NANYANG TECHNOLOGICAL UNIVERSITY

AE3002 Flight Mechanics

No

Semester 1 2008 - 2009.

1. (a) With full thrust

$P_a = T \cdot V_\infty = 900\, V_\infty$

Draw P_a VS V_∞ on figure 1 the intersection point with the power required is the max cruise speed point.

$\therefore V_{cruise} = 140\ m/s$

With half thrust

$P_a = \frac{1}{2} T V_\infty = 450\, V_\infty$

Draw P_a VS V_∞ on Figure 1.

$V_{cruise} = 86\, m/s$

(b) $R/C = \frac{T-D}{W} \cdot V_\infty = \frac{P_a - P_r}{W}$

$(R/C)_{max} = \frac{(P_a - P_r)_{max}}{W}$

From Figure 1. $(P_a - P_r)_{max} = (75000 - 25000) = 50000\ (W)$

at $V = 80\ (m/s)$

$\therefore (R/C)_{max} = \frac{50000}{4280} = 11.68\ (m/s)$

(c) From Figure 1.

$V_{stall} = 30\ m/s.$

$\therefore W = L = \frac{1}{2} \rho_\infty V_{stall}^2\, S \cdot C_{L\,max} \Rightarrow C_{L\,max} = 2.04$

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(d) At maximum cruise speed

$V_{c\_max} = 140\ m/s \qquad T_{max} = 900\ N$

$$\left.\begin{array}{l} L = W = \frac{1}{2}\rho_\infty V_{c\_max}^2 S C_L \Rightarrow C_L = 0.094 \\ T = D = \frac{1}{2}\rho_\infty V_{c\_max}^2 S C_D \Rightarrow C_D = 0.0197 \end{array}\right\} \Rightarrow 0.0197 = C_{D_0} + k\, 0.094^2 \quad \cdots\ ①$$

With half thrust

$V_c = 86\ m/s \qquad T = 450\ N$

$$\left.\begin{array}{l} L = W = \frac{1}{2}\rho_\infty V_c^2 S C_L \Rightarrow C_L = 0.249 \\ T = D = \frac{1}{2}\rho_\infty V_c^2 S C_D \Rightarrow C_D = 0.026 \end{array}\right\} \Rightarrow 0.026 = C_{D_0} + k\, 0.249^2 \quad \cdots\ ⑤$$

From ① and ⑤

$C_{D_0} = 0.019$

$k = 0.12$

(e) At $h = 3000\ m$, $\rho = 0.91\ kg/m^3$

For jet engine driven aircraft,

$$R = \frac{2}{C_t}\sqrt{\frac{2}{\rho_\infty S}} \cdot \frac{C_L^{\frac{1}{2}}}{C_D}\left(W_0^{\frac{1}{2}} - W_1^{\frac{1}{2}}\right)$$

For R_{max}, $\frac{C_L^{\frac{1}{2}}}{C_D}$ is max and $W_0^{\frac{1}{2}} - W_1^{\frac{1}{2}}$ is max

$$\left(\frac{C_L^{\frac{1}{2}}}{C_D}\right)_{max} = \frac{3}{4}\left(\frac{1}{3kC_{D_0}^3}\right)^{\frac{1}{4}} = \cancel{7.02}\ 18.9$$

$$\left(W_0^{\frac{1}{2}} - W_1^{\frac{1}{2}}\right)_{max} = \left[W_0^{\frac{1}{2}} - \left(W_0 - W_{fuel\_max}\right)^{2}\right] = 4280^{\frac{1}{2}} - \left(4280 - W_{fuel}\right)^{\frac{1}{2}}$$

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$$R_{max} = 1300000 = \frac{2}{0.00036} \times \sqrt{\frac{2}{0.91 \times 3.8}} \times \frac{18.9}{7.02} \times \left[4280^{\frac{1}{2}} - (4280 - W_{fuel\_max})^{\frac{1}{2}} \right]$$

$$\therefore \quad W_{fuel\_max} = 1865.1 \,(N)$$

$$E = \frac{1}{c_t} \cdot \left(\frac{C_L}{C_D}\right) \cdot \ln \frac{W_0}{W_1}$$

