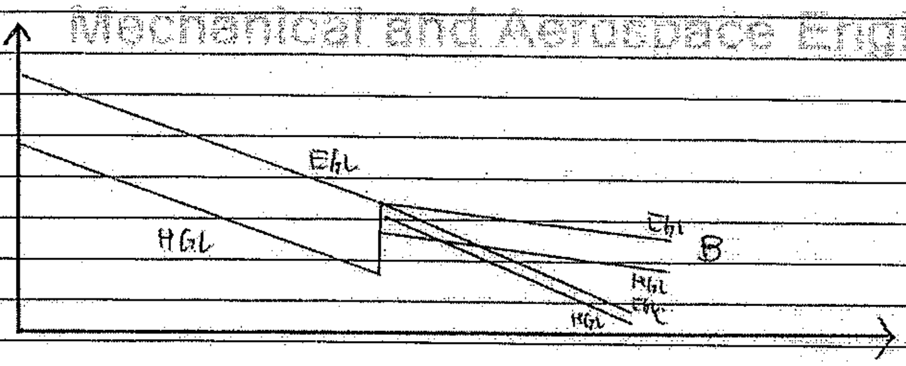
$D_2 = 0.451092795 m$

A to C: $\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + z_C + H_{L_1} + H_{L_3}$

$40 = f_1 (l_1/D_1) \frac{V_1^2}{2g} + f_4 (l_3/D_3) \frac{V_3^2}{2g}$

$40 = 0.04 \times \frac{200}{0.5} \times \frac{5.09^2}{2g} + 0.04 \times \frac{150}{D_3} \times \frac{0.5^2}{2g(\pi/4 \times D_3^2)^2}$

$D_3 = 0.366093177 m$



4i) $H = h_{L1} + h_{L2}$

$= \left[\left(f_1 \, l_1/D_1 + K_1\right) + \left(f_2 \, l_2/D_2 + K_2\right)\right] \frac{V^2}{2g}$

$= \left[\left(0.025 \times \frac{400}{0.2} + 5\right) + \left(0.025 \times \frac{100}{0.2} + 2\right)\right] \frac{2^2}{2 \times 9.81}$

$= 14.16921509 \, m$

4ii) $h_p = h_L + 40 = 54.16921509 \, m$

$Q = \pi\left(D^2/4\right) V$

$= \pi\left(0.2^2/4\right)(2)$

$= 0.02\pi \, m^3/s$

$C_H = 1 - 2000 \, C_Q^2$

$\frac{g h_p}{w^2 D^2} = 1 - 2000\left(\frac{Q}{wD^3}\right)^2$

$\frac{g h_p}{w^2 D^2} = 1 - 2000\left(\frac{0.02\pi}{wD^3}\right)^2$

$\frac{9.81 \times 54.169}{w^2 \times 0.5^2} = 1 - 2000\left(\frac{0.02\pi}{w \times 0.5^3}\right)^2$

$w = 51.29253109 \, rad/s$

4iii) $h_p + \cancel{\frac{P_A}{\rho g}} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_L$

$54.169 = -\frac{90\,000}{9.81} + \frac{2^2}{2g} + z_B + 14.16921519$

$z_B = 48.97043833 \, m$

4iv) $\frac{g h_p}{w^2 D^2} = 1 - 2000\left(\frac{Q}{wD^3}\right)^2$

$h_p = \frac{w^2 D^2}{g} - 2000\left(\frac{w^2 D^2}{g}\right)\left(\frac{1}{wD^3}\right)^2 Q^2$

$= \frac{w^2 D^2}{g} - \left(\frac{2000}{g D^4}\right) Q^2$

↑ Q, ↑ head loss, ↑ required pump head, ↑ w

A to C: $E = \Delta z + H_L$

$= 40 + \left[\left(0.025 \times \frac{400}{0.2} + 5\right) + \left(0.025 \times \frac{100}{0.2} + (2)\right)\right] \frac{Q^2}{2g\left[\pi/4 \times 0.2^2\right]^2}$

$= 40 + \frac{\left[\left(0.025 \times \frac{400}{0.2} + 5\right) + \left(0.025 \times \frac{100}{0.2} + (2)\right)\right]}{2 \times 9.81 \times \left[\pi/4 \times 0.2^2\right]^2} \times Q^2$

$= 40 + 3589.10411 \, Q^2$

$H_p = \frac{w^2 D^2}{g} - \left(\frac{2000}{9.81 \times 0.5^2}\right) Q^2$

$= \frac{w^2 D^2}{g} - 3261.977574 \, Q^2$

Relatively steeper system curve => pump should be organised in series

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INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED BOOK** examination.

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1 (a) Water flows through a vertical pipe-nozzle and exits into the atmosphere as shown in Figure 1. The volume flow rate at the nozzle exit is 0.02 m^3/s and the nozzle exit diameter is 5 cm. At section 1 just before flow enters into the nozzle, the pressure is 80 kPa (gauge pressure) and the pipe diameter is 10 cm. The weight of the nozzle is 20 kg and the weight of volume of water inside the nozzle is to be neglected. The flow is assumed to be frictionless throughout. Changes in height in the flow (within the pipe and nozzle) can be neglected. The density of water is 1000 kg/m^3. Determine the resultant force and its direction to secure the nozzle at the flange of the pipe.

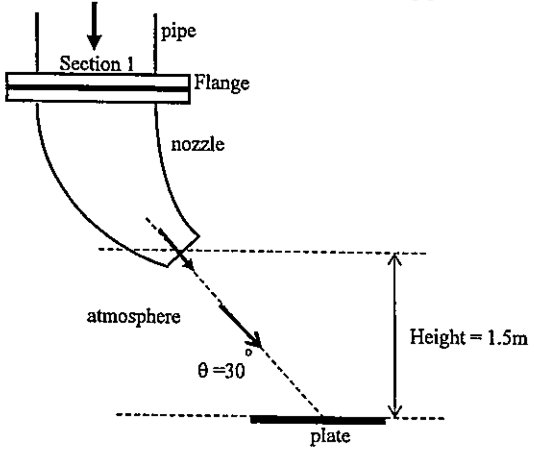

Figure 1

(10 marks)

Note: Question 1 continues on page 2.

(b) The water jet leaving the nozzle strikes a horizontal flat plate on the ground as shown in Figure 1. The plate is 1.5m below the nozzle exit. Upon striking the plate, the water splits along the plate such that 70% of the water flows towards the Right and the remaining 30% flows towards the Left.
Determine the horizontal force and direction required to hold the plate stationary.
Assume that the flow is steady and frictionless throughout its path and the weight of water in contact with the plate is negligible.

(5 marks)

(c) Water flows steadily through two pipes (in series) as shown in Figure 2. The static pressure indicated on the 2 cm diameter pipe is h = 1.0 m. The static pressure indicated on the 4 cm diameter pipe is h = 2.0 m.
Determine the flow direction (upwards or downwards) and total pressure drop (in terms of velocity V_1 or V_2) between Point 1 and Point 2.

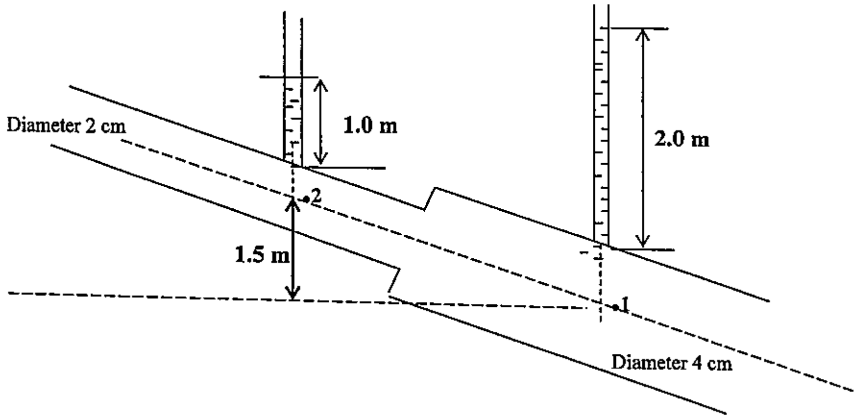

Figure 2

(10 marks)

2 (a) Develop an equation to illustrate the working principle of flow measurement for orifice meter, nozzle meter and venture meter.

The pressure drop across an orifice meter is measured to be 2.5 kPa. The diameter of the orifice is 24.5 mm and the diameter of the pipe is 35 mm. The density of the working fluid is 900 kg/m^3 and dynamic viscosity is 0.38 N.s/m^2. If the initial estimate of the coefficient of discharge of the orifice is 0.61, what is the expected flow rate? What should be the coefficient of discharge?

Figure 3 shows the coefficient of discharge for an orifice meter.

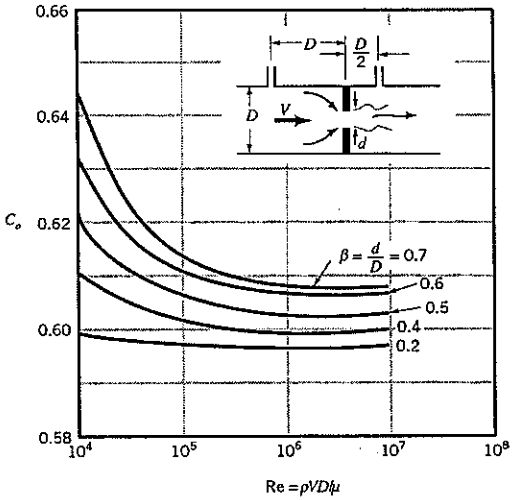

Figure 3

(10 marks)

(b) The drag force F_D of a spinning golf ball depends on its diameter D, flight velocity V, density of air ρ, dynamics viscosity μ, dimeter of dimple d and spin ω (radian per second).
Using ρ, V, D as repeating variables, determine suitable dimensionless parameters.

Typically, the flight velocity of a golf ball travels at 80 m/s with a spin of about 8500 rpm. For model testing in a wind-tunnel capable of generating 40 m/s in the test section, what should be the size of the model with respect to the golf ball and the corresponding spin to be generated on the model?

(10 marks)

(c) Explain the difference between geometric similarity and dynamic similarity. Give example in each case to support your answer

(5 marks)

3 (a) Figure 4 shows water being discharged from a reservoir A to the environment through pipes 1, 2 and 3 respectively. The EGL and HGL from A to D are shown. What do you think are at B, C and D that will yield the EGL and HGL shown? If the friction factor of pipes 1, 2 and 3 are identical, what additional information can be deduced from the given EGL and HGL? You may assume that all minor losses are negligible. Explain your answers.

(10 marks)

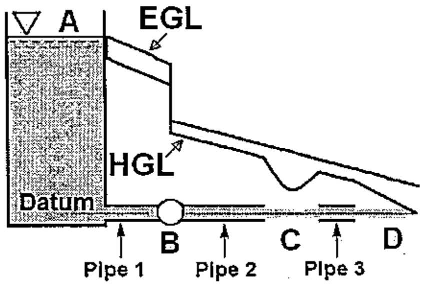

Figure 4

(b) Beer is delivered from a beer keg which is pressurized to a pressure P_A (gauge) as shown in Figure 5. The keg is connected to two taps D and E through pipes BCD and BCE respectively. The density of beer is 1000 kg/m^3. You may make use of the following information:

Description	Diameter (m)	Length (m)	Friction factor	ΣK_L
Pipe BC	0.01	3	0.01	3
Pipe CD	0.01	1	0.01	2
Pipe CE	0.01	2	0.01	3

The minor losses in the table include the valve loss coefficient when it is fully open. You may also assume that the elevations of taps D and E above A remain constant.

(i) When tap D is fully open and tap E is close, a 2 liter jug can be filled in 10 seconds. Determine the pressure P_A.

(8 marks)

(ii) If both taps are fully open, write down the governing equations to solve the flow rates in taps D and E if pressure P_A is maintained constant. You are not required to solve the equations. Based on the equations written, which tap will take a shorter time to fill a 2 liter jug? Explain your answers.

(7 marks)

Note: Question 3 continues on page 5.
Figure 5 appears on page 5.

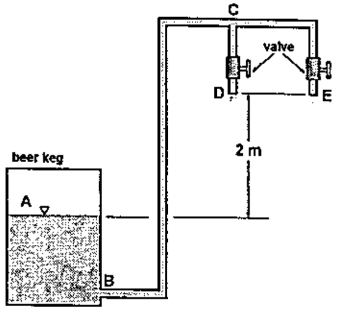

Figure 5

4 (a) A pump draws water from a pond and delivers it to a tank as shown in Figure 6. The elevations of the A, B and C are 0 m, 2 m and 42 m respectively. The fluid velocity in pipe 1 is 4 m/s and the pump characteristics is given as:

$$H_p = K - 200Q^2$$

The $NPSH_R$ of the pump is given by:

$$NPSH_R = 2 + 100Q^2$$

(i) Determine the total head loss for flow through the system
(ii) Determine the value of *K*.
(iii) What is the $NPSH_A$? Is the pump operating under cavitating condition?

You may make use of the following information.

Description	Diameter (m)	Length (m)	Friction factor	ΣK_L
Pipe 1	0.25	20	0.02	5
Pipe 2	0.2	100	0.025	10

The density of water is 1000 kg/m^3. The atmospheric pressure is 100 kPa and the vapour pressure of water is 2340 Pa and *NPSH* available is defined as

$$NPSH_A = \frac{P_s - P_v}{\rho g} + \frac{V_s^2}{2g}$$

(15 marks)

Note: Question 4 continues on page 6.
Figure 6 appears on page 6.

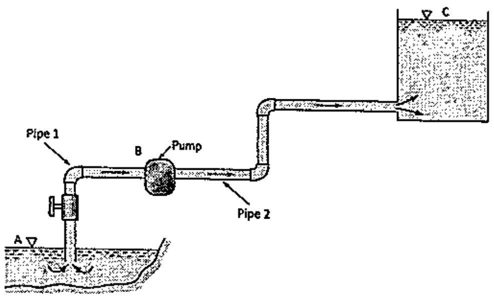

Figure 6

(b) The system demand curve for a pipe network is given as:

$$E = 50 + 500Q^2$$

and the pump characteristics is given as:

$$H_p = 200 - 100Q^2$$

(i) Determine the operating flow rate and the head developed across the pump when a single pump is used.

(2 marks)

(ii) When two identical pumps are connected in series, what is the operating flow rate and the head developed across a single pump?

(3 marks)

(iii) When two identical pumps are connected in parallel, what is the operating flow rate in the system and the head developed across a single pump?

(3 marks)

(iv) If two pumps are to be connected to the system, should a series or parallel arrangement be adopted? Explain your answers.

(2 marks)

END OF PAPER

April/May 2017

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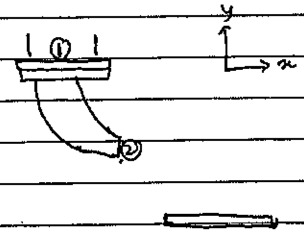
1a)



$Q_2 = 0.02\,m^3s^{-1}$, $D_2 = 0.05m$ Weight of nozzle $= 20kg$

$P_1 = 80000\,Pa$ (gauge), $D_1 = 0.1m$

By conservation of mass $A_1 = 0.00785398\,m^2$

$\dot{m}_1 = \dot{m}_2 = 0.02(1000) = 20kgs^{-1}$ $A_2 = 0.0019634954\,m^2$

Since flow is incompressible,

$Q_1 = Q_2 = 0.02\,m^3s^{-1}$

$V_1 = V_{1y} = 0.02 \div 0.00785398 = 2.5465ms^{-1}$

$V_2 = 0.02 \div 0.0019634954 = 10.1859ms^{-1}$

$V_{2x} = 10.1859\cos 30° = 8.82126ms^{-1}$

$V_{2y} = -10.1859\sin 30° = -5.0929582ms^{-1}$

$V_{1y} = -2.5465ms^{-1}$ (Taking ↑ as +ve)

$V_{1x} = 0ms^{-1}$

$\Sigma F_x = \dot{m}(V_{2x} - V_{1x}) = 20(8.82126 - 0) = 176.4252N$

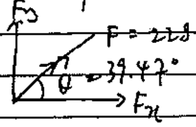
$\therefore F_x = 176.4252N$ (→)

$\Sigma F_y = \dot{m}(V_{2y} - V_{1y})$

$F_y - 20(9.81) = 20(-5.0929582 + 2.5465) = -50.929164N$

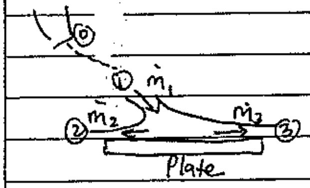
$F_y = 145.270836N$ (↑)

F = Resultant force $= [176.4252^2 + 145.270836^2]^{1/2} = 228.54N$,



$F = 228.54N$, $\theta = \tan^{-1}\left(\frac{145.270836}{176.4252}\right) = 39.47°$

1b)



$\dot{m}_3 = 0.7\dot{m}_1 = 14kgs^{-1}$ $\dot{m}_2 = 6kgs^{-1}$

Using Bernoulli Equation,

$$\cancel{\frac{P_0}{\rho g}} + \frac{V_0^2}{2g} + z_0 = \cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g} + \cancel{z_1}$$

$$\frac{10.1859^2}{2(9.81)} + 1.5 = \frac{V_1^2}{2(9.81)} \Rightarrow V_1 = 11.5405ms^{-1}$$

Using Bernoulli Equation from ① to ② $V_{1x} = 11.5404\cos 30° = 10mS^{-1}$

$$\cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g} + \cancel{z_1} = \cancel{z_2} + \frac{V_2^2}{2g} + \cancel{\frac{P_2}{\rho g}} \Rightarrow V_1 = V_2 = V_3 = 11.5405ms^{-1}$$

(likewise)

$F_x = \dot{m}_3(V_{3x} - V_{1x}) + \dot{m}_2(V_{2x} - V_{1x})$

$= 14(11.5405 - 10) + 6(-11.5405 - 10) = -107.676N$ (←)

Pls note! Question changed! → head loss

1c) Determine the flow direction (upwards / downwards) and ~~total pressure drop~~ head loss (in terms of velocity V_1 or V_2) between point 1 & point 2.

Using energy equation from ① to ②.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} - h_L = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$2 + \frac{V_1^2}{2g} - h_L = 1 + \frac{16V_1^2}{2g} + 1.5$$

$$-h_L = 0.5 + \frac{15V_1^2}{2g}$$

$$h_L = -0.5 - \frac{15V_1^2}{2g}$$ (Not possible as $h_L > 0$)

$Q_1 = Q_2$ (Conservation of mass)

$$V_1 A_1 = V_2 A_2$$

$$V_1 = V_2\left(\frac{A_2}{A_1}\right) = V_2\left(\frac{2^2}{4^2}\right) = V_2/4.$$

$$V_2 = 4V_1$$

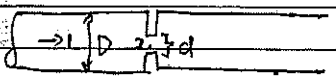
Hence flow is from ② to ①.

From ② to ①,

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = h_L + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1}$$

$$h_L = 1 - 2 + 1.5 + \frac{16V_1^2}{2g} - \frac{V_1^2}{2g} = 0.5 + \frac{15V_1^2}{2g} \,//$$

2. Using the orifice meter as an example.



Assume $z_1 = z_2$, Assume no loss of energy ⇒ $h_L = 0$.

Using Bernoulli's equation.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + 0 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{V_2^2}{2} - \frac{V_1^2}{2} = \frac{p_1}{\rho} - \frac{p_2}{\rho}$$

$$V_2^2\left(1 - \frac{V_1^2}{V_2^2}\right) = \frac{2(p_1 - p_2)}{\rho} \quad -①$$

subst. ② into ①

$$V_2^2(1 - \beta^4) = \frac{2(p_1 - p_2)}{\rho} \Rightarrow V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}} \,//$$

Continuity equation,

$$Q_1 = Q_2 \Rightarrow V_1 A_1 = V_2 A_2$$

$$\frac{V_1}{V_2} = \frac{d^2}{D^2}$$

$$\frac{V_1^2}{V_2^2} = \frac{d^4}{D^4} \quad ② \quad = \beta^4$$

$\therefore Q_{ideal} = A_2 V_2 = A_2 \sqrt{\dfrac{2(P_1-P_2)}{\rho(1-\beta^4)}}$

Since flow is not ideal, to account for that, a discharge coefficient is introduced for the case of orifice (subscript is 'o')

$\therefore Q_{actual} = C_o Q_{ideal} = C_o A_2 \sqrt{\dfrac{2(P_1-P_2)}{\rho(1-\beta^4)}}$

$A_2 = \dfrac{\pi\left(\frac{24.5}{1000}\right)^2}{4} = 4.7143\times10^{-4}\,m^2$

$Q_{actual} = (0.61)(4.714352476\times10^{-4})\sqrt{\dfrac{2(2500)}{900(1-0.7^4)}}$

$= 7.77566265\times10^{-4}\,m^3s^{-1}$

$\beta = \left(\dfrac{24.5}{35}\right) = 0.7$

$V_1 = \dfrac{Q_{actual}}{A_1} = 0.808186\,ms^{-1}$

$A_1 = \dfrac{\left(\frac{35}{1000}\right)^2\pi}{4} = 9.621127502\times10^{-4}\,m^2$

$Re = \dfrac{\rho V_1 D_1}{\mu} = \dfrac{(900)(0.808186)(0.035)}{0.38}$

$= 67$

Using Figure 3, $C_o \simeq 0.644$.

$Q_{actual} = 0.644\,(4.714352746\times10^{-4})\sqrt{\dfrac{2(2500)}{900(1-0.7^4)}}$

$= 8.209\times10^{-4}\,m^3s^{-1}$

~~$V_1 = 0.85323\,ms^{-1}$~~

$Re = (900)(0.85323)(0.035) \div 0.38 = 70.728$

From Figure 3, $C_o \doteq 0.644$.

$\therefore C_o = 0.644$ $Q_{expected} = 8.209\times10^{-4}\,m^3s^{-1}$

2b) $F_d = f(D, V, \rho, \mu, d, \omega)$.

7 variables

$m = 3$

No of π groups = 4 π groups

$\Pi_1 = F_d \rho^a V^b D^c \Rightarrow \Pi_1 = \dfrac{F_d}{\rho V^2 D^2}$

$\Pi_2 = \mu \rho^a V^b D^c \Rightarrow \Pi_2 = \dfrac{\mu}{\rho V D}$

$\Pi_3 = d \rho^a V^b D^c \Rightarrow \Pi_3 = \dfrac{d}{D}$

$\Pi_4 = \omega \rho^a V^b D^c$

$\Rightarrow \Pi_4 = \dfrac{\omega D}{V}$

$\left(\dfrac{\mu}{\rho V D}\right)_{model} = \left(\dfrac{\mu}{\rho V D}\right)_{prototype}$

$\dfrac{V_{prototype}}{V_{model}} = \dfrac{D_{model}}{D_{prototype}} \Rightarrow \dfrac{80}{40} = \dfrac{D_{model}}{D_{prototype}} = 2$

$\left(\frac{\omega D}{V}\right)_{model} = \left(\frac{\omega D}{V}\right)_{prototype}$

$\omega_{model} = \omega_{prototype}\left(\frac{D_{prototype}}{D_{model}}\right)\left(\frac{V_{model}}{V_{prototype}}\right) = \left(\frac{8500(2\pi)}{60}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= 222.53 \text{ rads}^{-1}$

2c) Geometric similarity indicates that the model & prototype must be of the same shape i.e. if the model is a golf ball (of spherical shape), the prototype must be of spherical shape too.

Dynamic similarity indicates the same flow conditions (boundary conditions) and is usually done by matching of dimensionless parameter such as the Froude no. or Reynolds' number in fluid ~~mechanics~~ mechanics.

3a) B – Turbine (since minor losses are neglected)
C – Venturi meter (Since HGL drops and rises again, cross section constricts & expands).
D – Nozzle (Since HGL ↓ ⇒ ↓ in cross-sectional area).

0 Slope of EGL & HGL = $h_L = f\frac{v^2}{2g}\frac{L}{D}$ if f is constant

⇒ Pipes 2 & 3 has the same pipe length over diameter ratio

⇒ Pipes 1 has a larger pipe to diameter ratio.