For E_{max},

$$\left(\frac{C_L}{C_D}\right)_{max} = \sqrt{\frac{1}{4 C_{D0} K}} = 10.47$$

$$\left(\frac{W_0}{W_1}\right)_{max} = \frac{4280}{4280 - 1865.1} = 1.77$$

$$\therefore \quad E_{max} = \frac{1}{0.00036} \times 10.47 \times \ln 1.77 = 16644.3 \,(s) = 4.6 \,(h)$$

2 (a) $dp = -\rho g \, dz$ ------- ①

At the same altitude

$$P = C \, T^{\frac{\gamma}{\gamma-1}} \quad C \text{ is a constant} \quad \text{------ ②}$$

$$\therefore \, dp = C \cdot \frac{\gamma}{\gamma-1} T^{\frac{1}{\gamma-1}} \cdot dT \quad \text{------ ③}$$

$$\rho = \frac{P}{RT} = \frac{C \, T^{\frac{\gamma}{\gamma-1}}}{RT} = \frac{C}{R} T^{\frac{1}{\gamma-1}} \quad \text{------ ④}$$

From ① ② ③ ④

$$C \cdot \frac{\gamma}{\gamma-1} T^{\frac{1}{\gamma-1}} dT = -\frac{C}{R} T^{\frac{1}{\gamma-1}} \cdot g \, dz$$

$$dz = -\frac{R \cdot \frac{\gamma}{\gamma-1}}{g} dT$$

$$\therefore \; z = \frac{R \frac{\gamma}{\gamma-1}}{g} (T_0 - T) = \frac{287 \times \frac{1.4}{0.4}}{9.81} (288.16 - 271) = 1757 \,(m)$$

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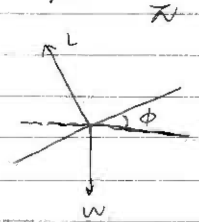
(b) The turning rate

$$w = \frac{\pi}{t} = \frac{3.14}{15} = 0.21 \text{ rad/s}$$

turning radius

$$R = \frac{V \cdot t}{\pi} = \frac{120 \times 15}{3.14} = 573.2 \text{ (m)}$$

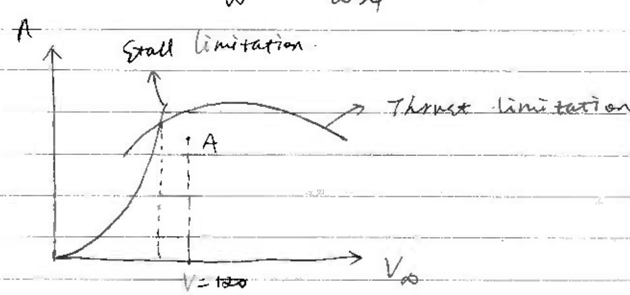


$$\left.\begin{aligned} L \cdot \sin\phi &= \frac{m V^2}{R} = \frac{W V^2}{g R} \\ L \cos\phi &= W \end{aligned}\right\} \Rightarrow \tan\phi = \frac{V^2}{gR} = 2.56 \Rightarrow$$

$$\phi = 68.67^\circ$$

$\therefore$ ~~$\tan\phi =$~~

$$\therefore n = \frac{L}{W} = \frac{1}{\cos\phi} = 2.75$$



$$n_{stall} = \frac{\frac{1}{2} \rho_\infty U_\infty^2 S C_{L\,max}}{W} \qquad \rho_\infty = 1.058 \text{ kg/m}^3$$

at $U_\infty = 120$ m/s $\quad n_{stall} = 3.57 > n = 2.75$

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For thrust limitation at $V = 120\ m/s$

$$n_{max} = \left[\frac{\frac{1}{2}\rho_\infty V_\infty^2}{k\left(\frac{W}{S}\right)} \left(\left(\frac{T}{W}\right)_{max} - \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{D0}}{\left(\frac{W}{S}\right)} \right) \right]^{\frac{1}{2}}$$

$$= 3.14 > n = 2.75$$

$\therefore$ this turn ~~does not~~ is within the aircraft capability.