3b) $Q_D = \frac{2000\text{cm}^3}{10} = 200\text{cm}^3/\text{s}$

$= 200 \times 0.01^3 = 2\times10^{-4}\text{m}^3\text{s}^{-1}$ $\quad V_D = \frac{Q_D}{A_D} = \frac{2\times10^{-4}}{\frac{0.01^2}{4}\pi} = 2.54648\text{ms}^{-1}$

Using the energy equation

$\frac{P_A}{\rho g} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} - \text{Total head loss} = \cancel{\frac{P_D}{\rho g}} + \frac{V_D^2}{2g} + z_D$

$\frac{P_A}{\rho g} = f_{BC}\frac{L}{D}\frac{V_{BC}^2}{2g} + f_{CD}\frac{L}{D}\frac{V_{BC}^2}{2g} + \underset{(BC)}{\sum K_L\frac{V^2}{2g}} + \underset{(CD)}{\sum K_L\frac{V^2}{2g}} + \frac{V_D^2}{2g} + z_D$

$\frac{P_A}{\rho g} = (0.01)\left[\frac{4}{0.01}\right]\left[\frac{2.54648^2}{2(9.81)}\right] + 5\left(\frac{2.54648^2}{2(9.81)}\right) + \frac{2.54648^2}{2(9.81)} + 2$

$\frac{P_A}{\rho g} = 5.305076652 \Rightarrow P_A = 52042.80195 \text{ Pa (gauge)}$

3b(ii) Governing equations:

$$Q_B = Q_D + Q_E \quad \text{—①}$$

from A to D,

$$\frac{P_A}{\rho g} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} - \text{Total head loss} = \cancel{\frac{P_D}{\rho g}} + \frac{V_D^2}{2g} + z_D$$

$$\frac{P_A}{\rho g} = \text{Total head loss from BC} + \frac{V_D^2}{2g} + 2 + \overbrace{(0.01)\frac{L}{D}\frac{V_D^2}{2g} + 2\left(\frac{V_D^2}{2g}\right)}^{\text{Total loss in CD.}} \quad \text{—②}$$

from A to E.

$$\text{similarly} \quad \frac{P_A}{\rho g} = \text{Total head loss from BC} + \frac{V_E^2}{2g} + 2 + \underbrace{0.01\frac{L}{D}\frac{V_E^2}{2g} + 3\left(\frac{V_E^2}{2g}\right)}_{\text{Total loss in CE}} \quad \text{—③}$$

Comparing ② & ③,

$$\frac{P_A}{\rho g} = \text{Total head loss in BC} + \frac{V_D^2}{2g} + 2 + \frac{V_D^2}{2g} + 2\left(\frac{V_D^2}{2g}\right)$$

$$\frac{P_A}{\rho g} = \text{Total head loss in CE} + \frac{V_E^2}{2g} + 2 + \cancel{(0.01)}\left(\frac{2}{0.01}\right)\frac{V_E^2}{2g} + 3\left(\frac{V_E^2}{2g}\right)$$

$$\therefore \frac{P_A}{\rho g} = \text{Total head loss in BC} + 2 + 4\frac{V_D^2}{2g}$$

$$\frac{P_A}{\rho g} = \text{Total } \cancel{\text{loss}} \text{ head loss in CE} + 2 + 5\frac{V_E^2}{2g}$$

$\therefore V_E < V_D$.

Since ~~A_E~~ Diameter of CE < Diameter of CD.

$A_{CE} < A_{CD}$

$\Rightarrow Q_E < Q_D$.

$\therefore$ Tap D will take a shorter time to fill the jug.

4a) $Q_1 = Q_2 = 4\left(\frac{0.25^2}{4}\right)\pi = 0.1963495408\ m^3s^{-1}$

$$V_2 = \frac{Q_2}{A_2} = \frac{0.1963495408}{\pi\left(\frac{0.2^2}{4}\right)} = 6.25\ ms^{-1}$$

Total head loss = Head loss in Pipe 1 + Head loss in Pipe 2

$$= (0.02)\left(\frac{20}{0.25}\right)\left(\frac{4^2}{2(9.81)}\right) + 5\left(\frac{4^2}{2(9.81)}\right) + (0.025)\left(\frac{100}{0.2}\right)\left(\frac{6.25^2}{2(9.81)}\right) + 10\left(\frac{6.25^2}{2(9.81)}\right)$$

$$= 50.179\ m.$$

③

How I got this.

$$\frac{P_A}{\rho g}+\cancel{\frac{V_A^2}{2g}}+\cancel{z_A}-h_{L(\text{pipe}1)}=\frac{P_S}{\rho g}+\frac{V_S^2}{2g}+z_S$$

$$\frac{P_{atm}-P_v}{\rho g}+\Delta z-h_{L(\text{pipe}1)}=\frac{P_s-P_v}{\rho g}+\frac{V_s^2}{2g}$$

Using energy equation from A to C,

$$\cancel{\frac{P_A}{\rho g}}+\cancel{\frac{V_A^2}{2g}}+\cancel{z_A}+h_{TL}-h_p=\cancel{\frac{P_C}{\rho g}}+\cancel{\frac{V_C^2}{2g}}+z_C$$

$$h_{TL}=h_p+42$$

$$50.179+200Q^2-42=K$$

$$K=50.179+200(0.1963495^2)-42=15.89$$

(ii) $NPSH_A=\dfrac{P_s-P_v}{\rho g}+\dfrac{V_s^2}{2g}=\dfrac{P_{atm}-P_v}{\rho g}+\Delta z-$ Total head loss in pipe 1!

$$=\frac{100000-2340}{1000(9.81)}+2-5.3822622987.$$

$$=6.57289\,m.$$

~~AP~~ $NPSH_R=2+100Q^2=2+100(0.196350^2)=5.855\,m.$

$NPSH_A > NPSH_R$. Pump is not operating under cavitation.

b) $E=H_p \Rightarrow 50+500Q^2=200-100Q^2$

$$Q=\left(\frac{150}{600}\right)^{\frac{1}{2}}=0.5\,m^3s^{-1},\quad H_p=175\,m$$

bii) $E=2H_p \Rightarrow 50+500Q^2=400-200Q^2$

$$Q=0.707106\,m^3s^{-1},\quad H_p=150\,m$$ (across each pump)

biii) $E=H_{p(\text{new})}$ $\quad 50+500Q^2=200-100\left(\frac{Q}{2}\right)^2$

$$\therefore 50+500Q^2=200-25Q^2$$

$$Q=0.5345\,m^3s^{-1}$$

Head across each pump $=200-100\left(\dfrac{\cancel{0.535}\,0.5345}{2}\right)^2$

$=193\,m$

biv) For parallel arrangement, Total head generated $=193\,m$, $Q=0.5345\,m^3s^{-1}$

For series arrangement, Total head generated $=300\,m$, $Q=0.707106\,m^3s^{-1}$

Hence since total head generation (energy) & flow rate for the series arrangement is higher a series arrangement should be adopted.

ALL THE BEST FOR FINALS!

Note: When in doubt, look for your prof! THE END IS NEAR! 加油! After the paper can forget everything liao!

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

MA3006 – FLUID MECHANICS

November/December 2017 Time Allowed: $2^1/_2$ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL FOUR (4)** Questions.
3. All questions carry equal marks.
4. This is a **CLOSED BOOK** examination.

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1 (a) A water jet from a nozzle strikes a semi-hemispherical vane of a cart which is mounted on wheels as shown in Figure 1. The water leaves the 4 cm diameter nozzle exit at a velocity of 12 m/s. The water is diverted through 180^0 upon leaving the vane. The mass of the cart and the vane is 200kg.

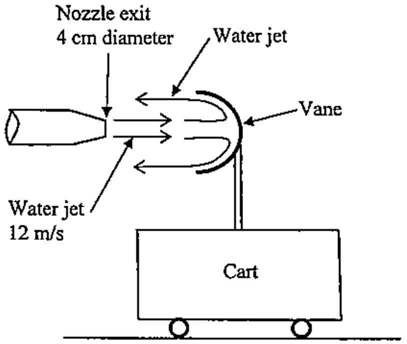

Figure 1

(i) Determine the applied force to prevent the cart from moving.

(ii) If the restraining force is removed, determine the time taken for the cart to move from rest to a velocity of 3 m/s.

The density of water is 1000 kg/m^3. Assume the flow is frictionless throughout and the weight of water in the vane to be negligible. Neglect any frictional forces between the wheels and the ground. Changes in height in the flow can also be neglected.

(13 marks)

Note: Question 1 continues on page 2.

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(b) Water from two dams flows into a turbine through two pipes which are then merged into a common pipe, just before entering the turbine as shown in Figure 2. The water leaving the turbine flows through a pipe which has an orifice plate installed at the pipe exit. At the 0.5 m diameter orifice exit, the water is discharged into the atmosphere. The discharge coefficient of the orifice is 0.62. At section 1, the water flows at 8 m/s through a 0.4 m diameter pipe at 300 kPa (absolute pressure) while at section 2, the water pressure in the 0.3 m diameter pipe is 400 kPa (absolute pressure). At section 3 which is just upstream of the orifice plate, the water pressure in the 0.8 m diameter pipe is 200 kPa (absolute pressure). The total power loss for flows between sections 1 and 2 to section 3 is 50 kW. Sections 1, 2 and 3 are at the same height. The atmospheric pressure is 100 kPa (absolute pressure) and the density of water is 1000 kg/m^3. Determine the turbine power output if the turbine efficiency is 80%.

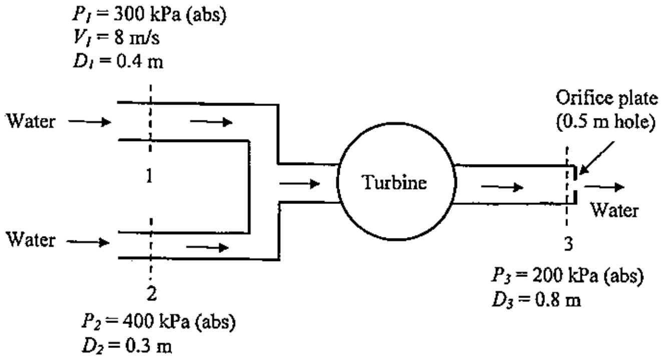


Figure 2

(12 marks)

2 (a) The lift force F_L of an aircraft depends on the aircraft speed V, the chord length of the wing l, the wing span S, the angle of attack α, the speed of sound c, the fluid density ρ and the dynamic viscosity μ of the fluid. Using dimensional analysis, derive the dimensionless groups and hence show their functional relationship.

(9 marks)

(b) A 1:20 scale model of an aircraft is tested in a wind tunnel where the temperature, density and dynamic viscosity of air are 20^0 C, 1.204 kg/m^3 and 1.82 $\times 10^{-5}$ N·s/m^2, respectively. The lift force of the model aircraft in the wind tunnel is found to be 1.25 kN. The prototype aircraft is flying at an altitude of 8,500 m above sea-level where the temperature, density and dynamic viscosity of air are -40^0 C, 1.514 kg/m^3 and 1.57 $\times 10^{-5}$ N·s/m^2, respectively. Using suitable dimensionless groups derived in part 2(a) above for dynamic similarity between the model and prototype aircrafts, determine the lift force of the prototype aircraft.

(6 marks)

(c) For steady laminar pipe flow, the local velocity in the pipe is given as:

$$u_r = V_c\left[1-\left(\frac{2r}{D}\right)^2\right]$$

where V_c is the centre-line velocity.

$$V_c = \left(\frac{\Delta p D^2}{16\mu l}\right)$$

Show that the volume flow rate is : $Q = \left(\frac{\pi D^4 \Delta p}{128\mu l}\right)$

The pipe diameter is 1 cm and the Reynolds number is 800, determine the centre-line velocity and the pressure gradient required for the flow

Density of fluid = 1050 kg/m^3 and dynamic viscosity is 0.00025 N.s/m^2.

(10 marks)

3 (a) Show that the velocity in a pipe can be expressed as:

$$V = \sqrt{\frac{2\Delta p}{f}\frac{d}{l}\frac{1}{\rho}}$$

where

Δp = pressure drop in the pipe
f = frictional factor of pipe
d = diameter of pipe
l = length between 2 points where pressure drop is measured
ρ = density of fluid

Water flows in a smooth pipe of diameter d = 0.01m. The pressure drop over a length of 10 m is 500 Pa. If the flow is laminar, determine the flow rate in the pipe. (Water: ρ = 1000 kg/m^3, and μ = 0.001 Ns/m^2)

(10 marks)

(b) Water from a pressurized tank is discharged to the atmosphere through a 100 mm diameter pipe and nozzle as shown in Figure 3. The total loss in the piping system, (including nozzle and other minor losses) is H_L=2.5V_1^2/2g.

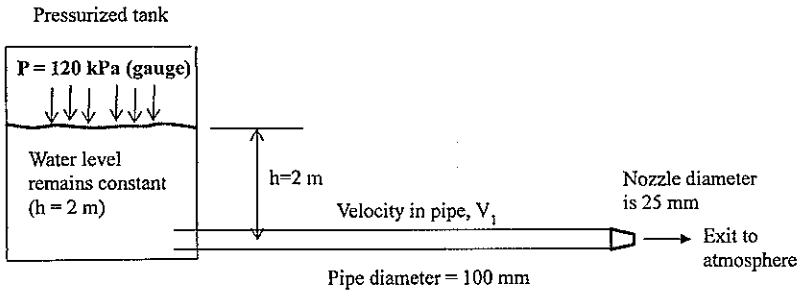


Figure 3

(i) Determine the volume flow rate in the system and velocity head at the nozzle outlet.

(ii) If the pressurized tank is to be replaced by a pump, determine the power input required by pump to produce the same volume flow rate. Total loss coefficient and h = 2 m remains unchanged. Pump efficiency is 70%.

(iii) Sketch the EGL and HGL for above (i) and (ii) and indicate clearly the pressure head at pipe inlet, velocity head at nozzle exit and pump head.

(15 marks)

4 (a) In a desalination plant, a mix-flow pump draws seawater from the coast to a storage tank as shown in Figure 4. The piping system comprises of the following:

Suction
Carbon steel 12" pipe, internal diameter = 305 mm
Overall length, inclusive of equivalent length for minor losses = 125 m
Frictional factor = 0.016

Discharge
Carbon steel 10" pipe, internal diameter = 250 mm
Overall length, inclusive of equivalent length for minor losses = 65 m
Frictional factor = 0.016

Show that the system characteristic can be expressed as:

$$E = (Z_2 - Z_1) + 0.08262\left(\frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5}\right)Q^2$$

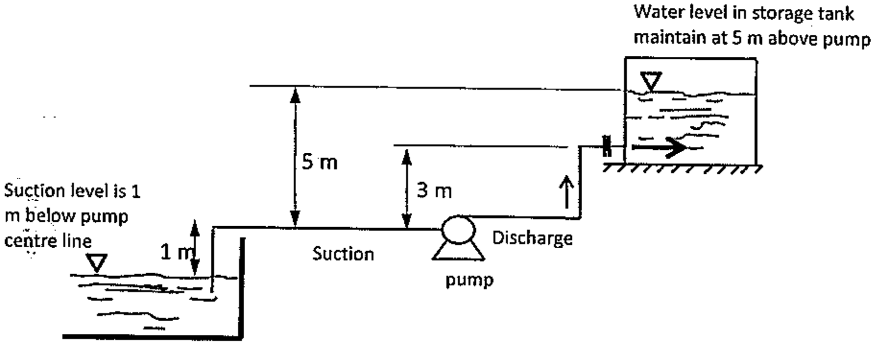


Figure 4

(4 marks)

Note: Question 4 continues on page 6.

where Z_2-Z_1 is the difference in elevation between suction and discharge surface (m)
f is the frictional factor of the pipe
l is length (m)
d is the internal diameter of pipe (m)
Q is the flow rate in m^3/s

(b) The pump characteristic, $NPSH_R$ and efficiency is given in the graph.

(i) Determine the operating flow rate and the power required by the pump. (8 marks)

(ii) Where in the piping system is cavitation most likely to occur? Explain. (2 marks)

(iii) Will cavitation occurs in the piping? (5 marks)

(iv) During low tide, the suction level is expected to drop. What is the limiting suction level for cavitation free operation? Is this pump suitable for this operation? (6 marks)

Given

$$NPSH_A = \frac{P_s - p_v}{\rho g} + \frac{V_s^2}{2g}$$

where P_s is pressure at pump suction,
p_v is vapour pressure of fluid,
V_s is velocity at pump suction,

Seawater
Density = 1020 kg/m^3
Dynamic viscosity = 0.00086 Ns/m^2
Vapour pressure of seawater = 2750 Pa
Atmospheric pressure is 100 kPa

END OF PAPER

Last Question is not answered as graph was not given

1ai) Nozzle Area $= \frac{\pi \times 0.04^2}{4} = 0.0012566\ m^2$

$\dot{m} = \rho V A$

$= 1000 \times 12 \times 0.0012566$

$= 15.0792\ kg/s$

$F_x = \dot{m}(V_{x\,out} - V_{x\,in})$

$= 15.0792(-12-12)$ (Taking → as +ve)

$= -361.9\ N$

$= 361.9\ N\ (\leftarrow)$.

Force of ground on cart $= 200 \times 9.81$

$= 1962\ N\ (\uparrow)$

ii) Relative velocity, $W = 12 - V$

$\dot{m} = \rho W A$

$= 1.2566\,W$

Net force on cart $= \dot{m}(W_{in} - W_{out})$

$= 1.2566\,W\,(W - (-W))$

$= 2.5132\,W^2$

$a = \frac{F_{net}}{m} = \frac{2.5132W^2}{200} = 0.012566\,W^2 = \frac{dV}{dt}$

$t = \int_0^3 \frac{1}{a}\,dV$

$= \int_0^3 \frac{1}{0.012566W^2}\,dV$.

$= \int_0^3 \frac{1}{0.012566(12-V)^2}\,dV$

$= \left[\frac{1}{0.012566(12-V)}\right]_0^3 = 2.21s$

b) $\beta = \frac{d}{D} = \frac{0.5}{0.8} = 0.625$

$Q = V_4 A_4 = C_0 Q_{ideal}$

$V_4 = C_0 \sqrt{\frac{2(p_3 - p_4)}{\rho(1-\beta^4)}} = 0.62 \times \sqrt{\frac{2(200\,000 - 100\,000)}{1000(1-0.625^4)}}$

$= 9.5249\ m/s$

$V_3 = V_4 \times \frac{A_4}{A_3}$

$= 9.5249 \times 0.625^2$

$= 3.7207\ m/s$

$\dot{m}_3 = \dot{m}_4 = 1000 \times 9.5249 \times \frac{\pi \times 0.5^2}{4}$

$= 1870.2\ kg/s$

$\dot{m}_1 = 1000 \times 8 \times \frac{\pi \times 0.4^2}{4} = 1005.3\ kg/s$

P(1)#

$\dot{m}_2 = \dot{m}_3 - \dot{m}_1$

$= 1870.2 - 1005.3$

$= 864.9\ \text{kg/s}$

$V_2 = \dfrac{864.9}{1000\left(\frac{\pi \times 0.3^2}{4}\right)} = 12.236\ \text{m/s}$

Energy Equation:

$$\sum_{in} \dot{m}\left(\frac{p}{\rho} + \frac{V^2}{2}\right) - \sum_{out} \dot{m}\left(\frac{p}{\rho} + \frac{V^2}{2}\right) = \dot{W}_{out} + \text{Power loss}$$

$$1005.3\left(\frac{300\,000}{1000} + \frac{8^2}{2}\right) + 864.9\left(\frac{400\,000}{1000} + \frac{12.236^2}{2}\right) - 1870.2\left(\frac{200\,000}{1000} + \frac{3.7207^2}{2}\right)$$

$= 333760 + 410706 - 386985 = 357481 \Rightarrow W_{out} = 357481 - 50000$

$= 307481$

Turbine output $= 307\,481 \times 80\%$

$= 245\,985\ \text{W} \approx 246\ \text{kW}$

2a) $[F_L] = \frac{ML}{T^2}$ $\quad [\alpha] = 1$

$[V] = \frac{L}{T}$ $\quad [c] = \frac{L}{T}$

$[l] = L$ $\quad [\rho] = \frac{M}{L^3}$

$[s] = L$ $\quad [\mu] = \frac{M}{LT}$

Repeating variables : ρ, V, l

$\pi_1 = F_L \rho^a V^b l^c$

$= \frac{ML}{T^2} \left[\frac{M}{L^3}\right]^a \left[\frac{L}{T}\right]^b [L]^c$

$a = -1,\ b = -2$

$L:\ 1 + 3 - 2 + c = 0 \Rightarrow c = -2$

$\pi_1 = \dfrac{F_L}{\rho V^2 l^2}$

$\pi_2 = \frac{s}{l},\ \pi_3 = \alpha,\ \pi_4 = \frac{c}{V}$

$\pi_5 = \mu \rho^a V^b l^c$

$= \frac{M}{LT} \left[\frac{M}{L^3}\right]^a \left[\frac{L}{T}\right]^b [L]^c$

$a = -1,\ b = -1$

$L:\ -1 + 3 - 1 + c = 0 \Rightarrow c = -1$

$\pi_5 = \dfrac{\mu}{\rho V l}$

$$\frac{F_L}{\rho V^2 l^2} = f\left(\frac{s}{l}, \alpha, \frac{c}{V}, \frac{\mu}{\rho V l}\right)$$

b) $\left(\dfrac{\mu}{\rho V l}\right)_m = \left(\dfrac{\mu}{\rho V l}\right)_p$

$$\frac{1.82 \times 10^{-5}}{1.204 \times V_m \times 1} = \frac{1.57 \times 10^{-5}}{1.514 \times V_p \times 20} \Rightarrow \frac{V_p}{V_m} = 0.0343$$

$$\left(\frac{F_L}{\rho V^2 \ell^2}\right)_m = \left(\frac{F_L}{\rho V^2 \ell^2}\right)_p$$

$$\frac{1.25\times10^3}{1.204\times1^2\times1^2} = \frac{(F_L)_p}{1.514\times0.0343^2\times20^2}$$

$$(F_L)_p = 740\,N$$

c) $Q = \int u_r \, dA$

$$= \int u_r \, 2\pi r dr$$

$$= 2\pi V_c \int_0^{\frac{D}{2}} \left[1 - \left(\frac{2r}{D}\right)^2\right] r \, dr$$

$$= 2\pi V_c \int_0^{\frac{D}{2}} r - \frac{4r^3}{D^2} \, dr$$

$$= 2\pi V_c \left[\frac{r^2}{2} - \frac{r^4}{D^2}\right]_0^{\frac{D}{2}}$$

$$= 2\pi V_c \left(\frac{D^2}{8} - \frac{D^2}{16}\right) = \frac{\pi V_c D^2}{8}$$

$$= \left(\frac{\pi D^2}{8}\right)\left(\frac{\Delta p D^2}{16\mu \ell}\right)$$

$$= \frac{\pi D^4 \Delta p}{128 \mu \ell} \quad \text{(shown)}$$

$D = 0.01m$

$Re = \frac{\rho V D}{\mu} = \frac{(1050)(V)(0.01)}{0.00025} = 800$

$V = 0.019048 \, m/s$

$V_c = 2V = 0.038095 \, m/s$

$\frac{\Delta p}{\ell} = \frac{16 \mu V_c}{D^2} = \frac{16(0.00025)(0.038095)}{0.01^2}$

$= 1.524 \, N/m^3$

3a) From energy equation,

$$\frac{p_1}{\rho g} - h_L = \frac{p_2}{\rho g}$$

$$h_L = f \frac{\ell}{d} \frac{V^2}{2g} = \frac{p_1 - p_2}{\rho g} = \frac{\Delta p}{\rho g} \Rightarrow V = \sqrt{\frac{2\Delta p}{f} \frac{d}{\ell} \frac{1}{\rho}} \quad \text{(shown)}$$

$f = \frac{64}{Re} = \frac{64\mu}{\rho V d}$

$V = \sqrt{\frac{2\Delta p \rho V d}{64\mu} \frac{d}{\ell \rho}}$

$\sqrt{V} = \sqrt{\frac{2(500)(0.01^2)}{64(0.001)(10)}}$

$V = 0.15625 \, m/s$

$Q = AV = \pi\left(\frac{0.01^2}{4}\right)(0.15625)$

$= 1.227 \, m^3/s$

P② #

b) i) $A_1 = \pi \times \frac{0.1^2}{4}$

$= 0.0078540 m^2$

$A_2 = \pi \times \frac{0.025^2}{4}$

$= 0.00049087 m^2$

$$\frac{P_0}{\rho g} + \cancel{\frac{V_0^2}{2g}} + z_0 - \frac{2.5V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$\frac{120\times10^3}{(1000)(9.81)} + 2 - \frac{2.5\left(\frac{Q}{0.0078540}\right)^2}{2(9.81)} = \frac{\left(\frac{Q}{0.00049087}\right)^2}{2(9.81)}$$

$$14.232 - 2065.7Q^2 = 211528Q^2$$

$$213594Q^2 = 14.232$$

$$Q = 0.0081628 m^3/s$$

Velocity head at outlet $= 211528 (0.0081628^2)$

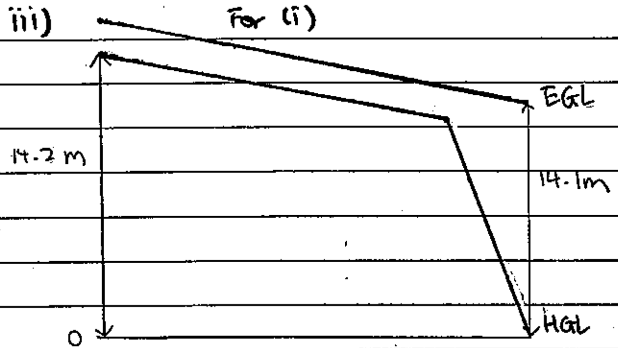
$= 14.1 m$

ii) Power input $= \frac{\rho g Q H}{\eta}$

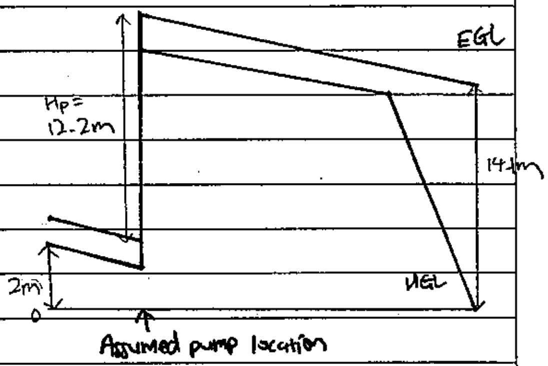
$$= \frac{(1000)(9.81)(0.0081628)(12.232)}{0.7}$$