(c) For this turn

$$L = \frac{W}{\cos\phi} = \frac{1}{2}\rho_\infty U_\infty^2 S C_L$$

$$\Rightarrow C_L = \frac{2W}{\cos\phi\ \rho_\infty U_\infty^2 S} = 1.15$$

$$C_L = \frac{5}{\frac{1}{3.14}\times 180} (\alpha + 2) = 1.15$$

$$\therefore \alpha = 11.2°$$

$$C_D = C_{D0} + kC_L^2 = 0.018 + 0.08 \times 1.15^2 = 0.1238$$

$$\therefore \frac{C_D}{C_L} \cancel{=} \frac{C_L}{C_D} = \frac{1.15}{0.1238} = 9.29$$

(d) the maximum load factor at stall limit is at the intersection point between stall limit and thrust limit.

$$n_{stall} = \frac{\frac{1}{2}\rho_\infty U_\infty^2 S C_{Lmax}}{W} = 2.48\times10^{-4}\ U_\infty^2 \quad \cdots\cdots ①$$

due to thrust limit

$$n_{max} = \left[\frac{\frac{1}{2}\rho_\infty V_\infty^2}{k\left(\frac{W}{S}\right)} \left(\left(\frac{T}{W}\right)_{max} - \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{D0}}{\left(\frac{W}{S}\right)} \right) \right]^{\frac{1}{2}}$$

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$$n_{max} = \left[7.76\times10^{-4} U_\infty^2 - 6.15\times U_\infty^4 \times 10^{-9} \right]^{\frac{1}{2}} \quad \cdots\cdots \ (2)$$

$(1) = (2)$

$$\therefore\ 2.48\times10^{-4} U_\infty^2 = \left[7.76\times10^{-4} U_\infty^2 - 6.15\times10^{-9} U_\infty^4\right]^{\frac{1}{2}}$$

$$U_\infty = 107.1\ (m/s)$$

$$n_{stall\_max} = 2.48\times10^{-4} U_\infty^2 = 2.84$$

(e) $$n_{w max} = \sqrt{\frac{T/W}{\sqrt{KC_{D_0}}} - 1} = \sqrt{\frac{\frac{30000}{80000}}{\sqrt{0.08\times0.018}} - 1} = 2.98$$

$$\cos\phi = \frac{1}{n_{w max}} \Rightarrow \phi = 70.4^\circ$$

3 (a) (i) $$C_m = C_{m0} + \frac{dC_m}{dC_L}\cdot C_L + C_{m\delta e}\cdot\delta e$$

At $\delta e = 0$. $C_{m0} = 0.08$. $\frac{dC_m}{dC_L} = -0.15$

$$\therefore\ C_m = 0.08 - 0.15\,C_L$$

when the aircraft is at trim state,

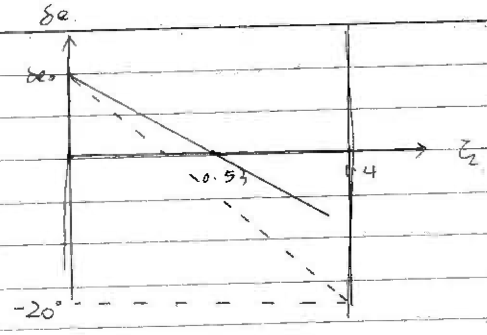
$$C_m = 0$$

$$\therefore\ C_{L\,trim} = \frac{0.08}{0.15} = 0.53$$

(ii) $$\frac{dC_m}{dC_L} = \frac{X_{cg}}{\bar{c}} - \frac{X_{NP}}{\bar{c}} = -0.15$$

$$\therefore\ \frac{X_{NP}}{\bar{C}} = 0.45$$

(iii)



$$\frac{dC_m}{dC_L} = -0.15 = \frac{\frac{dC_m}{d\delta e}}{\frac{dC_L}{d\delta e}} = \frac{-1.146}{\frac{dC_L}{d\delta e}} \Rightarrow$$

$$\frac{dC_L}{d\delta e} = 7.64 = \frac{0.53}{\delta e_0} \Rightarrow \delta e_0 = \cancel{14.4^\circ}\ 3.98^\circ$$