$= 1399 W$

iii) For (i)



For (ii)



4a) $E = (z_2 - z_1) + \frac{f_1 l_1}{d_1}\frac{Q^2}{2gA_1^2} + \frac{f_2 l_2}{d_2}\frac{Q^2}{2gA_2^2}$

$= (z_2 - z_1) + \frac{f_1 l_1}{d_1}\frac{Q^2}{2g(\frac{\pi d_1^2}{4})^2} + \frac{f_2 l_2}{d_2}\frac{Q^2}{2g(\frac{\pi d_2^2}{4})^2}$

$= (z_2 - z_1) + \frac{1}{2g(\frac{\pi}{4})^2}\left[\frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5}\right]$

$= (z_2 - z_1) + 0.08262\left(\frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5}\right)$ (shown)

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

MA3006 – FLUID MECHANICS

April/May 2019 Time Allowed: $2^1/_2$ hours

INSTRUCTIONS

1. This paper contains **FOUR** (4) questions and comprises **SIX** (6) pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED BOOK** examination.

---

1 (a) A steady stream of water jet discharges from the nozzle with a uniform velocity of 10 m/s at angle 60° as shown in Figure 1. The jet strikes a horizontal plate 2 m above the nozzle exit. Upon impact, part of the flow is guided towards the left while the remainder towards the right, and the plate is suspended in the air. The weight of the plate is 25 N. Assume frictionless flow along the plate and weight of water to be neglected.

(i) Write down the momentum equation for x and y direction.

(ii) Determine the diameter of the nozzle (at the exit).

(iii) Determine the mass flow rate A and B.

(12 marks)

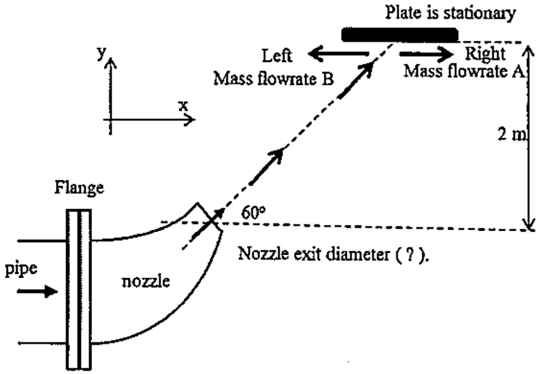


Figure 1

Note: Question 1 continues on page 2.

(b) Figure 2 shows a sprinkler system with 4 identical arms equally spaced. Water enters through the central collar and exits from the nozzles at radius 0.18 m. Each nozzle has a diameter of 0.008 m and water exits at an angle (θ) of 30 degree. The flow rate entering the sprinkler is 0.005 m^3/s

(i) develop an equation for the sprinkler torque and determine the angular speed ω when there is No resisting torque applied.

(ii) what is the torque required to reduce the angular speed to 10 radians per second.

(iii) with no resisting torque applied, show that when the nozzle exit angle (θ) is 90 degree, the sprinkler stops rotating.

(13 marks)

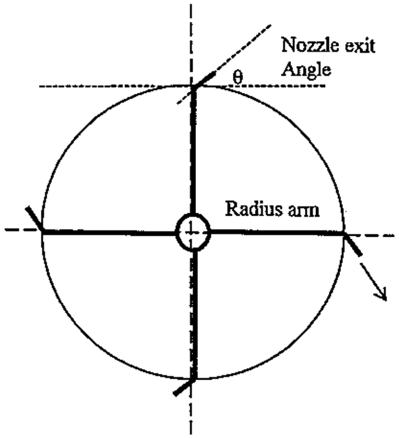


Figure 2

2 (a) An orifice plate is installed in a piping system to monitor the water flow rate. Develop a governing equation for volume flow rate as a function of pressure drop across the orifice plate and dimension of the orifice and pipe diameter.

The piping system carrying has a diameter of 80 mm with an orifice plate of diameter 48 mm is selected for monitoring volume flow rate. If the operating mass flow rate is 10 kg/s, determine the pressure difference across the orifice plate.

Dynamic viscosity of water, $\mu = 1.002 \times 10^{-3}$ Ns/m^2
Density of water, $\rho = 1000$ kg/m^3
Figure 3 shows the coefficient of discharge for an orifice plate.

(13 marks)

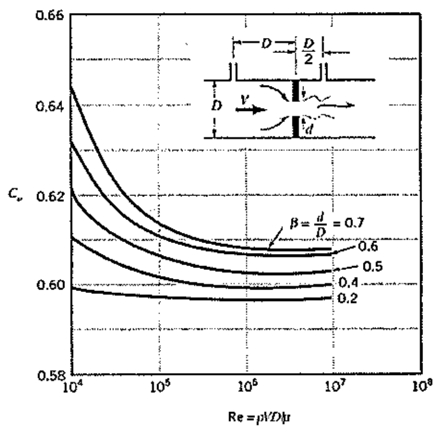


Figure 3

(b) A viscosity meter comprises of a rotating cylinder immersed in the testing fluid. The power W required to rotate the cylinder depends on diameter of the cylinder d, its length l, rotating speed ω, the fluid velocity V, the fluid density ρ and fluid dynamic viscosity μ.

Determine suitable dimensionless parameters relating to power W by using ρ, d, and ω as repeating variables.

(12 marks)

3 (a) A student conducted an experiment where a liquid of density 1200 kg/m^3 and dynamic viscosity 0.1 Ns/m^2 is passed through a horizontal pipe of diameter 0.1 m as shown in Figure 4. Given that the flow is laminar and the centerline velocity is 1 m/s, determine the length L if the manometer height is 0.2 m. The density of the manometer fluid is 1600 kg/m^3. Comment on the results obtained. If the flow is inviscid and the volume flow rate remains constant, do you expect the length L and the manometer height h to be different? Explain your answers. (You are not required to compute L and h).

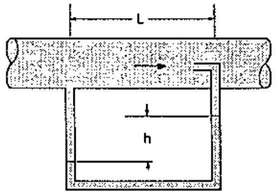


Figure 4

(12 marks)

(b) Water is pumped from reservoir A into reservoir B as shown in Figure 5 through smooth pipes 1 and 2. The elevations of the free surface of reservoirs A and B are equal and the pump characteristic is given as:

$$H_p = K - 2000Q^2 \quad \text{where } K \text{ is a constant}$$

If the volume flow rate into reservoir B is 0.02π m^3/s, determine the total frictional head loss and the constant K. You may make use of the following information:

Description	Diameter (m)	Length (m)	Friction factor	ΣK_L
Pipe 1	0.2	50	f	2
Pipe 2	0.2	400	f	5

For laminar flow, $f = 64/\text{Re}$
For turbulent flow in smooth pipe $f = 0.316/(\text{Re})^{0.25}$
The density of water is 1000 kg/m^3 and its dynamic viscosity is 0.001 Ns/m^2.

Sketch the EGL, taking into consideration of all frictional and minor losses. Indicate the location along the pipeline where cavitation is likely to occur and explain your answers.

If the pump is removed and the flow is inviscid, sketch the EGL for flow from A to B. Comment on your results.

Note: Figure 5 appears on page 5.

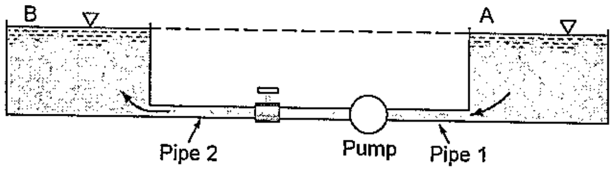


Figure 5

(13 marks)

4 (a) Water is pumped from reservoir A to reservoir B through a pipe of diameter D and length L. The diameter of the pump impeller is 0.5 m. The pump characteristic is given as:

$$H_p = 100 - 625Q^2$$

and its efficiency (in percent) is given as:

$$\eta = 600Q - 3750Q^3$$

Given that the system demand is $E = 60 + 500Q^2$

(i) Determine the difference in elevation between the free surfaces of reservoirs A and B.

(ii) If the rotational speed of the pump impeller is 2000 rpm, use Figure 6 to select the correct type of pump.

(iii) Determine the pump input power.

(iv) If 2 pumps are connected in parallel, determine the pump head, volume flow rate and the input power of each pump.

The pump specific speed N_s is given as $\quad N_s = \dfrac{\omega\sqrt{Q}}{(gh_p)^{3/4}}$

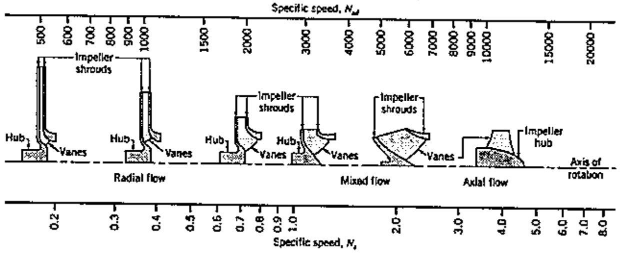


Figure 6

(13 marks)

Note: Question 4 continues on page 6.

(b) Water is drawn from a pond A to a tank B as shown in Figure 7. The volume flow rate is 0.1 m^3/s. The diameter of the suction pipe is 0.2 m and the head loss in the suction pipe is 2 m. Given that the NPSH required at this flow rate is 5 m, determine the minimum allowable pressure at the pump inlet and the maximum height of the pump above the free surface of the pond to avoid cavitation. The atmospheric pressure is 100kPa and the vapour pressure is 2340 Pa.

An identical pump with the same pipe network is to be installed at an altitude Z m above the sea level. The entire pipe network arrangement is the same as shown in Figure 7. The volume flow rate, head loss in the suction pipe and the NPSH required are assumed to remain constant. It is noticed that the maximum possible height of the pump above the free surface of the pond is reduced to 1.6 m. Determine the altitude Z where this pump network is installed. The atmospheric pressure at an altitude Z m is P_{atm} and the vapour pressure is 2340 Pa.

The density of water is 1000 kg/m^3. *NPSH* available is defined as

$$NPSH_A = \frac{P_s - P_v}{\rho g} + \frac{V_s^2}{2g}$$

The atmospheric pressure at an altitude Z m above the sea level is given by the expression below:

$$P_{atm} = 100\left[1 - \left(\frac{0.0065}{288}\right)Z\right]^{5.2586} \text{ kPa}$$

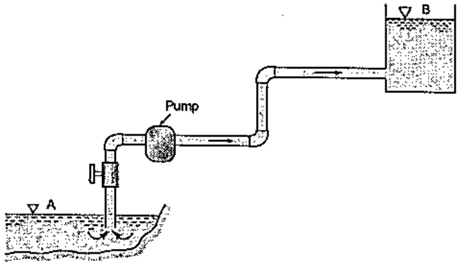
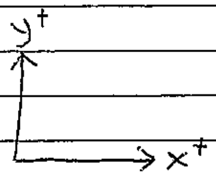


Figure 7

(12 marks)

END OF PAPER

1a) i) $\Sigma F_x = \Sigma(\dot m \vec V)_{out} - \Sigma(\dot m \vec V)_{in}$



$0 = (\dot m_A - \dot m_B)V - \dot m \cos 60^\circ V$

$\dot m_A - \dot m_B = \frac{1}{2}\dot m$

$\dot m = 2(\dot m_A - \dot m_B) \ldots (1)$

$\Sigma F_y = \Sigma(\dot m \vec V)_{out} - \Sigma(\dot m \vec V)_{in}$

$-W = 0 - \dot m V \sin 60^\circ$

$\dot m V \cdot \frac{1}{2}\sqrt{3} = W \ldots (2)$

ii) From (2),

$\dot m V \cdot \frac{1}{2}\sqrt{3} = W$

$\rho \cdot A \cdot V^2 \cdot \frac{1}{2}\sqrt{3} = W$

$\rho \cdot \frac{\pi}{4} \cdot d^2 \cdot V^2 \cdot \frac{1}{2}\sqrt{3} = W$

$d = \frac{2}{V}\sqrt{\frac{2W}{\pi\rho\sqrt{3}}} = \frac{2}{10} \cdot \sqrt{\frac{2 \cdot 25}{\pi \cdot 10^3 \sqrt{3}}} = 0.0192\,m$

iii) $\dot m = \dot m_A + \dot m_B \ldots (3)$

Subs (1), (3)

$\dot m_A + \dot m_B = 2\dot m_A - 2\dot m_B$

$\dot m_A = 3\dot m_B \ldots (4)$

Subs (1), (4), (2)

$\frac{2W}{V\sqrt{3}} = 4\dot m_B$

$\dot m_B = \frac{W}{2V\sqrt{3}} = \frac{25}{20\sqrt{3}} = 0.722\ Kg/s$

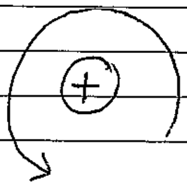
$\dot m_A = \frac{3W}{2V\sqrt{3}} = \frac{3 \cdot 25}{20\sqrt{3}} = 2.165\ Kg/s$

1b) i) $\Sigma \tau = \Sigma(\dot{m}(\vec{r} \times \vec{V}))_{out} - \Sigma(\dot{m}(\vec{r} \times \vec{V}))_{in}$

$0 = \Sigma(\dot{m}(\vec{r} \times \vec{V}))_{out} - 0$

$\cos 30^{\circ} \cdot V_{out} = V_{cv}$

$\frac{1}{2}\sqrt{3} \cdot \frac{\frac{1}{4}Q}{\frac{1}{4}\pi d^2} = \omega_{cv} r$



$\omega_{cv} = \frac{Q}{\pi r d^2} \cdot \frac{\sqrt{3}}{2} = \frac{0.005\sqrt{3}}{2\pi \cdot 0.18 \cdot 0.008^2} = 119.646 \text{ rad/s}$

ii) $\Sigma \tau = \Sigma(\dot{m}(\vec{r} \times \vec{V}))_{out} - \Sigma(\dot{m}(\vec{r} \times \vec{V}))_{in}$

$\tau = \dot{m} r (V \cos 30^{\circ} - \omega_{cv} \cdot r)$

$= 5 \cdot 0.18 \left(\frac{0.005\sqrt{3}}{2 \cdot \pi \cdot 0.008^2} - 10 \cdot 0.18 \right) = 17.76 \text{ Nm}$

iii) $\Sigma \tau = \Sigma(\dot{m}(\vec{r} \times \vec{V}))_{out} - \Sigma(\dot{m}(\vec{r} \times \vec{V}))_{in}$

$0 = \Sigma(\dot{m}(\vec{r} \times \vec{V}))_{out} - 0$

$0 = r(V \cos 90^{\circ} - \omega_{cv} \cdot r)$

$0 = \omega_{cv} \cdot r$

$\omega_{cv} = 0 \text{ rad/s}$

2a) $P_1+\rho g z_1+\frac{1}{2}\rho V_1^2 = P_2+\rho g z_2+\frac{1}{2}\rho V_2^2$

$P_1 + 0 + \frac{1}{2}\rho V_1^2 = P_2 + 0 + \frac{1}{2}\rho V_2^2$

$\frac{1}{2}\cdot\rho\,(V_2^2-V_1^2) = P_1 - P_2$

$\frac{1}{2}\cdot\rho\cdot Q^2\cdot\left(\frac{1}{(\frac{\pi}{4}\cdot d^2)^2}-\frac{1}{(\frac{\pi}{4}\cdot D^2)^2}\right) = P_1 - P_2$

$\frac{8}{\pi^2}\cdot\rho\cdot\frac{Q^2}{d^4}\cdot\left(1-\frac{d^4}{D^4}\right) = P_1 - P_2$

$Q = \frac{\pi d^2}{4}\sqrt{\frac{2\,(P_1-P_2)}{\rho\left(1-\frac{d^4}{D^4}\right)}}$

$\beta = \frac{d}{D} = \frac{48}{80} = 0.6$

$Re = \frac{\rho\cdot V_{actual}\cdot D_{inlet}}{\mu} = \frac{\rho\cdot\frac{Q_{actual}}{\frac{1}{4}\pi D^2}\cdot D}{\mu} = \frac{4\dot{m}}{\mu\cdot\pi D} = \frac{4\cdot 10}{1.002\cdot 10^{-3}\cdot\pi\cdot 0.08}$

$= 1.588\cdot 10^5$

$C_n = f(\beta, Re)$

$= 0.61$

$Q_{actual} = C_n\cdot Q_{ideal}$

$= C_n\cdot\frac{\pi d^2}{4}\sqrt{\frac{2\cdot\Delta P}{\rho\cdot(1-\beta^4)}}$

$0.01 = 0.61\cdot\frac{\pi\cdot 0.048^2}{4}\sqrt{\frac{2\cdot\Delta P}{10^3\cdot(1-0.6^4)}}$

$\Delta P = 35.717\ kPa$

2b) $W = f(d, l, \omega, V, \rho, \mu)$

$d = L \qquad V = LT^{-1} \qquad W = ML^2T^{-3}$

$l = L \qquad \rho = ML^{-3}$

$\omega = T^{-1} \qquad \mu = ML^{-1}T^{-1}$

$\Pi_1 = W \cdot (\rho)^a (d)^b (\omega)^c$

$= ML^2T^{-3} (ML^{-3})^a (L)^b (T^{-1})^c$

$a = -1$

$b = -5$

$c = -3$

$\Pi_2 = l \cdot (\rho)^a (d)^b (\omega)^c$

$= L \ (ML^{-3})^a (L)^b (T^{-1})^c$

$a = 0$

$b = 1$

$c = 0$

$\Pi_3 = V \ (\rho)^a (d)^b (\omega)^c$

$= LT^{-1} (ML^{-3})^a (L)^b (T^{-1})^c$

$a = 0$

$b = -1$

$c = -1$

$\Pi_4 = \mu \ (\rho)^a (d)^b (\omega)^c$

$= ML^{-1}T^{-1} (ML^{-3})^a (L)^b (T^{-1})^c$

$a = -1$

$b = -2$

$c = -1$

$$\frac{W}{\rho d^5 \omega^3} = f\left(\frac{l}{d}, \frac{V}{\omega d}, \frac{\mu}{\rho \omega d^2}\right)$$

3a) $P_1 + gh(\rho_l - \rho_m) = P_2$

$P_1 - P_2 = \Delta P = -9.81 \cdot 0.2\,(1200 - 1600)$

$= 784.532$ Pa

$$V_c = \frac{\Delta P D^2}{16 \mu l}$$

$$l = \frac{\Delta P D^2}{16 \mu V_c} = \frac{784.532 \cdot 0.01}{16 \cdot 0.1 \cdot 1} = 4.903 \text{ m}$$

For every 4.903 m along the pipe, there will be a pressure loss with an amount of 0.2m of the manometer fluid due to the viscous flow.

If the flow is to be inviscid, Bernoulli's principles will be applicable. Thus, the manometer height, h, will be 0 since there will not be any pressure loss along the pipe. L will be fixed if the manometer is installed in the same position.

3b) $P_A + \rho g h_A + \frac{1}{2}\rho V_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho V_B^2 - \rho g H_p + \rho g\,(h_{L1} + h_{L2})$

$$0 + 0 + 0 = 0 + 0 + 0 - H_p + h_{L1} + h_{L2}$$

$$H_p = f_1 \cdot \frac{l_1}{d_1} \cdot \frac{V_1^2}{2g} + f_2 \cdot \frac{l_2}{d_2} \cdot \frac{V_2^2}{2g} + K_{L1} \cdot \frac{V_1^2}{2g} + K_{L2} \cdot \frac{V_2^2}{2g}$$

$$Re = \frac{\rho V D}{\mu} = \frac{10^3 \cdot \frac{Q}{\frac{\pi}{4}D}}{\mu} = \frac{10^3 \cdot \frac{0.08}{0.2}}{10^{-3}} = 400000 \text{ (Turbulent)}$$

$$\text{Total head loss} = \frac{f}{d} \cdot \frac{V^2}{2g}\left(l_1 + l_2\right) + \frac{V^2}{2g}(K_1 + K_2)$$

$$= \frac{V^2}{2g}\left[\frac{f}{d}(l_1 + l_2) + K_1 + K_2\right]$$

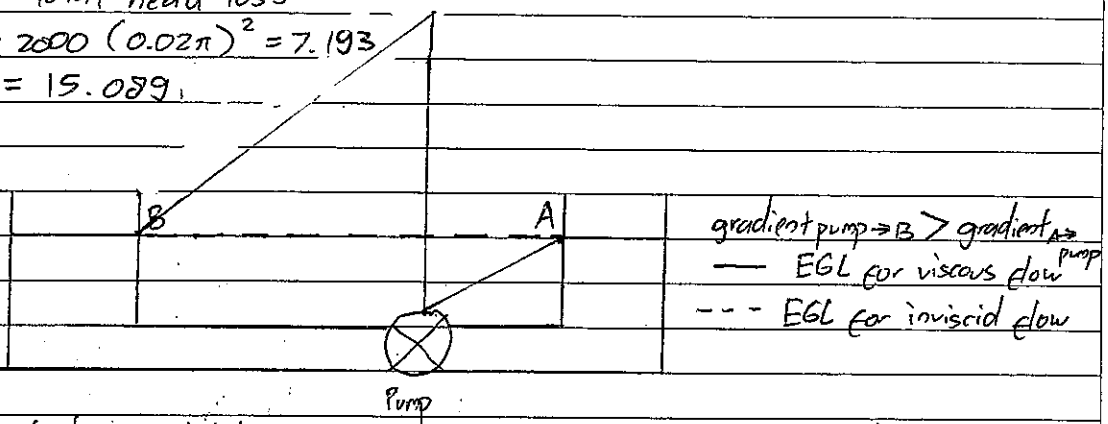
$$= \frac{Q^2}{2\left(\frac{\pi}{4}D^2\right)^2 g}\left[\frac{f}{d}(l_1 + l_2) + K_1 + K_2\right] =$$

$$= \frac{0.02^2}{2 \cdot 9.81\left(\frac{1}{4} \cdot 0.2^2\right)^2}\left[\frac{0.316}{0.2 \cdot (4 \cdot 10^5)^{\frac{1}{4}}}(400 + 50) + 2 + 5\right] = 7.193 \text{ m}$$

H_p = Total head loss

$$K - 2000\,(0.02\pi)^2 = 7.193$$

$$K = 15.089$$



Cavitation is likely most to occur at at point just right before the pump because the flow will have the lowest pressure head at that point for this system.

If the pump is removed and the flow is inviscid, there will be no pressure at any points in between A and B. Hence, the EGL is a straight line.