For most forward cg position the $\frac{dC_L}{d\delta e}$ is

$$\frac{dC_L}{d\delta e} = \frac{1.4}{\frac{(3.98+20)}{180} \times 3.14} = 3.35$$

$$\therefore \frac{dC_m}{dC_L} = \frac{\frac{dC_m}{d\delta e}}{\frac{dC_L}{d\delta e}} = \frac{-1.146}{3.35} = -0.34$$

$$\frac{dC_m}{dC_L} = \frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}} = -0.34$$

$$\frac{x_{cg}}{\bar{c}} = \frac{x_{NP}}{\bar{c}} - 0.34 = 0.11$$

(b)

(i) $W = L = \frac{1}{2} \rho_\infty V_{stall}^2 S C_{L max}$

$$V_{stall} = \sqrt{\frac{2W}{S \rho_\infty C_{L max}}} = 47.6 \text{ (m/s)}$$

$$V_{unstick} = 1.2 V_{stall} = 57 \text{ (m/s)}$$

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$C_{n\beta_{-v}}$ due to the vertical fin is

$$N = \frac{1}{2}\rho_\infty (V_{unstick}^2 + V_{side}^2) \cdot S_v \cdot C_{L_v} \cdot \eta \, l$$

$$\beta = \frac{V_{side}}{V_{unstick}} = \frac{8}{57} = 8^\circ$$

$$C_{L_v} = \beta \cdot \frac{dC_{L_v}}{d\beta} = 0.16$$

$$\eta = 0.9 \quad l = 15$$

$\Rightarrow$

$$\cancel{N =} \; C_N = \frac{N}{\frac{1}{2}\rho_\infty (V_{unstick}^2 + V_{side}^2) \cdot S_w \cdot b}$$

$$= \eta \frac{S_v \cdot l}{S_w \cdot b} \cdot C_{L_v}$$

$$C_{n\beta_{-v}} = \frac{dC_N}{d\beta} = \eta \frac{S_v l}{S_w b} \frac{dC_{L_v}}{d\beta} = 0.9 \times \frac{30 \times 15}{70 \times 25} \times 0.08$$

$$= 0.0185 \ /deg$$

$\therefore C_{n\beta}$ for the whole aircraft is

$$C_{n\beta} = 1.2\, C_{n\beta_{-v}} = 0.022 \ /deg$$

rudder control power

$$= \frac{dC_{L_v}}{d\delta_r} = \frac{dC_{L_v}}{d\alpha} \frac{d\alpha}{d\delta_r} = 0.08 \times 0.4 = 0.032$$

Yaw moment balance

$$\Sigma N = \frac{1}{2}\rho_\infty (V_{unstick}^2 + V_{side}^2) \cdot S_w \cdot b \cdot C_{n\beta} \; \cancel{\frac{1}{2}\rho_\infty (V_{unstick}^2 + V_{side}^2}$$

$$- \frac{1}{2}\rho_\infty (V_{unstick}^2 + V_{side}^2) \cdot S_{v0} \cdot l \cdot \frac{dC_{L_v}}{d\delta_r} \cdot \delta_r = 0 \quad \cdots\cdots \; ①$$

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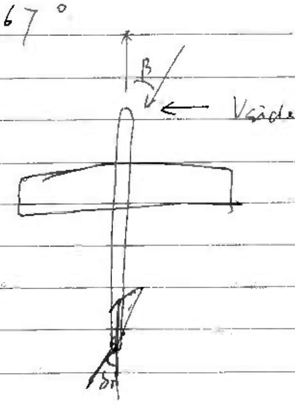
From ①.