4a) i) $E = 60 + 500Q^2$

$h_2 - h_1 + \frac{1}{2gA^2}\left(\frac{f\ell}{D} + \Sigma k\right)Q^2 = 60 + 500Q^2$

$h_2 - h_1 = 60\ m$

ii) $H_p = E$

$100 - 625Q^2 = 60 + 500Q^2$

$Q = 0.189\ m^3/s$

$N_s = \frac{\omega\sqrt{Q}}{(gh_p)^{3/4}} = \frac{2000 \cdot 2\pi \cdot \frac{1}{60} \cdot \sqrt{0.189}}{(9.81 \cdot (100 - 625 \cdot 0.189^2))^{3/4}} = 0.627$ (radial or centrifugal pump)

iii) $P_{in} = \frac{P_{out}}{\eta} = \frac{\rho Q g H_p}{\eta} = \frac{10^3 \cdot 0.189 \cdot 9.81 \cdot (100 - 625 \cdot 0.189^2)}{0.01(600 - 3750 \cdot 0.189^2)0.189} = 163.44\ kW$

iv) $E = H_p$

$60 + 500(2Q)^2 = 100 - 625Q^2$

$Q = 0.123\ m^3/s$

$H_p = 100 - 625Q^2 = 100 - 625(0.123)^2 = 90.476\ m$

$P_{in} = \frac{P_{out}}{\eta} = \frac{\rho Q g H_p}{\eta} = \frac{10^3 \cdot 0.123 \cdot 9.81(100 - 625 \cdot 0.123^2)}{0.01 \cdot 0.123(600 - 3750 \cdot 0.123^2)} = 163.44\ kW$

4b) $P_A + \rho g h_A + \frac{1}{2}\rho V_A^2 = P_s + \rho g h_s + \frac{1}{2}\rho V_s^2$

$-P_v + P_A + 0 + 0 = P_s + \rho g h_s + \frac{1}{2}\rho V_s^2 - P_v - \rho g h_L$

$$\frac{P_A - P_v}{\rho g} = \frac{P_s - P_v}{\rho g} + \frac{V_s^2}{2g} + h_s - h_L$$

$$-h_L + \frac{P_A - P_v}{\rho g} - h_s = NPSH_A$$

$$-2 + \frac{10^5 - 2340}{10^3 \cdot 9.81} - h_s = 5$$

$h_s = 2.959 \ m$

$$NPSH_A = \frac{P_s - P_v}{\rho g} + \frac{V_s^2}{2g}$$

$$5 = \frac{P_s - 2340}{10^3 \cdot 9.81} + \frac{0.01}{(\frac{\pi}{4} \cdot 0.2^2)^2 \cdot 2 \cdot 9.81}$$

$P_s = 46307.19 \ Pa$

$$NPSH = \frac{P_A - P_v}{\rho g} - h_s - h_L$$

$$5 = \frac{P_{atm} - 2340}{10^3 \cdot 9.81} - 1.6 - 2$$

$P_{atm} = 86677.19 \ Pa$

$$10^5 \left[1 - \left(\frac{0.0665}{288}\right) z\right]^{5.2586} = 86677.19$$

$z = 116.167 \ m$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

MA3006 – FLUID MECHANICS

November/December 2019

Time Allowed: $2^1/_2$ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **CLOSED BOOK** examination.

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1 (a) Figure 1 shows a water jet discharging from a nozzle at 12 m/s. The nozzle is inclined at an angle of 30 degrees to the horizontal and has a diameter of 0.008 m. The water jet strikes a plate inclined at an angle of 60 degrees to the horizontal.
The height difference between the nozzle and the plate is negligible. The height difference between the plate ends is also negligible and the flow is assumed frictionless. The weight of the plate can be neglected.

(i) Determine the force F required to hold the inclined plate stationary

(ii) If the plate is vertical, determine the force F required to hold the plate stationary and calculate the mass flow rates of water, m_2 and m_3.

(iii) What is the power required to move the vertical plate horizontally towards the water jet at a steady speed of 3 m/s?

(20 marks)

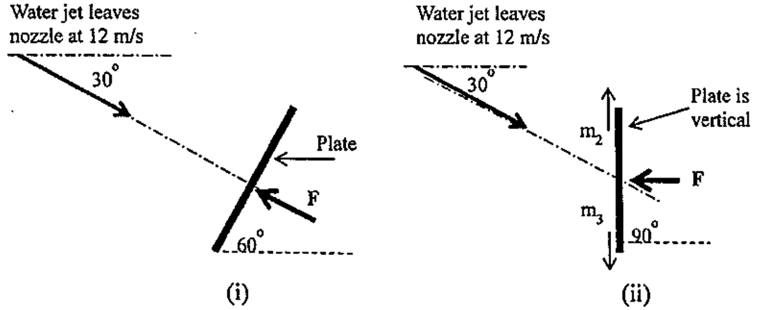

Figure 1

Note: Question 1 continues on page 2.

(b) Water flows from a smaller to a larger diameter pipe as shown in Figure 2. The volume flow rate is 0.002 m^3/s and head loss between point 1 to point 2 is 0.22 m. Determine the pressure head difference, h.

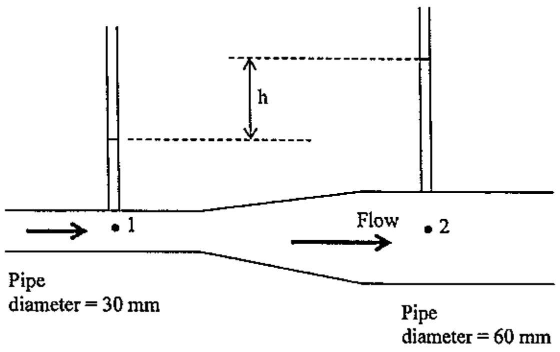

Figure 2

(5 marks)

2 (a) An orifice plate of diameter d is installed in a pipe of diameter D to monitor the volume flow rate.
Assuming ideal flow, derive an expression for the pressure difference between upstream pressure (P_1) in the pipe and the pressure (P_2) at the orifice in terms of the following:

Q_{ideal} = ideal flow rate,
A_2 = area at orifice,
ρ = density of fluid, and
$\beta = \dfrac{d}{D}$.

An orifice plate is assembled in a piping system to monitor the water flow rate. The diameter of the pipe is 80 mm and the orifice plate diameter is 56 mm. If the velocity of water in the pipe is 1.8 m/s, what is the pressure difference between the upstream pressure (P_1) and the pressure at orifice (P_2)?
Dynamic viscosity of water, $\mu = 1.002 \times 10^{-3}$ Ns/m^2
Density of water, $\rho = 1000$ kg/m^3
Figure 3 shows the coefficient of discharge for an orifice plate.

(12 marks)

Note: Question 2 continues on page 3
Figure 3 appears on page 3.

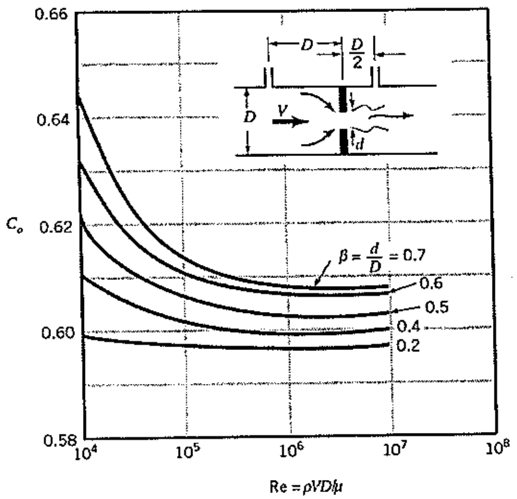

Figure 3

(b) The pressure rise Δp across a fan blade depends on the rotor rotational speed ω, the external diameter of fan D, the volume flow rate Q, the density ρ and dynamic viscosity μ of the fluid. Using dimensional analysis, determine suitable dimensionless groups.
From the established dimensionless group, derive the power coefficient.
You may use ρ, ω and D as repeating variables.

(10 marks)

(c) The external diameter of a prototype blower fan operating at 500 rpm is 0.8m. The power of the blower fan is 200 W and delivers a volume flow rate of 1.5 m^3/s. A geometrical similar model fan which is 80% smaller than the prototype is used for model testing.
What should be the speed of the model fan to achieve dynamic similarity?

(3 marks)

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3 (a) A student conducted an experiment whereby fluids A and B having the same volume flow rate, flowed through two identical horizontal pipes 1 and 2 respectively as shown in Figure 4. If the diameter of each pipe is 0.1 m and the flow rate through each pipe is 0.01 m^3/s, determine the ratio of the centerline velocities, V_c, in pipe 2 to that in pipe 1. Comment on the results obtained with the aid of sketches, if necessary.

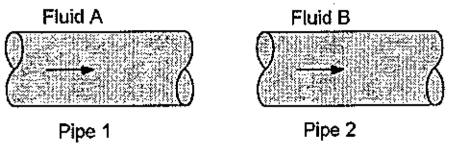

Figure 4

The properties of the fluids are given below:

Fluid	Density (kg/m^3)	Dynamic viscosity (kg/m.s)
A	800	0.1
B	7	0.00001

The expressions for volume flow rate are:

Laminar flow: $Q = \pi R^2 V_c / 2$, and

Turbulent flow: $Q = 2\pi R^2 V_c \dfrac{n^2}{(n+1)(2n+1)}$

where n is a constant to be determined from Figure 5.

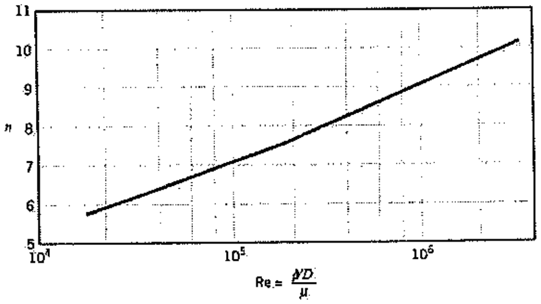

Figure 5

(10 marks)

(b) Water flows from reservoirs A and B into reservoir C as shown in Figure 6. The elevations of the free surface of reservoirs A, B and C are 40 m, 45 m and 0 m respectively. If the volume flow rates from reservoirs A and B are each equal to 0.2 m^3/s, determine the diameter D_3 and the length L_2. All minor losses are neglected. Which pipe has the largest frictional head loss per unit length? Information on the pipes are given below.

Note: Question 3 continues on page 5.
Figure 6 appears on page 5.

Description	Diameter (m)	Length (m)	Friction factor	ΣK_L
Pipe 1	0.5	1000	0.02	0
Pipe 2	0.4	L_2	0.02	0
Pipe 3	D_3	500	0.02	0

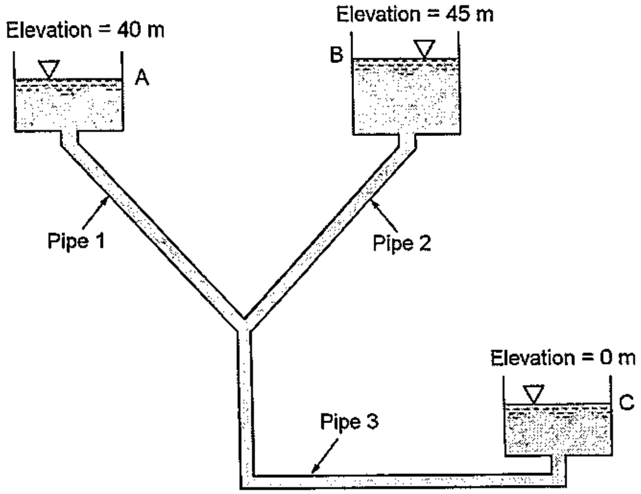

Figure 6

(15 marks)

4 Water is pumped from a closed tank A to reservoir C as shown in Figure 7. The elevations of the free surface of A and C are 0 m and 50 m respectively. The air pressure in the closed tank is 200 kPa (gauge) and the pump characteristics is given as:

$$H_p = 100 - 400Q^2$$

where H_p is the pump head and Q is the volume flow rate.

The *NPSH* required is given by: $NPSH_R = 3 + 100Q^2$

(i) Determine the volume flow rate through the system.

(ii) If the rotational speed of the pump impeller is 2000 rpm, what type of pump would you recommend?

(iii) Given that the elevation of point B is 70 m, do you think cavitation will occur at point B when the volume rate is that obtained in part (i)? Explain your answer.

Note: Question 4 continues on page 6.
Figure 7 appears on page 6.

(iv) On a certain day, the air pressure in the closed tank drops to -80 kPa (gauge) and the corresponding flow rate in the system is 0.18 m^3/s. Given that the pump is located 2 m below the free surface of closed tank A, determine the *NPSH* available. Do you think cavitation will occur at the pump inlet? Explain your answer.

(v) Are there any differences between the cavitations that may occur in part (iii) and (iv)? Explain your answer.

Information on the pipes are given below.

Description	Diameter (m)	Length (m)	Friction factor	ΣK_L
Pipe 1	0.25	10	0.02	0
Pipe 2	0.25	240	0.02	4
Pipe 3	0.25	200	0.02	2

The density of water is 1000 kg/m^3. The atmospheric pressure is 100 kPa and the vapour pressure (p_v) of water is 2340 Pa and *NPSH* available is defined as

$$NPSH_A = \frac{P_s - p_v}{\rho g} + \frac{V_s^2}{2g}$$

where P_s is the pump suction pressure and V_s is the flow velocity in the pipe.

The pump specific speed N_s is given as $$N_s = \frac{\omega\sqrt{Q}}{(gH_p)^{3/4}}$$

where ω is the pump rotational speed.

For $N_s \le 1$,	Centrifugal pump
For $1 < N_s < 4$,	Mixed flow pump
For $N_s \ge 4$,	Axial flow pump

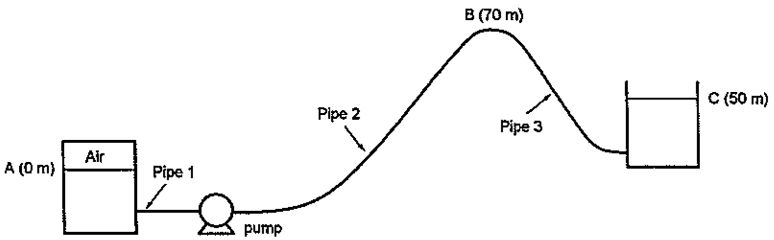

Figure 7

(25 marks)

END OF PAPER

1\. a. i. $\Sigma F = \dot{m}(V_{out} - V_{in})$ $\qquad V_1 = V_2 = 12 m/s$ (bernoulli)

$V_1 = 12 m/s$, $\theta = 30°$

$d = 0.008 m$

$Q = AV = \frac{\pi(0.008)^2}{4}(12)$

$= 0.000603 m^3/s$

$\dot{m} = \rho Q = 1000(0.000603)$

$= 0.6032 kg/s$

$\Sigma F_x = \dot{m}(V_{x2} - V_{x1})$

$= 0.6032(0 - 12\cos 30°)$

$= -6.269N$

$\Sigma F_y = \dot{m}(V_{y2} - V_{y1})$

$= 0.6032(0 - 12\sin 30°)$

$= -3.6192N$

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-6.269)^2 + (-3.6192)^2}$

$= 52.4N$

ii. $\Sigma F_x = \dot{m}(V_{x2} - V_{x1})$

$= 0.6032(0 - 12\cos 30°)$

$= -6.259N$

$F = -F_x$

$= 6.259N$

$\Sigma F_y = 0$

$\Sigma F_y = \dot{m}(V_{y2} - V_{y1})$

$0 = \dot{m}_2(12\cos 60° - 12\sin 30°) + \dot{m}_3(12\cos 120° - 12\sin 30°)$

$0 = -7\dot{m}_3^2$

$\dot{m}_3 = 0 kg/s$

$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

$\dot{m}_2 = \dot{m}_1 = 0.6032 kg/s$

iii. relative speed of CV = $12\cos 30° - 3$

$W_1 = 7.39 m/s$

$W_1 = W_2 = W_3 = 7.39 m/s$

$\dot{m}_1 = 1000\left(\frac{\pi(0.008)^2}{4}\right)(7.39)$

$\approx 0.371 kg/s$

$P = F.V = 0.6032(0.739\cos 30°)(7.39)$

$= 28.53 W$

b. $Q = 0.002\ m^3/s$ $\qquad V_1 = 0.002 \div \left(\frac{\pi(0.03)^2}{4}\right)$

$h_L = 0.22m$ $\qquad = 2.83 m/s$ $\qquad Z_1 = Z_2$

$D_1 = 30mm = 0.03m$ $\qquad V_2 = 0.02 \div \left(\frac{\pi(0.06)^2}{4}\right)$

$D_2 = 60mm = 0.06m$ $\qquad = 0.707 m/s$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1}^{0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}^{0}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\text{pressure head} = \frac{P_1 - P_2}{\rho g} = \frac{0.707^2 - 2.83^2}{2(9.81)}$$

$$= -0.383m$$

2. a $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}$

$$\frac{V_2^2}{2g} = \frac{P_1 - P_2}{\rho g} + \frac{V_1^2}{2g}$$

$$V_2^2 = \frac{2(P_1 - P_2)}{\rho} + V_1^2$$

$$= \frac{2(P_1 - P_2)}{\rho} + (\beta^2 V_2)^2$$

$$V_2^2(1 - \beta^4) = \frac{2(P_1 - P_2)}{\rho}$$

$$V_2^2 = \frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}$$

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$

$Q_1 = Q_2$

$A_1 V_1 = A_2 V_2$

$V_1 = \frac{A_2}{A_1} V_2$ $\qquad \beta = \frac{d}{D}$

$= \left(\frac{d}{D}\right)^2 V_2$

$= \beta^2 V_2$

$Q = A_2 V_2$

$Q_{ideal} = C_0 A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$

$P_1 - P_2 = \left(\frac{Q_{ideal}}{A_2 C_0}\right)^2 \frac{\rho(1 - \beta^4)}{2}$

$D = 80mm = 0.08m$ $\qquad V = 1.8 m/s$ $\qquad \rho = 1000 kg/m^3$

$d = 56mm = 0.056m$ $\qquad \mu = 1.002 \times 10^{-3} Ns/m^2$ $\qquad \beta = 0.7$

$Re = 1.43 \times 10^5$

$$P_1 - P_2 = \left(\frac{Q_{ideal}}{A_2 C_0}\right)^2 \left(\frac{\rho(1 - \beta^4)}{2}\right)$$

$$= \frac{1.8\left(\frac{\pi(0.08)^3}{4}\right)}{\frac{\pi(0.056)^2}{4}(0.613)} \left(\frac{1000(1 - 0.7^4)}{2}\right) = 13.6 kPa$$

b. $\Delta p = f(W, D, Q, \rho, \mu)$

using MLT,

$\Delta p \doteq ML^{-1}T^{-2}$, $W \doteq T^{-1}$, $D \doteq L$, $Q \doteq L^3T^{-1}$, $\rho \doteq ML^{-3}$, $\mu \doteq ML^{-1}T^{-1}$

min no. of reference dimension $= 3$ (M, L, T)

select 3 repeating variables as ρ, ω, D & form π terms:

$\pi_1 = \Delta p\,\rho^a \omega^b D^c \doteq ML^{-1}T^{-2} M^a L^{-3a} T^{-b} L^c$

$$\left.\begin{array}{l} 1+a=0 \\ -1-3a+c=0 \\ -2-b=0 \end{array}\right\} a=-1,\ b=-2,\ c=-2$$

$\pi_1 = \dfrac{\Delta p}{\rho \omega^2 D^2}$

$\pi_2 = Q\rho^a \omega^b D^c \doteq L^3T^{-1}M^a L^{-3a} T^{-b} L^c$

$$\left.\begin{array}{l} a=0 \\ 3-3a+c=0 \\ -1-b=0 \end{array}\right\} a=0,\ b=-1,\ c=-3$$

$\pi_2 = \dfrac{Q}{\omega D^3}$

$\pi_3 = \mu\rho^a \omega^b D^c \doteq ML^{-1}T^{-1}M^a L^{-3a} T^{-b} L^c$

$$\left.\begin{array}{l} 1+a=0 \\ -1-3a+c=0 \\ -1-b=0 \end{array}\right\} a=-1,\ b=-1,\ c=-2$$

$\pi_3 = \dfrac{\mu}{\rho \omega D^2}$

$\therefore \pi_1 = \phi(\pi_2, \pi_3)$

$$\frac{\Delta p}{\rho \omega^2 D^2} = \phi\left(\frac{Q}{\omega D^3}, \frac{\mu}{\rho \omega D^2}\right)$$

power coefficient, $C_p = \dfrac{P}{\rho \omega^3 D^5}$

c. prototype model $P = 200W$

500rpm $Q = 75 m^3/s$

52.36rad/s $\mu_p = \mu_m$

0.8m 0.64m $\rho_p = \rho_m$

By dynamic similarity,

$$\left(\frac{\mu}{\rho \omega D^2}\right)_p = \left(\frac{\mu}{\rho \omega D^2}\right)_m$$

$$\omega_m D_m^2 = \omega_p D_p^2$$

$$\omega_m = \frac{52.36(0.8)^2}{0.64^2}$$

$$= 81.81 \text{ rad/s}$$

$$n_m = 81.81 \div \frac{2\pi}{60}$$

$$= 781.25 \text{ rpm}$$

3. a. $D = 0.1m$ $V_A = V_B$

$Q = 0.01 m^3/s$ $V = 1.27 m/s$

$Re_A = \frac{800(V_A)(0.1)}{0.1}$ $Re_B = \frac{7(V_B)(0.1)}{0.00001}$

$= 800 V_A$ $= 70\ 000 V_B$

$= 1018.59$ (laminar) $= 88900$ (turbulent) , $n = 7$

$0.01 = \pi\left(\frac{0.1}{2}\right)^2 V_{CA}/2$ $0.01 = 2\pi\left(\frac{0.1}{2}\right)^2 V_{CB}\left(\frac{7^2}{(7+1)(14+1)}\right)$

$V_{CA} = 2.55 m/s$ $V_{CB} = 1.56 m/s$

$$\frac{V_{CB}}{V_{CA}} = \frac{1.56}{2.55} = \frac{52}{85}$$

b. Q6: $Q_A = 0.02\ m^3/s$ $\quad V_1 = 1.02\ m/s$

$Q_C = 0.4\ m^3/s$ $\quad V_3 = \frac{0.509}{D_3^2}\ m/s$

$V_2 = 1.59\ m/s$

$$\cancel{\frac{P_A}{\rho g}} + \cancel{\frac{V_A^2}{2g}} + Z_A - h_L = \cancel{\frac{P_C}{\rho g}} + \cancel{\frac{V_C^2}{2g}} + \cancel{Z_C}$$

$$h_L = Z_A$$

$$h_{L_1} + h_{L_3} = Z_A$$

$$0.02\left(\frac{1000}{0.5}\right)\left(\frac{1.02^2}{2(9.81)}\right) + 0.02\left(\frac{500}{D_3}\right)\left(\frac{\left(\frac{0.509}{D_3^2}\right)^2}{2(9.81)}\right) = 40$$

$$D_3 = 0.322m$$

$$\frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B - h_L = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + Z_C$$

$$Z_B = h_L$$

$$h_{L_2} + h_{L_3} = Z_B$$

$$0.02\left(\frac{L_2}{0.4}\right)\left(\frac{1.59^2}{2(9.81)}\right) + 0.02\left(\frac{500}{0.322}\right)\left(\frac{\left(\frac{0.509}{0.322}\right)^2}{2(9.81)}\right) = 45$$

$$L_2 = 6370.78m$$

$$h_{L_1} = 0.02\left(\frac{1000}{0.5}\right)\left(\frac{1.02^2}{2(9.81)}\right)$$
$$= 2.12m$$

$$h_{L_2} = 0.02\left(\frac{6370.78}{0.4}\right)\left(\frac{1.59^2}{2(9.81)}\right)$$
$$= 41.04m$$

$$h_{L_3} = 0.02\left(\frac{500}{0.322}\right)\left(\frac{\left(\frac{0.509^2}{0.322}\right)^2}{2(9.81)}\right)$$
$$= 3.96m$$

Largest friction loss = Pipe 2.

4. i. $H_p = NPSH_R$ $\qquad V_1 = V_2 > V_3 = 8.96\,m/s$

$100 - 400Q^2 = 3 + 100Q^2$

$500Q^2 = 97$

$Q = 0.44\,m^3/s$

ii. $n > 2000\,rpm$

$\omega = 209.44\,rad/s$

$$N_s = \frac{\omega\sqrt{Q}}{(gH_p)^{3/4}} = \frac{209.44\sqrt{0.44}}{[(9.81)(100-400(0.44)^2)]^{3/4}}$$

$= 2.42$ (mixed flow pump)

iii. $\frac{\cancel{P_B}}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L - h_p = \frac{P_S}{\rho g} + \frac{V_S^2}{2g} + \cancel{Z_S}$ $\qquad NPSH_R = 3 + 100(0.44)^2$

$= 22.36m$

$\frac{P_S - P_V}{\rho g} + \frac{V_S^2}{2g} = \frac{V_B^2}{2g} - h_p + h_L$

$NPSH_A > NPSH_R$ (no cavitation)

$NPSH_A = \frac{8.96^2}{2(9.81)} - 22.56 + 98.2$

$= 79.73m$

iv. $Q = 0.18\,m^3/s$ $\qquad V_1 = V_2 = V_3 = 3.67\,m/s$ $\qquad h_L = 0.02\left(\frac{10}{0.25}\right)\left(\frac{3.67^2}{2(9.81)}\right)$

$\frac{P_A}{\rho g} + \cancel{\frac{V_A^2}{\rho g}} + Z_A - h_L = \frac{P_S}{\rho g} + \frac{V_S^2}{2g} + \cancel{Z_S}$ $\qquad = 0.549m$

$\frac{P_S - P_V}{\rho g} + \frac{V_S^2}{2g} = \frac{P_A - P_V}{\rho g} + Z_A - h_L$

$NPSH_A = \frac{20\times10^3 - 2340}{1000(9.81)} + 2 - 0.549$ $\qquad NPSH_R = 3 + 100(0.18)^2$

$= 3.25m$ $\qquad = 6.24m$

$NPSH_R > NPSH_A$ (cavitation occurs)

v. Cavitation in part (iii) will take longer, to occur while cavitation in part (iv) has already occured.