$\delta_r = 2.67^\circ$



the rudder should deflection positively $\delta_r = 2.67^\circ$

~~to~~ balance the yaw moment.

4 (a)

(i) For phugoid

$\Delta w = 0$

$M_u = 0$

$$\left.\begin{aligned} \frac{d\Delta u}{dt} - X_u \Delta u + g\Delta\theta &= 0 \\ -Z_u \Delta u - u_0 \frac{d\theta}{dt} &= 0 \end{aligned}\right\} \Rightarrow$$

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \\ -\frac{Z_u}{u_0} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\theta \end{bmatrix}$$

(ii) $A = \begin{bmatrix} X_u & -g \\ -\frac{Z_u}{u_0} & 0 \end{bmatrix}$

$I\lambda - A = \begin{bmatrix} \lambda - X_u & -g \\ -\frac{Z_u}{u_0} & \lambda \end{bmatrix}$

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$$\det(I\lambda - A) = (\lambda - X_u)\lambda - g\frac{Z_u}{u_0} = 0$$

$$\lambda^2 - X_u\lambda - g\frac{Z_u}{u_0} = 0$$

(iii) $$X_u = -\frac{2g}{u_0(\frac{L}{D})} = -\frac{2 \times 9.81}{100 \times 8} = -0.0245$$

$$Z_u = -\frac{2g}{u_0} = -\frac{2 \times 9.81}{100} = -0.1962$$

$$\left.\begin{aligned} \therefore\ 2\zeta\omega_n &= -X_u = 0.0245 \\ \omega_n^2 &= -g\frac{Z_u}{u_0} = -9.81 \times \frac{(-0.1962)}{100} = 0.0192 \end{aligned}\right\} \Rightarrow$$

$$\zeta = 0.088$$

$$T_{\frac{1}{2}} = \frac{0.693}{|\eta|} = 7.848\ (s)$$

(b) (i) $\sum l = I_{xx}\dfrac{d\Delta p}{dt} = \dfrac{dL}{dp}\Delta p + \dfrac{dL}{d\delta a}\Delta\delta a$

$$\Delta\dot{p} = L_p \cdot \Delta p + L_{\delta a}\cdot\Delta\delta a \quad \text{---} \quad (1)$$

$$L_p = \frac{dL}{dp}\Big/ I_{xx} \qquad L_{\delta a} = \frac{dL}{d\delta a}\Big/ I_{xx}$$

The solution of (1) is

$$\Delta p = -\frac{L_{\delta a}}{L_p}\left(1 - e^{L_p t}\right)\cdot\Delta\delta a$$

(ii) From figure 2.

At $t = 0.8$, $\Delta p = 22$

$$\therefore 22 = -\frac{L_{\delta a}}{L_p}\left(1 - e^{L_p\times 0.8}\right)\cdot 5 \quad \text{.....} \quad (3)$$

For steady state solution

$$\Delta p = 30 = -\frac{L_{\delta a}}{L_p}\cdot\Delta\delta a = -\frac{L_{\delta a}}{L_p}\cdot 5 \quad \text{....} \quad (4)$$

From (3) and (4)

$$L_p = -1.65$$
$$L_{\delta a} = 9.91$$

$$L_p = \frac{QSb^2 C_{lp}}{2 I_{xx} U_0} \Rightarrow C_{lp} = \frac{L_p \cdot 2 I_{xx} U_0}{QSb^2} = \frac{-1.65\times 2\times 565000\times 70}{\frac{1}{2}\rho U^2\times 50\times 16.5^2}$$

$$= -3.73$$

$$L_{\delta a} = \frac{QSb\, C_{l\delta a}}{I_{xx}} \Rightarrow C_{l\delta a} = 0.16$$